

JOHNSTONE, P. T., *Notes on logic and set theory* (Cambridge University Press, Cambridge, 1987) 120 pp., cloth: 0521 33502 7, £20; paper: 0 521 33692 9, £6.95.

This is a remarkable book. In only 107 pages of text, it includes all of the results of modern mathematical logic and set theory which are of basic significance. There are three chapters on logic, culminating in the Löwenheim–Skolem theorem and the completeness and compactness theorems for first-order predicate logic. There is a chapter on recursive functions, including discussion of register machines, Church's Thesis, recursive unsolvability and formal definability. The chapter on formal set-theory covers the Zermelo–Fraenkel axioms. This runs naturally on into a chapter on ordinals and well-orderings, and the book ends with chapters on the Axiom of Choice, cardinal arithmetic, and finally consistency, incompleteness and independence results.

In respect of its scope and length, the book is reminiscent of R. C. Lyndon, *Notes on logic* (Van Nostrand, 1964) and J. N. Crossley *et al.*, *What is mathematical logic?* (Oxford University Press, 1972). But it is a very different book from these. Although the author makes a point of rejecting a title such as "Logic for the working mathematician", in fact the content would be aptly described in that way. Furthermore, its presentation relies on a considerable mathematical sophistication on the part of the reader, so that anyone who is not a working mathematician would find it difficult. The treatment is definitely logic-as-mathematics, the flavour being that of D. W. Barnes and J. M. Mack, *An algebraic introduction to mathematical logic* (Springer-Verlag, 1975).

In his preface, the author says, "I have not been afraid to shoot a few of the sacred cows of logical tradition for the sake of rendering the exposition more smoothly compatible with mathematical practice". While this attitude is to be commended in general terms, it has two consequences which may detract from the book's usefulness. First, some of the details of the formalisms used (for example, restrictions on the *modus ponens* rule and on the formation of quantified formulas) serve to complicate matters and to shift the formalism further from the intuition. In this respect he seems to be prepared to forgo reliance on logical intuition, preferring to rely on mathematical intuition. Second, it is important when teaching this subject to keep lecture material and supporting textbook material compatible. Because of its brevity, this book would be difficult to learn from unless used in conjunction with a lecture course, but then it would be useful only if the lecturer were to adopt its particular conventions. Of course such a comment can be made about any mathematical text, but in this case the author seems to have invited it.

The style of presentation is concise and, for the most part, very clear. But there are points where clarity seems to have been sacrificed for the sake of brevity. Proofs which are short but technically demanding have been preferred to easier longer ones. In some places good motivation precedes the introduction of new ideas—in others the motivation seems inadequate. But the book does convey well the author's enthusiasm for the subject matter by a style which, despite the brevity, is admirably informal. The occasional "notes for worriers" are a nice feature.

Each chapter ends with a substantial set of (not insubstantial) exercises. These seem generally well-chosen, and include references to other branches of mathematics. There are no solutions provided.

There is no doubt that this is a valuable addition to the stock of available textbooks in this area. Besides being useful to those who have to learn, it will also stimulate thought in those who teach.

A. G. HAMILTON