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ON CERTAIN SEQUENCE SPACES

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ABSTRACT. In this paper define the spaces $l_\infty(\Delta)$, $c(\Delta)$, and $c_0(\Delta)$, where for instance $l_\infty(\Delta)=\{x=(x_k):\sup_k |x_k-x_{k+1}|<\infty\}$, and compute their duals (continuous dual, α-dual, β-dual and γ-dual). We also determine necessary and sufficient conditions for a matrix A to map $l_\infty(\Delta)$ or $c(\Delta)$ into l_∞ or c, and investigate related questions.

1. Introduction

Let l_∞, c and c_0 be the linear spaces of complex bounded, convergent and null sequences $x=(x_k)$, respectively, normed by

$$\|x\|_\infty=\sup_k |x_k|$$

where $k\in\mathbb{N}=\{1,2,\ldots\}$, the positive integers. If $\Delta x=(x_k-x_{k+1})$, we define

(i) $l_\infty(\Delta)=\{x=(x_k):\Delta x\in l_\infty\}$;

(ii) $c(\Delta)=\{x=(x_k):\Delta x\in c\}$;

(iii) $c_0(\Delta)=\{x=(x_k):\Delta x\in c_0\}$.

These spaces are Banach with norm $\|x\|_\Delta=|x_1|+\|\Delta x\|_\infty$. Here we prove that $(l_\infty(\Delta), \|\cdot\|_\Delta)$ is a Banach space.

Let (x^n) be a Cauchy sequence in $l_\infty(\Delta)$, where $x^n=(x_i^n)=(x_1^n, x_2^n, \ldots)\in l_\infty(\Delta)$, for each $n\in\mathbb{N}$. Then

$$\|x^n-x^m\|_\Delta=|x_1^n-x_1^m|+\|\Delta x^n-\Delta x^m\|_\infty\to 0 \qquad (n, m\to\infty).$$

Therefore we obtain $|x_k^n-x_k^m|\to 0$, for $n, m\to\infty$ and each $k\in\mathbb{N}$.

Hence $(x_k^n)=(x_k^1, x_k^2, \ldots)$ is a Cauchy sequence in $\mathbb{C}$ (complex numbers) whence by the completeness of $\mathbb{C}$, it converges to x_k say, i.e., there exists

$$\lim_n x_k^n=x_k, \quad \text{for each} \quad k\in\mathbb{N}.$$

Further, for each $\varepsilon>0$, there exists $N=N(\varepsilon)$, such that for all $n, m\geq N$ and, for all $k\in\mathbb{N}$,

$$|x_1^n-x_1^m|<\varepsilon, \qquad |x_{k+1}^n-x_{k+1}^m-(x_k^n-x_k^m)|<\varepsilon$$

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and

$$\lim_m |x_1^n - x_1^m| = |x_1^n - x_1| \le \varepsilon,$$

$$\lim_m |x_{k+1}^n - x_{k+1}^m - (x_k^n - x_k^m)| = |x_{k+1}^n - x_{k+1} - (x_k^n - x_k)| \le \varepsilon,$$

for all $n \ge N$. Since ε is not dependent on k,

$$\sup_k |x_{k+1}^n - x_{k+1} - (x_k^n - x_k)| \le \varepsilon.$$

Consequently we have $\|x^n - x\|_\Delta \le 2\varepsilon$, for $n \ge N$. Hence we obtain $x^n \to x$ $(n \to \infty)$ in $l_\infty(\Delta)$, where $x = (x_k)$.

Now we must show that $x \in l_\infty(\Delta)$. We have

$$|x_k - x_{k+1}| = |x_k - x_k^N + x_k^N - x_{k+1}^N + x_{k+1}^N - x_{k+1}| \le |x_k^N - x_{k+1}^N| + \|x^N - x\|_\Delta = O(1).$$

This implies $x = (x_k) \in l_\infty(\Delta)$.

