

The Caucus-Race: Teaching Cyclical Majorities

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Cyclical majorities (or, if you prefer, the paradox of voting) have been intensively studied since the publication of Kenneth Arrow's and Duncan Black's penetrating studies.¹ Yet, despite its undoubted importance, the subject is very dry: students tend to nod sleepily during lectures. Perhaps, therefore, an excursion into fairyland may be both helpful and informative.²

Lovers of Lewis Carroll's *Alice's Adventures In Wonderland* will recall that, after Alice's and the creatures' damp departure from "the pool of tears," the Dodo averred "that the best way to get us dry would be a Caucus-race."

"'What is a Caucus-race?' said Alice: not that she much wanted to know, but the Dodo had paused as if it thought that *somebody* ought to speak, and no one else seemed inclined to say anything.

"'Why,' said the Dodo, 'the best way to explain it is to do it.' (And, as you might like to try the thing yourself some winter day, I will tell you how the Dodo managed it.)

"'First it marked out a race-course, in a sort of circle, ('the exact shape doesn't matter,' it said) and then all the party were placed along the course, here and there. There was no 'One, two, three, and away!' but they began running when they liked, and left off when they liked, so that it was not easy to know when the race was over. However, when they had been running half-an-hour or so, and were quite dry again, the Dodo suddenly called out 'The race is over!' and they all crowded round it, panting, and asking 'But who has won?'

"'This question the Dodo could not answer without a great deal of thought, and it stood for a long time with one finger pressed upon its forehead (the position in which you usually see Shakespeare, in the pictures of him), while the rest waited in silence. At last the Dodo said '*Everybody* has won, and *all* must have prizes.'"³

Carroll's divertissements, as Warren Weaver and others have often noted, frequently cloak puzzling or profound questions.⁴ The caucus-race is no exception: without mentioning ordinary ties, what sort of races can sensibly be said to be won by everybody? Carroll aficionados have been perplexed to explain it. Martin Gardner's *Annotated Alice*, for example, suggests that "Carroll may have intended his caucus-race to symbolize the fact that committee members generally do a lot of running around in circles, getting nowhere, and with everybody wanting a political plum."⁵

Despite the Dodo's practical instruction, we may well echo Alice: What is a caucus-race? The details of the caucus-race occupy about 1/3 of Chapter 3 in *Alice's Adventures in Wonderland*. Since an early manuscript version of that chapter contains no reference to a caucus-race and since the published version is entitled "A Caucus-Race and a Long Tale," the caucus-race is obviously the result of careful, perhaps considered and reflective, writing. A surmise that the caucus-race is a serious subject is reinforced by noting that it is a "dry" subject, not something that Alice "much wanted to know."

The key to understanding the caucus-race is to be found in the writings of Charles L. Dodgson rather than those of Lewis Carroll. Between 1861 and 1885, Dodgson repeatedly wrote on the subject of voting and elections. These writings included a brilliant and original pamphlet published in 1876, *A Method of Taking Votes on More than Two Issues*. That pamphlet, reprinted and discussed by Duncan Black, contains a perceptive analysis of "cyclical majorities."⁶

What is a cyclical majority? The best way to explain it is to give an example. Consider a trio of electors X, Y, Z voting on three competing candidates (or motions) A, B, C. Assume X most prefers A and least prefers C, Y most prefers B and least prefers A, and Z most prefers C and least prefers B. This situation is summarized in the following matrix:

1 Kenneth J. Arrow, *Social Choice and Individual Values* (2nd ed.; New York: John Wiley & Sons, Inc., 1963). Duncan Black, *The Theory of Committees and Elections* (Cambridge: Cambridge University Press, 1958).

2 This illustration of the pedagogical uses of Lewis Carroll's *Alice* books suggests many other applications in political science teaching. It is taken from my lectures on systematic political analysis at Oakland University, which were developed during my tenure on a National Science Foundation Science Faculty Fellowship for the study of pure and applied mathematics, 1968-69.

3 Roger Lancelyn Green (ed.), *The Works of Lewis Carroll* (London: Paul Hamlyn, 1965).

4 Warren Weaver, "Lewis Carroll," *Scientific American* (April, 1956); reprinted in *Lives in Science: a Scientific American Book* (New York: Simon and Schuster, Inc.; 1957), pp. 241-256.

5 Martin Gardner, *The Annotated Alice* (New York: Clarkson N. Potter, Inc., 1960).

6 Black, *The Theory of Committees and Elections*, *op. cit.*

		Preferences		
		1st	2nd	3rd
Voters	X	A	B	C
	Y	B	C	A
	Z	C	A	B

If the candidates are considered two at a time (a common practice with motions and, for Dodgson, a *desideratum* with candidates) and if the electors vote according to their preference schedules, then no candidate receives a majority over each and every other candidate. A cycle is formed with no natural beginning or end: A beats B, B beats C, C beats A, etc. The term "cyclical majority," coined by Dodgson, vividly describes this type of situation where no candidate receives a majority, in pairwise voting, against each and every other candidate.

The similarities between Dodgson's cyclical majority and Carroll's caucus-race are striking. With both, the course was a cycle, "a sort of circle," beginning where one liked and leaving off where one liked. And "It was not easy to know when the race was over." Who won? "This question the Dodo could not answer without a great deal of thought," and Dodgson also confessed its difficulty. The Dodo answered, "Everybody has won." Dodgson was inclined to say, "Nobody has won." The difference between these answers, of course, is more apparent than real: When everybody is the most charming person in a room, then nobody is the most charming person in the room.

Given these similarities, the plausibility of the thesis that the caucus-race is a cycle, a literary example of a cyclical majority, depends upon the presumption that Lewis Carroll was familiar with the scholarly works of Charles L. Dodgson. Fortunately for the purposes of this note, Carroll's intimate familiarity with Dodgson's writings is well known.