SOME ASPECTS OF CONVECTION IN METEOROLOGY

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ABSTRACT

Various aspects of convection in meteorology which may have some relevance for astrophysics are discussed. In particular the role of convection in determining the gross thermal structure of the atmosphere, the treatment of convective turbulence in the boundary layer, and the larger scale organization of convection are dealt with.

1. INTRODUCTION

As noted by Prof. Biermann during this conference, a number of seminal approaches to convection in astrophysics such as convective adjustment and mixing-length theory originated in meteorology. The purpose of this paper is to briefly review the current status of such notions in meteorology as well as to report some relatively recent approaches to meteorological convection which may prove useful in astrophysics.

Since most of what I discussed at the conference has appeared in the meteorological literature, I will tend to use this written version of my lecture as a selective annotated directory to this literature rather than a complete version of the lecture. Nothing remotely approximating a complete review of convection in the meteorological literature will be attempted.

In section 2, I will describe the observed thermal structure of earth's atmosphere and explain why simple radiative models with convective adjustment prove inadequate — qualitatively and quantitatively. I will also outline our current understanding of the observed structure — though this understanding is by no means complete.

In section 3, I will introduce a current phenomenological approach to penetrative convection in the atmosphere which may prove a more consistent alternative to convective adjustment in astrophysics. In both sections 2 and 3, allusions will be made to the fact that, in the atmosphere, convection often occurs in relatively narrow plumes, and that such convection is generally associated with local static stability. The point which may be relevant to astrophysics is that convection is not necessarily related to local temperature gradients in a simple way.

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Section 4 deals with a particular observed feature of atmospheric convection : namely, when broad regions (for example the maritime tropics equatorwards of 20-30°) are uniformly unstable (or conditionally unstable) convection does not occur in a uniformly distributed manner. Rather, the convecting system itself appears to be unstable to so-called mesoscale systems (physically akin to internal gravity waves) which in turn organize the convection on scales much larger than the scale of the convective elements. The mechanism for this organization appears to be different when the convection is deep, extending almost from the ground to the tropopause (cumulonimbus convection), and when convection occurs within the middle levels of the troposphere; both are discussed in section 4. The point, however, is that in the earth's atmosphere convection rarely if ever manifests itself in the appearance of a pattern characterized by the scale of the convective elements alone; larger scales of organization and motion almost invariably appear. This may offer some insight into such phenomena as supergranules on the sun.

2. THE LARGE SCALE THERMAL STRUCTURE OF THE EARTH'S ATMOSPHERE

In Figure 1 a somewhat smoothed picture of the height and latitude structure of longitudinally averaged temperature is presented (only heights up to 16 km are shown). In Figure 2 several height profiles of temperature appropriate to various latitudes are shown. From Figure 2 we may note a certain similarity of profiles from various latitudes. In most cases temperature decreases with height above the ground with a gradient of about 6°/km until some level (known as the tropopause) where the temperature gradient goes to zero or becomes positive. The relatively uniform lapse rate $(-\frac{\partial T}{\partial z})$ of the lower atmosphere as well as the observation that radiative equilibrium profiles for the lower atmosphere are statically unstable led early on to the use of a convective adjustment model to explain the thermal structure of the lower atmosphere (Gold, 1909; Emden, 1913; Goody, 1949). Several important difficulties have, however, arisen in connection with such models (some were recognized almost immediately) :

i) Convective adjustment would lead to the adiabatic lapse rate in the lower atmosphere. This is 9.8°/km not 6°/km. It is sometimes suggested that the atmosphere adjusts instead to the moist saturated adiabatic lapse rate. While this lapse rate is on the order of 6°/km near the ground, by 4km the atmosphere, because of its low temperature, saturates with very small amounts of water vapor and the saturated lapse rate differs little from the dry value. Thus, the use of the saturated lapse rate is no solution to this problem — even ignoring the fact that the atmosphere is rarely saturated. It is interesting to note that the application of convective adjustment to the earth's atmosphere leads to errors on the order of 30°K or 10% in absolute temperature.

ii) Convective adjustment in no way can account for the abrupt change in tropopause height near 30° latitude: equatorwards it is near 16 km, polewards it is near 12 km.