Furthermore, since $l_\infty(\Delta)$ is a Banach space with continuous coordinates (that is, $\|x^n - x\|_\Delta \to 0$ implies $|x_k^n - x_k| \to 0$, for each $k \in \mathbb{N}$, as $n \to \infty$), it is a BK-space.

Now we define $s: l_\infty(\Delta) \to l_\infty(\Delta)$, $x \to sx = y = (0, x_2, x_3, \ldots)$. It is clear that s is a bounded linear operator on $l_\infty(\Delta)$ and $\|s\| = 1$. Also

$$s[l_\infty(\Delta)] = sl_\infty(\Delta) = \{x = (x_k) : x \in l_\infty(\Delta), x_1 = 0\} \subset l_\infty(\Delta)$$

is a subspace of $l_\infty(\Delta)$ and

$$\|x\|_\Delta = \|\Delta x\|_\infty \text{ in } sl_\infty(\Delta).$$

On the other hand we can show that

$$\Delta : sl_\infty(\Delta) \to l_\infty,$$
(1.1)
$$x = (x_k) \to y = (y_k) = (x_k - x_{k+1})$$

is a linear homeomorphism. So $sl_\infty(\Delta)$ and l_∞ are equivalent as topological spaces [1]. Δ and Δ^{-1} are norm preserving and $\|\Delta\| = \|\Delta^{-1}\| = 1$.

Let l_∞^* and $[sl_\infty(\Delta)]^*$ denote the continuous duals of l_∞ and $sl_\infty(\Delta)$, respectively. We can prove that

$$T : [sl_\infty(\Delta)]^* \to l_\infty^*$$
$$f_\Delta \to f = f_\Delta \circ \Delta^{-1}$$

is a linear isometry. Thus $[sl_\infty(\Delta)]^*$ is equivalent [1] to l_∞^*. In the same way, we can show that $sc(\Delta)$ and c, $sc_o(\Delta)$ and c_0 are equivalent as topological spaces and $[sc(\Delta)]^* \cong [sc_o(\Delta)]^* \simeq l_1$($l_1$ absolutely convergent series).

2. Dual spaces

In this section we determine the α-, β-, and γ-duals of $sl_\infty(\Delta)$, and obtain some results useful in the characterization of certain matrix maps.

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LEMMA 1. $\sup_K |X_K - X_{K+1}| < \infty$ *if and only if*

$$\text{(i)}\ \sup_k k^{-1}|x_k| < \infty \quad \text{and} \quad \text{(ii)}\ \sup_k |x_k - k(k+1)^{-1}x_{k+1}| < \infty.$$

Proof. Let $\sup_k |x_k - x_{k+1}| < \infty$. Then

$$|x_1 - x_{k+1}| = \left|\sum_{\nu=1}^{k} (x_\nu - x_{\nu+1})\right| \le \sum_{\nu=1}^{k} |x_\nu - x_{\nu+1}| = O(k).$$

This implies $\sup_k k^{-1}|x_k| < \infty$,

$$|x_k - k(k+1)^{-1}x_{k+1}| = |k(k+1)^{-1}(x_k - x_{k+1}) + (k+1)^{-1}x_k| = O(1).$$

Now suppose (i) and (ii) hold. Then

$$|x_k - k(k+1)^{-1}x_{k+1}| \ge k(k+1)^{-1}|x_k - x_{k+1}| - (k+1)^{-1}|x_k|.$$

This implies $\sup_k |x_k - x_{k+1}| < \infty$.

Now let (P_n) be a sequence of positive numbers increasing monotonically to infinity.

