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Varieties and section closed classes of groups

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A class of groups is said to be *section closed* if it is closed with respect to taking factor groups and subgroups. The central concept of this thesis is the relationship between a locally finite variety of groups and the section closed classes of groups which generate it. Bryant and Kovács [1] defined the *skeleton* $S(\underline{V})$ *of a variety* $\underline{V}$ of groups to be the intersection of the section closed classes of groups which generate $\underline{V}$. Varieties generated by their skeletons are of particular interest, for they are generated by a unique minimal section closed class of groups. Since a locally finite variety $\underline{V}$ is generated by its finite monolithic groups, $S(\underline{V})$ is always contained in the section closure $Q\,sM(\underline{V})$ of the class $M(\underline{V})$ of finite monolithic groups in $\underline{V}$. For a positive integer m, let $\underline{A}_m$ denote the variety of all abelian groups of exponent dividing m. Bryant and Kovács showed that, for $m > 1$ and a locally finite variety $\underline{V}$, the skeleton $S(\underline{A}_m\underline{V})$ of the product variety $\underline{A}_m\underline{V}$ is equal to $Q\,sM(\underline{A}_m\underline{V})$. A *variety of* *A-groups* is defined to be a locally finite variety whose nilpotent groups are abelian. Earlier Cossey [2] showed that the skeleton $S(\underline{U})$ of a variety $\underline{U}$ of A-groups is $Q\,sM(\underline{U})$.

These results are generalized here by showing that for a nontrivial variety $\underline{U}$ of A-groups and a locally finite variety $\underline{V}$, the skeleton $S(\underline{UV})$ is $Q\,sM(\underline{UV})$. As a corollary necessary and sufficient conditions are given for $S(\underline{UV})$ to consist of all finite groups in $\underline{UV}$. Examples are given to show that a product of two nontrivial locally finite varieties need not be generated by its skeleton, or, even if it is, the skeleton need

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not contain all the critical groups in the variety.

In proving the main theorem above, we are led to consider a variety which, for some prime p, is generated by finite monolithic groups each of which is an extension of a nontrivial abelian p-group by a p'-group. In the appendix [3], knowledge of the skeleton of such a variety is applied to show that if $\underline{\underline{U}}$ is a variety of A-groups, $\underline{\underline{V}}$ a locally finite variety whose lattice of subvarieties is distributive and the exponents of $\underline{\underline{U}}$ and $\underline{\underline{V}}$ are coprime, then the lattice of subvarieties of $\underline{\underline{UV}}$ is distributive.

The consideration of such extensions of abelian p-groups by p'-groups leads to an interesting question. Can such a group ever be contained in a locally finite variety $\underline{\underline{V}}$ without being contained also in $S(\underline{\underline{V}})$? Bryant and Kovács have shown the answer to be negative, provided the p-group is cyclic or elementary abelian. If the p-group is not cyclic and has sufficiently large exponent then, it is shown here, there is a locally finite variety $\underline{\underline{V}}$ containing the group, but the group is not in $S(\underline{\underline{V}})$. In particular if the p-group has exponent at least p^3 and the p'-group is cyclic this is true. Further special cases of the problem are considered.

References

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[2] John Cossey, "Critical groups and the lattice of varieties", *Proc. Amer. Math. Soc.* 20 (1969), 217-221.

[3] L.F. Harris, "A product variety of groups with distributive lattice", *Proc. Amer. Math. Soc.* (to appear).