The book under review concentrates more on the general theory of J-symmetric operator algebras and representations of *-algebras on Krein spaces, which are the indefinite analogues of C^*-algebras and their representations on Hilbert space. Many of the results here have been known for more than twenty years, but they have been scattered in the literature. The authors present these results in a coherent fashion, making reference to the modern theory of C^*-algebras where relevant, and they include some new results in representation theory. The theory of representations on general Krein spaces is quite weak, but it is much stronger in the case of those Krein spaces, known as Π_κ-spaces or Pontryagin spaces, where H_- or H_+ is finite-dimensional.

The final one-third of the book is devoted to relatively recent applications to the study of *-derivations of C^*-algebras. Let $\mathcal{A}$ be a C^*-algebra of operators on a Hilbert space H, and δ be a closed, densely-defined (unbounded) *-derivation from $\mathcal{A}$ into $\mathcal{B}(H)$. One seeks closed operators S on H (with good properties) which implement δ in the sense that

$$\delta(A)x = SAx - ASx \quad (x \in \mathrm{Dom}(S), A \in \mathrm{Dom}(\delta)).$$

It is easy to construct a J-symmetric representation of the Banach *-algebra $\mathrm{Dom}(\delta)$ on a Krein space H' in such a way that closed operators S implementing δ correspond to closed subspaces of H' invariant under the representation, and moreover Hilbert space properties of S correspond neatly to Krein space properties of the subspace. However, H' is not a Π_κ-space, so the results obtained from representation theory are quite weak. One can obtain stronger results if one already has a skew-symmetric operator S which implements δ and one of the deficiency indices of S is finite, a situation which is not unusual in practice. Then there is a representation of $\mathrm{Dom}(\delta)$ on a Π_κ-space, and the stronger representation theory can be used to show that δ can also be implemented by extensions of S with better Hilbert space properties. Moreover, these constructions enable index theory of semigroups of *-endomorphisms of $\mathcal{B}(H)$, as developed by Powers and Arveson, to be derived from the representation theory of Krein spaces.

Since little material on representations on Krein spaces or implementation of derivations has previously appeared in books, this book will be of great interest to those specialising in Krein spaces, and also those specialising in C^*-algebras with interests in derivations. A novice who wishes to master Krein spaces may prefer to start with a more leisurely introduction to the basic geometry and operator theory, and a reader who has not already studied the basic theory of C^*-algebras may not appreciate Chapter 6 and some earlier sections. The background material summarised in the first section varies enormously in level of difficulty, but much of the book can be read without knowledge of the more demanding topics. It is the nature of this subject that the proofs require verification of many routine properties, and the authors have used fine judgment to steer a course between too much detail and too little detail. The text has been written carefully, with only a few typographical errors. Indeed, this book succeeds in describing a subject which was well worth writing about.

C. J. K. BATTY

OLLERENSHAW, K. and BRÉE, D. *Most-perfect pandiagonal magic squares: their construction and enumeration* (Institute of Mathematics and its Applications, Southend-on-Sea, 1998), xiii + 152 pp., 0 905091 06 X, £19.50.

This is a marvellous book. It is very readable, carefully planned, and contains fascinating material.

Magic squares have a long history, and well-known constructions exist for squares of all sizes greater than two. However, the enumeration problem of counting the different magic squares of a particular type has so far remained unresolved. The authors consider a special type of magic square which they call "most-perfect pandiagonal", and succeed in constructing all such squares and in counting them by means of a one-one correspondence between such squares and what they call "reversible squares". Let me explain these ideas.

In a magic square based on the numbers $0, 1, \ldots, n^2-1$, the rows, columns and main diagonals all have the same sum, namely $\frac{1}{2}n(n^2-1)$. If *all* (broken) diagonals have this sum, the square is *pandiagonal* (or Nasik, or diabolic). It is easy to show that no pandiagonal square of order $n \equiv 2 \pmod 4$ can exist, so the authors concentrate on $n \equiv 0 \pmod 4$. They further call the square *most-perfect* if each 2×2 subsquare has entries summing to $2(n^2-1)$, and pairs of integers a distance $\frac{1}{2}n$ apart along a (broken) diagonal add up to n^2-1. Here for example is a most-perfect pandiagonal square of order 4:

0	14	3	13
7	9	4	10
12	2	15	1
11	5	8	6

The integers 3, 10, 12, 5 lie along a broken diagonal and add up to 30; those integers two apart along the diagonal add up to $n^2-1=15$, viz. $3+12=10+5=15$.

It is shown that these most-perfect pandiagonal squares are in a one-one correspondence with a family of square arrays called reversible squares. An $n \times n$ reversible square contains the numbers $0, 1, \ldots, n^2-1$; each row and each column have reverse symmetry (i.e. added to its reverse it gives a constant row or column) and, in any rectangular array within the square, the sums of the integers in opposite corners are equal. These reversible squares are nicely constructed, and, importantly, can be enumerated. The enumeration is a *tour de force* of manipulation of binomial coefficients, and the numbers of such squares turn out to be astronomical.

The major part of this work was carried out over several years by Dame Kathleen Ollerenshaw, who completed the work for $n = 2^r p^s$ ($r \geq 2$, p prime). The work was then extended to all $n \equiv 0 \pmod 4$ by David Brée. It should perhaps be pointed out that the formula obtained in the final section can in fact be obtained more neatly by using the inclusion-exclusion principle.

The exposition is very clear throughout. Often the reader is reminded of a definition; key points from one chapter are often repeated in a later one. After a look at some earlier constructions, such as those of McClintock (1897) and of Rosser and Walker (1939) on whose ideas the present authors build, we are led very carefully through the construction and enumeration of the new squares. This detailed exposition covers 90 pages; the only major flaw I found was in the definition of McClintock squares, where the definition does not fit with the example, but this is peripheral to the main stream of the argument. There then follow a glossary of 8 pages, an appendix on binomial identities (5 pages), detailed proofs of some of the properties of most-perfect and reversible squares which had been stated without proof (10 pages), a historical appendix of 16 pages on earlier constructions of pandiagonal magic squares, detailed algorithms for constructing reversible squares and hence magic squares, and finally a complete list of principal (canonical) reversible squares of order 12.

One passage in the Preface sticks out, demanding a response: "When a serious effort was made to provide proof, the argument became increasingly complex and involved numerous diversions that had their own interest. The full proof thus became unsuitable for publication as an article in a recognised journal and better suited to publication as a book." What a sad condemnation of what we tend now to expect from learned journals – articles written tersely for a select few, with little thought for the general reader's understanding or enjoyment.

Read this book, and enjoy it!

I. ANDERSON