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iii) Radiative-convective equilibrium does not account for the fact that in the neighbourhood of the tropopause, a temperature minimum exists at the equator.

iv) Finally and revealingly, close scrutiny of Figures 1 and 2 indicates that the lapse rate of the tropopause is not really uniform,especially at high latitudes.

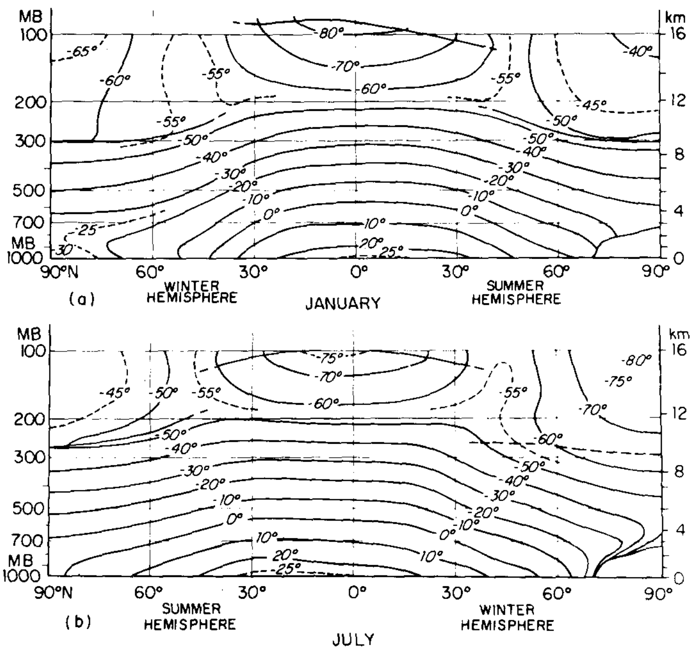

Figure 1. Zonally (longitudinally) averaged temperature as a function of height and latitude. Contours are lines of constant temperature (°C). After Palmen and Newton (1969).

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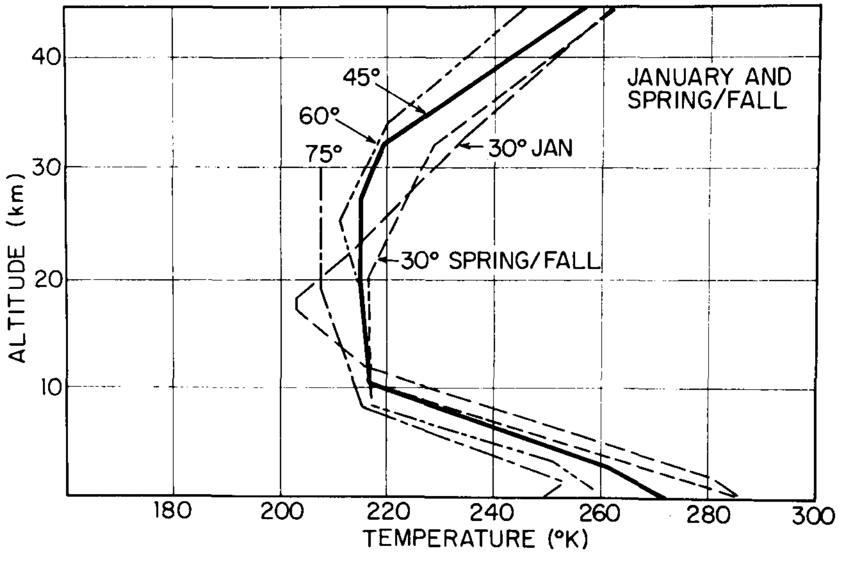

Figure 2. Zonally averaged temperature as a function of height for various latitudes. After U.S. Standard Atmosphere Supplements (1966).