LEMMA 2. *If*

$$\sup_n \left|\sum_{\nu=1}^{n} c_\nu\right| < \infty, \quad \textit{then} \quad \sup_n \left(p_n \left|\sum_{k=1}^{\infty} \frac{c_{n+k-1}}{P_{n+k}}\right|\right) < \infty.$$

Proof. Using Abel's partial summation, we get

$$\sum_{k=1}^{\infty} \frac{c_{n+k-1}}{P_{n+k}} = \sum_{k=1}^{\infty} \left(\sum_{\nu=1}^{k} c_{n+\nu-1}\right)\left(\frac{1}{P_{n+k}} - \frac{1}{P_{n+k+1}}\right) \tag{2.1}$$

and

$$P_n \left|\sum_{k=1}^{\infty} \frac{c_{n+k-1}}{P_{n+k}}\right| = O(1).$$

LEMMA 3. *If the series* $\sum_{k=1}^{\infty} c_k$ *is convergent, then*

$$\lim_n \left(P_n \sum_{k=1}^{\infty} \frac{c_{n+k-1}}{P_{n+k}}\right) = 0.$$

Proof. Since $|\sum_{\nu=1}^{k} c_{n+\nu-1}| = |\sum_{\nu=n}^{n+k-1} c_\nu| = o(1)$, for every $k \in \mathbb{N}$. Using (2.1) we get

$$P_n \left|\sum_{k=1}^{\infty} \frac{c_{n+k-1}}{P_{n+k}}\right| = o(1).$$

COROLLARIES. *Let* (P_n) *be as above*

(1) *If* $\sup_n |\sum_{\nu=1}^{n} P_\nu a_\nu| < \infty$, *then* $\sup_n |P_n \sum_{k=n+1}^{\infty} a_k| < \infty$.

Proof. We put $P_{k+1}a_{k+1}$ instead of c_k in Lemma 2. We get

$$P_n \sum_{k=1}^{\infty} \frac{c_{n+k-1}}{P_{n+k}} = P_n \sum_{k=n+1}^{\infty} a_k = O(1).$$

(2) *If* $\sum_{k=1}^{\infty} P_k a_k$ *is convergent, then* $\lim_n P_n \sum_{k=n+1}^{\infty} a_k = 0$.

Proof. We put $P_{k+1}a_{k+1}$ instead of c_k in Lemma 3.

(3) $\sum_{k=1}^{\infty} ka_k$ *is convergent if and only if* $\sum_{k=1}^{\infty} R_k$ *is convergent with* $nR_n = o(1)$, *where* $R_n = \sum_{k=n+1}^{\infty} a_k$.

Proof. Use Abel's summation formula and put $P_n = n$ in Corollary (2), we get

$$\sum_{k=1}^{n} ka_{k+1} = \sum_{k=1}^{n} R_k - nR_{n+1}.$$

DEFINITION 2.1. If X is a sequence space we define [2]:

(i) $X^\alpha = \{a = (a_k) : \sum_{k=1}^{\infty} |a_k x_k| < \infty$, for each $x \in X\}$;

(ii) $X^\beta = \{a = (a_k) : \sum_{k=1}^{\infty} a_k x_k$ is convergent, for each $x \in X\}$;

(iii) $X^\gamma = \{a = (a_k) : \sup_n |\sum_{k=1}^{n} a_k x_k| < \infty$, for each $x \in X\}$.

X^α, X^β, and X^γ are called the α- (or Köthe-Toeplitz), β- (or generalized Köthe-Toeplitz), and γ-dual spaces of X, respectively. We can show that $X^\alpha \subset X^\beta \subset X^\gamma$. If $X \subset Y$ then $Y^+ \subset X^+$, for $+ = \alpha$, β, or γ.

THEOREM 2.1.

(1) $(SL_\infty(\Delta))^\alpha = \{a = (a_k) : \sum_{k=1}^{\infty} k\, |a_k| < \infty\} = D_1$,

(2) $(sl_\infty(\Delta))^\beta = \{a = (a_k) : \sum_{k=1}^{\infty} ka_k$ *is convergent*, $\sum_{k=1}^{\infty} |R_k| < \infty\} = D_2$,

