CORRESPONDENCE.

ON THE FORMULÆ FOR COMPLETE ANNUITIES.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—I have often been struck by the difficulty of the proofs given in Part II, Chap. XI, of the *Text-Book* for the correction to be made to complete the value of an annuity. This difficulty seems to me to arise from our confining ourselves too much to the idea of a year as the unit of time, forgetting that, so far as life assurance is concerned, a year is an arbitrary interval taken for the sake of convenience in computation.

There are two principal formulæ proved in Chap. XI, from which all the others given may be at once derived. The first, on the assumption of a uniform distribution of deaths, is proved in sec. (5), and is:

$$\mathring{a}_x = a_x + \bar{A}_x \frac{i-\delta}{i\delta}.$$

In this equation change the unit of time from one year to $\frac{1}{m}$ of a year, then $\mathring{a}_x$ becomes $m\mathring{a}_x^{(m)}$, because 1 is payable at the end of each interval, a_x becomes $ma_x^{(m)}$, $\bar{A}_x$ is unchanged, i becomes $\frac{j_{(m)}}{m}$, and δ, which is equal to $\log(1+i)$, becomes $\log\left(1+\frac{j_{(m)}}{m}\right)=\frac{1}{m}\log(1+i)=\frac{\delta}{m}$, and the equation becomes

$$m\mathring{a}_x^{(m)} = ma_x^{(m)} + \bar{A}_x \frac{\frac{j_{(m)}}{m} - \frac{\delta}{m}}{\frac{j_m\delta}{m^2}}$$

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or
$$\mathring{a}_x^{(m)} = a_x^{(m)} + \bar{A}_x \frac{j_{(m)} - \delta}{j_m \delta},$$

which is equation (7) in section (8).

The second formula, which is independent of any assumption as to the distribution of deaths, is proved in sec. (11), and is:

$$\mathring{a}_x = a_x + \frac{\bar{M}_x}{2D_x} - \frac{\bar{M}_x - \bar{M}_{x+1}}{12D_x}.$$

Changing the unit of time in this equation, we get

$$m\mathring{a}_x^{(m)} = ma_x^{(m)} + \frac{\bar{M}_x}{2D_x} - \frac{\bar{M}_x - \bar{M}_{x+\frac{1}{m}}}{12D_x}.$$

Take as a first approximation to the value of $\bar{M}_{x+\frac{1}{m}}$,

$$\bar{M}_x + \frac{1}{m}(\bar{M}_{x+1} - \bar{M}_x);$$

then, dividing the above equation by m, we get at once

$$\mathring{a}_x^{(m)} = a_x^{(m)} + \frac{1}{2m}\bar{A}_x - \frac{1}{12m^2}\bar{A}^1_{x\overline{1}|},$$

which is equation (9) of section (12).

A more accurate approximation to the value of $\bar{M}_{x+\frac{1}{m}}$, however, is

$$\bar{M}_x + \frac{1}{m}\frac{d\bar{M}_x}{dx} = \bar{M}_x - \frac{\mu_x D_x}{m}.$$

$$\therefore \quad \frac{\bar{M}_x - \bar{M}_{x+\frac{1}{m}}}{12D_x} = \frac{\mu_x}{12m},$$

and we get
$$\mathring{a}_x^{(m)} = a_x^{(m)} + \frac{1}{2m}\bar{A}_x - \frac{\mu_x}{12m^2},$$

which is equation (11) of section (18).

In this way the connection between the various formulæ is shown. I may mention, in passing, that in the proof of the second principal equation, on page 189, the value of the correction for the first tth part of the year is only $\frac{1}{t}(\bar{M} - \bar{M}_{\frac{1}{t}})$ in the limit when t is infinite, and there should be a foot-note to that effect, as there is on page 186.

I am, Sir,
Your obedient servant,
H. N. SHEPPARD.

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