Our current understanding of the atmosphere's structure suggests no uniform explanation for the whole globe. Recent work (Schneider and Lindzen, 1976; Schneider, 1976) shows that within a certain neighbourhood of the equator (extending to about 30° latitude) the atmosphere cannot sustain significant horizontal temperature gradients (in many respects this region is similar to a spherically symmetric atmosphere where rotation is not of great importance). Large scale dynamic effects in this region serve primarily to homogenize (horizontally) the temperature in this region, and as a result the vertical temperature structure of this region is indeed describable in terms of radiative-convective equilibrium. However, because the convection occurs in relatively narrow cumulonimbus towers, it leads to finite stability rather than neutral lapse rates. How this occurs is outlined in Appendix 1. From about 30-70° latitude, horizontal temperature gradients are significant and rotation is of basic importance. It is generally believed that convection in this region is due to baroclinic eddies whose energy is drawn from horizontal temperature gradients. These eddies tend to carry heat upwards, and the rate at which these eddies stabilize the atmosphere is much greater than the rate at which radiation acts to destabilize the atmosphere, so the question of convective adjustment does not arise. The stability achieved in this region is primarily related to the north-south temperature difference, and at the moment there does not appear to be any basic reason why temperature lapse rates at middle latitudes should be the same as they are in the tropics. A discussion of how baroclinic eddies act to establish the lapse rate in middle latitudes may be found in Stone (1972, 1973). The relevance of this process to astrophysics is not at all clear. Finally, the arctic-antarctic ice and snow cover lead to high surface albedos and radiation tends to stabilize rather than destabilize the atmosphere. This, in turn, tends to suppress baroclinic eddies. A comprehensive discussion of terrestrial atmospheric stability based on numerical simulation may be found in Held (1976).

3. PENETRATIVE CONVECTION AND MIXED LAYERS

One may reasonably ask, at this stage, whether convection in the earth's atmosphere ever leads to a neutral lapse rate. The answer is almost certainly yes, but it is not clear that even in these instances, convective adjustment is the correct approach.

We shall, in this section, look at one of the more extensively studied examples of convective mixing: namely the convective mixing of the air near the ground where the convection is forced by solar heating of the surface. A substantial number of phenomenological theories exist for this process and there is still a measure of controversy surrounding them. I will sketch one typical example of such theories due to Tennekes (1973). The geometry of the situation is shown in Figure 3 where profiles of both potential temperature and convective heat flux are presented. At the bottom of the mixed layer there is a thin superadiabatic layer dominated by mechanical turbulence. The nature of this layer is ignored except insofar as it delivers a heat flux $(\overline{\theta w})_o$ to the interior; this heating forces the convective mixing which proceeds over a finite layer of thickness, h, topped by an inversion layer with temperature jump, Δ. The region above this jump is stably stratified with $\frac{d\Theta}{dz} = \gamma$. As heating continues, h increases with time -- whence the name "penetrative convection". The picture thus far is reasonably well observed over land in middle latitudes. As the mixed layer rises into a warmer environment, the cooling of the entrained warmer air must give rise to a negative flux $(\overline{\Theta w})_i$ beneath the inversion. This is mathematically expressed as follows :

$$- (\Theta w)_i = \Delta \frac{dh}{dt} . \qquad (3.1)$$

An equation may be written for the time evolution of Δ, on noting that the penetration of the mixed layer into the stable interior tends to increase Δ, while the heating of the mixed layer tends to decrease Δ :

$$\frac{d\Delta}{dt} = \gamma \frac{dh}{dt} - \left(\frac{\partial\Theta}{\partial t}\right)_{b.l.} . \qquad (3.2)$$