(3) $(sl_\infty(\Delta))^\gamma = \{a = (a_k) : \sup_n |\sum_{k=1}^{n} ka_k| < \infty, \sum_{k=1}^{\infty} |R_k| < \infty\} = D_3$,

where

$$R_k = \sum_{\nu=k+1}^{\infty} a_\nu.$$

Proof. (1) If $a \in D_1$ then $\sum_{k=1}^{\infty} |a_k x_k| = \sum_{k=1}^{\infty} k\, |a_k|\, (|x_k|/k) < \infty$ (Lemma 1) for each $x \in sl_\infty(\Delta)$. This implies $a \in (sl_\infty(\Delta))^\alpha$. If $a \in (sl_\infty(\Delta))^\alpha$, then $\sum_{k=1}^{\infty} |a_k x_k| < \infty$, for each $x \in sl_\infty(\Delta)$. So we take

$$x_k = \begin{cases} 0, & k = 1 \\ k, & k \geq 2 \end{cases}$$

then

$$|a_1| + \sum_{k=1}^{\infty} |a_k x_k| = \sum_{k=1}^{\infty} k\, |a_k| < \infty.$$

(2) Suppose that $a \in D_2$. If $x \in sl_\infty(\Delta)$, then there exists one and only one $y = (y_k) \in l_\infty$, such that ((1.1))

$$x_k = -\sum_{\nu=1}^{k} y_{\nu-1}, \qquad y_0 = 0.$$

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Then

$$\sum_{k=1}^{n} a_k x_k = -\sum_{k=1}^{n} a_k\Big(\sum_{\nu=1}^{k} y_{\nu-1}\Big) = -\sum_{k=1}^{n-1} R_k y_k + R_n \sum_{k=1}^{n-1} y_k. \tag{2.2}$$

Since $\sum_{k=1}^{\infty} R_k y_k$ is absolutely convergent and $R_n \sum_{k=1}^{n-1} y_k \to 0 \quad (n \to \infty)$ (Corollary (3)) the series $\sum_{k=1}^{\infty} a_k x_k$ is convergent for each $x \in sl_\infty(\Delta)$; this yields $a \in (sl_\infty(\Delta))^\beta$.

If $a \in (sl_\infty(\Delta))^\beta$, then $\sum_{k=1}^{\infty} a_k x_k$ is convergent, for each $x \in sl_\infty(\Delta)$. We take

$$x_k = \begin{cases} 0, & k=1 \\ k, & k>1 \end{cases}$$

thus $\sum_{k=1}^{\infty} k a_k$ is convergent. This implies $nR_n = o(1)$ (Corollary (3)). If we use (2.2) we get

$$\sum_{k=1}^{\infty} a_k x_k = -\sum_{k=1}^{\infty} R_k y_k$$

convergent, for all $y \in l_\infty$. So we have $\sum_{k=1}^{\infty} |R_k| < \infty$ and $a \in D_2$.

(3) The proof of (3) is the same as above.

It is easy to check that $(sl_\infty(\Delta))^{\dagger} = (sc(\Delta))^{\dagger}$, for $\dagger = \alpha$, β, or γ.

Now let E be one of the sequence spaces $l_\infty(\Delta)$, $c(\Delta)$ or $c_0(\Delta)$. We can show that

$$(SE)^{\dagger} = E^{\dagger}, \quad \text{for } \dagger = \alpha,\ \beta, \text{ or } \gamma.$$

3. Matrix maps.

Let each of E and F denote one of the sequence spaces l_∞ and c, and let E' and F' denote one of the sequence spaces $l_\infty(\Delta)$ and $c(\Delta)$. Let (X, Y) denote the set of all infinite matrices A which map X into Y.

THEOREM 3.1. *$A \in (E', F)$ if and only if*

(i) $(\alpha_{nl}) \in F$, and $(A_n(k)) \in F$,

(ii) $R \in (E, F)$,

where

$$A_n(k) = \sum_{k=1}^{\infty} k a_{nk} \quad \text{and} \quad R = (r_{nk}) = \Big(\sum_{\nu=k+1}^{\infty} a_{n\nu}\Big).$$

