CORRESPONDENCE

Readers of the note 'Circular Seating Arrangements' (*J.I.A.* **108**, 405) may be interested to note that the problem can be solved by an inductive method.

The method involves seating n couples, adding an $(n+1)$th couple to the right of the first man, and then changing the $(n+1)$th wife with another wife.

For fixed men, if P_{n+1} represents the number of positions with no matches for $(n+1)$ couples, these can be produced from P_n type positions, Q_n type positions (one specified man's wife on a specified side, no other matches) if the $(n+1)$th couple split the pair either by separation or exchange, or from R_n type positions (one specified man's wife on a specified side, and exactly one other match in any position with the wife on either side) if the $(n+1)$th couple split one pair by separation and the other by exchange.

Consideration of the number of ways that will produce a P_{n+1} type position leads to the formula:

$$P_{n+1} = (n-2)P_n + (3n-4)\,Q_n + R_n \tag{1}$$

Consideration of the number of ways that a Q_{n+1} or R_{n+1} type position will arise *before* the $(n+1)$th wife changes, leads to the formulae:

$$Q_{n+1} = P_n + Q_n \tag{2}$$

and

$$R_{n+1} = R_n + (2n-1)\,Q_n. \tag{3}$$

By eliminating Q's and R's the following formula is then produced:

$$P_n = (n-3)P_{n-1} + \{(3n-7)P_{n-2} + (5n-12)P_{n-3} + (7n-19)P_{n-4} + \ldots + (n^2-n-19)P_4 + (n^2-n-12)P_3\} + 1. \tag{4}$$

Since $P_1 = P_2 = 0$ and $P_3 = 1$ by inspection we have

$$P_4 = P_3 + 1 = 2$$
$$P_5 = 2P_4 + 8P_3 + 1 = 13$$
$$P_6 = 3P_5 + 11P_4 + 18P_3 + 1 = 80$$
etc.

which reproduces the values in column (3) of Table 1 of the note.

For $n \geqslant 6$, formula (4) can be transformed to:

$$P_n = nP_{n-1} + 2P_{n-2} - (n-4)P_{n-3} - P_{n-4}.$$

I would be happy to provide a more detailed version of this solution to anyone who may be interested.

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