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If we ignore radiative processes, $\Theta_{b.l.}$ satisfies a simple budget :

$$C_p\rho\left(\frac{\partial\Theta}{\partial t}\right)_{b.l.} = -\frac{\partial}{\partial z}\left(C_p\rho\overline{\Theta w}\right), \tag{3.3}$$

or, integrating over the mixed layer, we find

$$\left(\frac{\partial\Theta}{\partial t}\right)_{b.l.} \simeq \frac{(\overline{\Theta w})_o - (\overline{\Theta w})_i}{h} . \tag{3.4}$$

Substitution of (3.4) into (3.2) yields

$$\frac{d\Delta}{dt} = \gamma\frac{dh}{dt} - \frac{(\overline{\Theta w})_o}{h} + \frac{(\overline{\Theta w})_i}{h} , \tag{3.5}$$

and (3.1) together with (3.5) are generally taken as the basic equations for the system. $(\overline{\Theta w})_o$ is given, and (3.1) and (3.5) then form 2 equations in 3 unknowns : Δ, h and $(\overline{\Theta w})_i$. Clearly another relation is needed (and it is at this point that the bulk of the controversy is engendered). Tennekes (1974) first considers the turbulent energy budget near the inversion:

$$\frac{g}{T_o}\ \overline{\Theta w} \simeq \frac{g}{T_o}(\overline{\Theta w})_i \simeq \frac{\partial}{\partial z}\left(\frac{1}{2}q^2 w\right) + \varepsilon , \tag{3.6}$$

where q is the magnitude of turbulent velocity fluctuations and ε is a dissipation rate which is empirically found to be negligible near the inversion. (T_o is a mean temperature.) Thus (3.6) suggests that the kinetic energy generated by buoyancy is consumed in bringing heat down through the inversion. Since buoyancy tends to generate vertical velocity, and buoyancy acts throughout the mixed layer, $\frac{\partial}{\partial z}\left(\frac{1}{2}q^2 w\right)$ ought to scale as follows:

$$\frac{\partial}{\partial z}\left(\frac{1}{2}q^2 w\right) \sim -O\left(\frac{\sigma_w^3}{h}\right), \tag{3.7}$$

where σ_w is the vertical velocity variance, and

$$-(\overline{\Theta w})_i \sim \frac{T_o}{g}\frac{\sigma_w^3}{h} . \tag{3.8}$$

In addition since σw is generated by $(\overline{\Theta w})_o$ we have

$$\sigma_w^2 \sim gh\frac{\Theta_o}{T_o}$$

and

$$\sigma_w^3 \sim \frac{gh}{T_o}(\overline{\Theta w})_o . \tag{3.9}$$

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Combining (3.8) and (3.9) we have

$$-\ (\overline{\Theta w})_i\ = k\ (\overline{\Theta w})_o\ : \qquad (3.10)$$

k is a constant which is empirically found to be about 0.2. Equation (3.10) closes the system described by (3.1) and (3.5). The resulting equations have been used (with moderate success) to describe a variety of convective boundary layers. For the diurnal boundary layer, surface heating during the day causes h and $\Theta_{b.l.}$ to increase; the heat thus deposited is carried away by radiation during the night when $(\overline{\Theta w})_o$ is zero. This incidentally explains how there can be a turbulent heat flux into the atmosphere in the mean even though the mean stability may be positive.

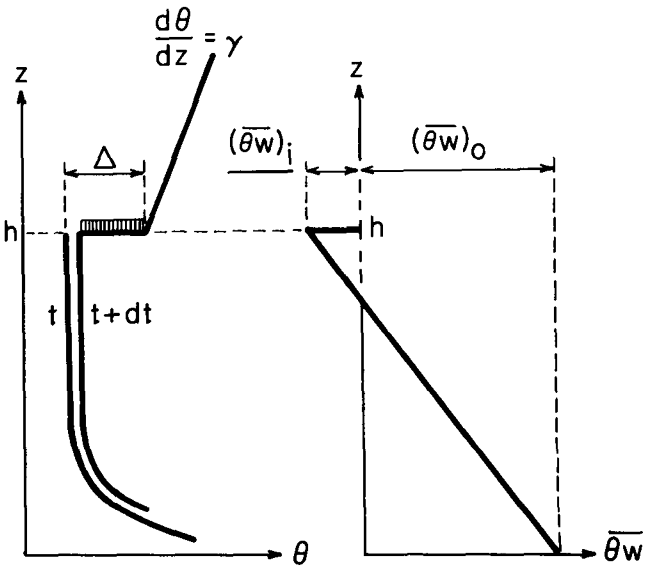

Figure 3. The vertical distributions of potential temperature and turbulent heat flux in and above a convective boundary layer, after Tennekes (1973).

