BULL. AUSTRAL. MATH. SOC. 05B05
VOL. 59 (1999) [175–176]

Trades and defining sets: theoretical and computational results

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Given a particular combinatorial structure, there may be many distinct objects having this structure. When investigating these, two natural questions to ask are:

▷ Given two objects, where and how do they differ?

▷ How much of an object is necessary to uniquely identify it?

These two questions are obviously related, with the first leading to the concept of a *trade*, and the second to that of a *defining set*. In this thesis we study trades and defining sets, in the context of t-(v, k, λ) *designs*. In our enquiries, we make use of both theoretical and computational techniques, with theoretical results leading to more efficient programmes and computational results suggesting lines of theoretical enquiry.

We investigate the *spectrum* of trades, and prove an extant conjecture regarding this [**1**]. Our results also suggest a more general version of this conjecture [**2**]. A t-(v, k, λ) design where $\lambda = 1$ is called a *Steiner* design, and the related trades are called Steiner trades. In the case $t = 2$, we establish the spectrum of Steiner trades for each value of k, except for a finite number of values in each case [**3, 4**].

The connection between trades and defining sets is used to obtain some new theoretical results on defining sets of designs, and is exploited throughout the thesis [**5, 6**]. We also consider the collections of all trades and all defining sets in a design.

A *simple* design is one which is a set, as opposed to a multiset. We present an algorithm to enumerate all the trades in simple designs. For non-simple designs we introduce the concept of a *discriminating set*, and present an algorithm to enumerate these. Output from these algorithms was used to investigate the trades and defining sets of a number of designs. An error in a published result was discovered, and some new results were obtained [**7**].

Given part of a design, its *completions* are all those designs that contain it. An existing algorithm to complete partial designs is examined, and a heuristic yielding a much improved algorithm for Steiner designs is discussed. This completion routine was used to investigate a number of designs, and new information on the size and distribution of their defining sets was obtained [**8**].

Received 5th November, 1998
Thesis submitted to The University of Queensland, February 1998. Degree approved, August 1998. Supervisors: Dr. G. Havas and Professor A.P. Street.

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