Proof. If $A \in (E', F)$ then the series $A_n(x) = \sum_{k=1}^{\infty} a_{nk} x_k$ are convergent and $(A_n(x)) \in F$, for each $n \in \mathbb{N}$ and all $x \in E'$. The necessity of (i) is trivial. We just put $x = (1, 0, 0, \ldots)$ and $x = (k)$. Furthermore we have $\sum_{k=1}^{\infty} |r_{nk}| < \infty$ for each $n \in \mathbb{N}$ (Theorem 2.1). Now let $x \in sE' \subset E'$.

$$A_n(m, x) = \sum_{k=1}^{m} a_{nk} x_k = -\sum_{k=1}^{m-1} r_{nk} y_k + r_{nm} \sum_{k=1}^{m-1} y_k \tag{3.1}$$

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where $y \in E$, $y_0 = 0$ such that

$$x_k = -\sum_{r=1}^{k} y_{r-1}.$$

Hence

$$\lim_m A_n(m, x) = A_n(x) = -\sum_{k=1}^{\infty} r_{nk} y_k,$$

for each $n \in \mathbb{N}$ (Corollary 3). Thus, we get $(R_n(y)) = (\sum_{k=1}^{\infty} r_{nk} y_k) \in F$, for each $y \in E$. This yields $R \in (E, F)$.

Now suppose (i) and (ii) hold.

If $x \in E'$,

$$x_k = \begin{cases} x_1, & k = 1 \\ x'_k, & k > 1 \end{cases}, \quad \text{where } x' = (x'_k) \in sE'.$$

We write again (3.1) and get

$$A_n(x) = a_{n1}x_1 - \sum_{k=1}^{\infty} r_{nk} y_k.$$

This implies the $A_n(x)$ exist for each $x \in E'$ and $A \in (E', F)$.

THEOREM 3.2. *$A \in (E, F')$ if and only if*

(i) $\sum_{k=1}^{\infty} |a_{nk}| < \infty$, *for each* $n \in \mathbb{N}$,

(ii) $B \in (E, F)$,

where

$$B = (b_{nk}) = (a_{nk} - a_{n+1,k}).$$

The proof is trivial.

THEOREM 3.3. (1) $l_\infty \cap c(\Delta) = l_\infty \cap c_0(\Delta) = M_0$

$$= \{x = (x_k) : x \in l_\infty, \lim_k (x_k - x_{k+1}) = 0\},$$

(2) $(M_0, l_\infty) = (l_\infty, l_\infty)$,

(3) $A \in (l_\infty, M_0)$ *if and only if*

(i) $\sup_n \sum_{k=1}^{\infty} |a_{nk}| < \infty$,

(ii) $\lim_n \sum_{k=1}^{\infty} |a_{nk} - a_{n+1,k}| = 0$.

Proof. (1) If $x \in l_\infty \cap c(\Delta)$, $x \in l_\infty$ and $x_k - x_{k+1} \to l$ $(k \to \infty)$, $x_k - x_{k+1} = l + \varepsilon_k$ $(\varepsilon_k \to 0,\ k \to \infty)$. This implies

$$x_{n+1} = x_1 - nl - \sum_{k=1}^{n} \varepsilon_k \quad \text{and} \quad l = \frac{x_1}{n} - \frac{x_{n+1}}{n} - \frac{1}{n}\sum_{k=1}^{n} \varepsilon_k.$$

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This yields $l=0$ and $x\in l_\infty \cap c_0(\Delta)$.

(2) The proof is trivial.

(3) The necessity of (i) follows follows from the fact that it is necessary for $A\in(l_\infty, l_\infty)$. Other parts are trivial

If we write

$$m_0=\{x=(x_k): x\in M_0 \text{ and } x_k\in\mathbb{R}\} \quad (\mathbb{R} \text{ real numbers})$$

we have that [3], [4], for any positive integer p and integer $0\le n_1<n_2<\cdots<n_p$,

$$\inf_{n_1,n_2,\ldots,n_p} \sup_k \frac{1}{p}\sum_{i=1}^{p} x_{k+n_i}=\limsup_k x_k \text{ on } m_0.$$

THEOREM 3.4. If $A\in(c, c)$ *and* $\sup_n \sum_{k=1}^\infty |r_{nk}|<\infty$, *then* $A\in(M_0, c)$, *where* $r_{nk}=\sum_{\nu=k+1}^\infty a_{n\nu}$.