To be sure, the concept of a diurnal boundary layer is hardly applicable in astrophysics. However, the above approach has also successfully accounted for the semi-permanent mixed layer of the tropical maritime atmosphere (Sarachik, 1974). In that particular case there exists a between-cloud subsidence which causes $\frac{dh}{dt}$ in equ. (3.1) and (3.5) to be replaced by $(\frac{dh}{dt} - w)$ and an equilibrium solution exists wherein

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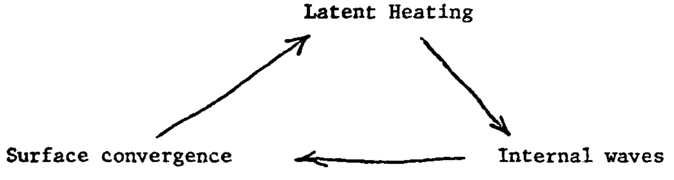
$\frac{dh}{dt} = 0$. More germain to astrophysics would be the inclusion of radiation in the above picture. Equ. (3.1), (3.3), (3.6) and (3.7) would all need modification since radiation would not only alter the gross budgets but would also act to dissipate buoyancy. It is also conceivable that convection, if it were to occur in plumes, would not lead to an adiabatic lapse (well mixed potential temperature) (see Appendix 1). This might affect the validity of (3.8) and (3.9) since the mean stability would inhibit buoyant acceleration. The above, of course, all remains to be done, but it might conceivably form a more satisfactory alternative to convective adjustment. The possibility of convection leading to inversion "discontinuities" etc. might have significant implications as well.

4. MESOSCALE ORGANIZATION OF CONVECTION

We turn now to a last and somewhat different aspect of atmospheric convection. Even when rather broad regions of the atmosphere are relatively uniformly unstable (or more typically conditionally unstable with respect to moist processes), convection (in the form of cumulus clouds) rarely if ever occurs in a uniformly distributed manner. Instead, the convection is almost always organized into systems whose scale is typically 1-2 orders of magnitude larger than the scale of the cumulus clouds themselves. The larger scale (100-400km) is referred to in meteorology as the mesoscale. Cloud clusters and squall systems are examples of mesoscale systems. Mesoscale organization appears to be an intrinsic feature of atmospheric convection. For certain types of atmospheric convection the relation to mesoscale organization seems reasonably clear. In these cases moisture is concentrated near the surface (in the first 2 kilometers of the atmosphere typically) and virtually the entire depth of the troposphere is conditionally unstable. Such situations tend to be characterized by intense cumulonimbus convection. The rainfall in such situations tends to satisfy a simple moisture budget where the rainfall (and hence latent heat release) is proportional to the convergence of moisture (plus evaporation where this is relevant). Moreover, since the moisture tends to be confined to $Z < Z_T$ (where Z_T is typically 2 km), the convergence of moisture tends to be proportional to the vertical velocity at Z_T. Finally the latent heat release is significant for the larger scale motions. In the presence of an internal wave perturbation (which produces convergence) one can imagine an interaction of the sort indicated below:



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If the internal waves produced by latent heating produce more surface convergence (in the proper phase) than is needed to maintain the wave, the system will be unstable. This mechanism is referred to as wave CISK (conditional instability of the second kind), and is described in greater detail in Lindzen (1974). CISK is used to describe any collective instability of cumulonimbus convection and larger scale motions. The concept was introduced by Charney and Eliassen (1964) in connection with hurricane generation. The mathematical problem in the present instance consists simply in the solution of the equation for thermally forced internal gravity waves which takes approximately the following form:

$$\frac{d^2w}{dz^2} + \lambda^2 w = Q(z) \tag{4.1}$$

where all fields are proportional to $e^{ik(x-ct)}$. w is the vertical velocity, Q is proportional to heating, x is a horizontal coordinate, k is a horizontal wavenumber, and c is a horizontal phase speed which may be complex (for unstable solutions). For our purposes

$$\lambda^2 \sim \frac{N^2}{c^2} \tag{4.2}$$

where N is the Brunt-Vaisala frequency. Now it is an easy matter to write the solution for w (satisfying suitable boundary conditions) as functional of Q(z):

$$w = F_c\ [Q] \tag{4.3}$$

where w depends on c (and z) as well as Q. But Q is proportional to $w(Z_T)$, and (4.3) becomes

$$w(Z) = F_c\ [q(Z')w(Z_T)] \tag{4.4}$$

where q(Z) is a specified function. At $Z = Z_T$ (4.4) becomes

$$w(Z_T) = F_c\ [q(Z')w(Z_T)] \tag{4.5}$$

which proves to be possible only for certain values of c--one of which is typically associated with the greatest degree of instability. Current calculations indicate that the imaginary part of c is much smaller than the real part and that for common terrestrial situations Re(c) $\sim$ 15m/s. Since solutions are of the form $e^{ik(x-ct)}$, growth rates are equal to k x Im(c) and one might infer that maximum growth rates are achieved as $k \to \infty$(and as the frequency k Rec $\to \infty$ also). This, however, is inconsistent with the fundamental premise of CISK: namely, that convection is organized by large scale convergence. Clearly such organization cannot be achieved on time scales shorter than characteristic development times for the clouds. For example in the tropics cumulonimbus clouds have a characteristic time scale of about 1 hour, which suggests a maximum frequency, ω, of about $(1\ \text{hour})^{-1}$. Now

$$\omega \sim kc \sim \frac{1}{3600\ \text{sec}} \sim k \times 15\ \text{m/s}.$$

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Hence $k \sim \frac{1}{3600 \times 15 \text{ m}}$

and horizontal wavelength $\sim \frac{2\pi}{k} \sim 2\pi \times 3600 \times 15 \text{ m} \sim 339$ km. (4.6)

In fact, both this wavelength, and the predicted phase speed are characteristic of tropical mesoscale disturbances, implying that the maximum frequency suggested above is, in fact, what is realized. A similar approach has been used by Raymond (1975) to account for the structure and evolution of intense convective storms in the midwestern United States.

The relevance of wave-CISK for astrophysics is questionable since there appears to be no astrophysical counterpart to rainfall. However, it is also observed in the earth's atmosphere that cumulus convection which is restricted to relatively shallow layers within the middle of the troposphere and which is associated with little (and sometimes no) rainfall is also organized into mesoscale patterns. Latent heat does not appear at first sight, to play a major role in forcing these mesoscale systems. In a recent paper, Lindzen and Tung (1976) have shown that the near neutral stability created by mid-level cumulus activity helps trap internal gravity waves in the stable region below the clouds, creating a duct wherein wave modes may exist without significant forcing. The phase speeds of these ducted modes (determined primarily by the thickness of the stable region below the clouds) are in good agreement with observations. Furthermore, observed periods appear to satisfy the relation

$$\tau_{wave} \sim 2\pi \, \tau_{cloud} \qquad (4.7)$$

just as in the case of wave CISK disturbances. Given a duct phase speed, c, and a characteristic cloud time scale τ_{cloud}, the mesoscale wavelength is again

$$\text{wavelength} \sim 2\pi \, c \, \tau_{cloud} \qquad (4.8)$$

The means for interaction between the waves and the cloud field are not entirely clear in this case. However, the period given by (4.7) is still the shortest period on which any interaction could take place. Moreover, the well known degeneracy of such features of convection as its plan form suggests that the organization of convection might be responsive to relatively weak perturbations. Similary, the waves, being ducted, call for only small forcing.