Proof.

$$\sum_{k=1}^{m} a_{nk}x_k=x_1\sum_{k=1}^{m} a_{nk}-\sum_{k=1}^{m-1} r_{nk}(x_k-x_{k+1})+(x_1-x_m)r_{nm},$$

$$\lim_m \sum_{k=1}^{m} a_{nk}x_k=A_n(x)=x_1\sum_{k=1}^{\infty} a_{nk}-\sum_{k=1}^{\infty} r_{nk}(x_k-x_{k+1}), \quad \text{for each} \quad x\in M_0.$$

Since $\sup_n \sum_{k=1}^\infty |r_{nk}|<\infty$ and $\lim_n r_{nk}$ exists, these imply $R=(r_{nk})\in(c_0, c)$ and $\lim_n \sum_{k=1}^\infty r_{nk}(x_k-x_{k+1})$ exists. Thus we get $A\in(M_0, c)$.

Now let E and F be sequence spaces. We define

$$E(F)=\{x: x_k=y_kz_k,\ y\in E,\ z\in F\}$$

by pointwise multiplication. Let M_s denote the space of all x for which $\sup_n |\sum_{k=1}^n x_k|<\infty$. It is easy to check that $M_s=\{y: y_k=x_k-x_{k-1},\ x\in l_\infty,\ x_0=0\}$.

A matrix is called strongly regular if it is regular and

$$\lim_n \sum_{k=1}^{\infty} |a_{nk}-a_{nk+1}|=0.$$

It is known [3] that, if A is regular, then for all $x\in l_\infty$,

$$\lim_n A_n(y)=\lim_n \sum_{k=1}^{\infty} a_{nk}y_k=0$$

if and only if A is strongly regular, where $y_k=x_k-x_{k+1}$.

Now we consider the set $M_s(M_0)$. It is clear that $M_s\subset M_s(M_0)$ and this inclusion is strict.

THEOREM 3.5. *Let A be a regular matrix.* $\lim_n A_n(y)=0$ *for all* $y\in M_s(M_0)$ *if and only if A is strongly regular.*

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Proof. If $\lim_n A_n(y)=0$ for all $y \in M_s(M_0)$ then $A \in (M_s(M_0), c_0)$, this implies $A \in (M_s, c_0)$. Hence we get $\lim_n \sum_{k=1}^{\infty} |a_{nk} - a_{n,k+1}| = 0$ [5].

Now let A be strongly regular. If $y \in M_s(M_0)$, then $y_k = \alpha_k x_k$, $\alpha \in M_s$ and $x \in M_0$.

$$\sum_{k=1}^{m} a_{nk} y_k = \sum_{k=1}^{m} a_{nk}(x_k - x_{k+1})\gamma_k + \sum_{k=1}^{m} (a_{nk} - a_{n,k+1})\gamma_k x_{k+1} + a_{n,m+1}\gamma_m x_{m+1}$$

where

$$\gamma_n = \sum_{k=1}^{n} \alpha_k.$$

Hence

$$\lim_m \sum_{k=1}^{m} a_{nk} y_k = A_n(y) = \sum_{k=1}^{\infty} a_{nk}(x_k - x_{k+1})\gamma_k + \sum_{k=1}^{\infty} (a_{nk} - a_{n,k+1})\gamma_k x_{k+1},$$

for each $n \in \mathbb{N}$. Thus we get $\lim_n A_n(y) = 0$.

THEOREM 3.6. $A \in (M_s(M_0), c)$ *if and only if*

(i) $\sup_n \sum_{k=1}^{\infty} |a_{nk}| < \infty$,

(ii) $\lim_n a_{nk}$ *exists, for each* $k \in \mathbb{N}$,

(iii) $\sum_{k=1}^{\infty} |a_{nk} - a_{n,k+1}|$ *converges uniformly in* n.

The proof is easy.

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