At this point it is worth noting that the earth's atmosphere can sustain a class of free oscillations (Lamb waves) which do not require explicit ducting. These waves are, essentially, horizontally propagating acoustic waves with $c \sim 319$ m/s. By the above arguments we ought to expect organization of convection with wavelengths given by (4.8) based on the speed of sound and τ_{cloud}. There is no clear cut observational evidence available for this suggestion. However, the wavelengths obtained are on the order of several thousand kilometers, and on the earth, regions on this scale with relatively uniform conditional instability are rare. The situation appears somewhat more congenial on the sun where a convective layer exists over the entire star. Identifying the convective elements with granules for which $\tau \sim 5$ minutes and taking c ∿ 10 km/sec one obtains from equ. (4.8) that the dominant wavelength ought to be 40,000 km. Whether it is purely an accident that this is also the scale of supergranules remains to be seen. Less arguably, the above discussion demonstrates rather clearly that the appearance of structures of a given horizontal scale need not imply vertical scales of the same order. Similarly, terrestrial experience suggests that convection rarely involves merely a single horizontal scale.

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APPENDIX 1. HEAT TRANSFER BY THIN PLUMES

The following discussion is based on work by Arakawa and Schubert (1974) concerning cumulonimbus clouds. The present discussion, however, ignores moisture (both for simplicity and because of its irrelevance to astrophysical problems). We shall consider convection which occurs in plumes which occupy a small fraction of the total horizontal area and which despite their small area contribute significantly to the mean vertical mass flow. By "mean" we shall always refer to an average over an area large compared to the cross-sectional area of plumes, but small compared to any large scale flow. Our aim will be to parameterize the effect of plumes on this large scale flow. Means will be indicated by overbars. The approach will be analogous to the use of Reynold's averaging where the eddies will be convective plumes.

We will first partition the mean vertical mass flux into plume and environmental (non plume) contributions:

$$\overline{\rho w} = Mp + \tilde{M}, \qquad (A.1)$$

where ρ = density, Mp = plume mass flux and M = environmental mass flux. For our purposes the following quasi-Boussinesq continuity equation will suffice:

$$\overline{\nabla \cdot (\rho \underset{\sim}{v})} + \frac{\partial}{\partial z}(\overline{\rho w}) = 0 ; \qquad (A.2)$$

$(\nabla \cdot \underset{\sim}{q})$ will here refer to horizontal divergence of $\underset{\sim}{q}$. It will also prove useful to consider an ensemble of plumes where

$$Mp = \sum_i Mi , \qquad (A.3)$$

Each plume may either be entraining mass from its environment in which case

$$E_i = \left(\frac{\partial Mi}{\partial z} + \rho \frac{\partial \sigma i}{\partial t}\right), \quad \frac{\partial Mi}{\partial z} + \frac{\partial \sigma i}{\partial t} > 0 \qquad \text{(A.4a)}$$

or detraining into the environment in which case

$$D_i = -\left(\frac{\partial Mi}{\partial z} + \frac{\partial \sigma i}{\partial t}\right), \quad \frac{\partial Mi}{\partial z} + \frac{\partial \sigma_i}{\partial t} < 0; \qquad \text{(A.4b)}$$

σ_i is the fractional area occupied by the i^{th} plume and the fractional area occupied by all plumes is

$$\sigma_p = \sum_i \sigma_i \, . \qquad \text{(A.5)}$$

Mp satisfies the following budget :

$$E - D = \frac{\partial Mp}{\partial z} + \rho \frac{\partial \sigma p}{\partial t} , \qquad \text{(A.6)}$$

where

$$E = \sum_{\substack{\text{entraining}\\ \text{plumes}}} Ei \, ,$$

$$D = \sum_{\substack{\text{detraining}\\ \text{plumes}}} Di$$

The static energy:

$$s = c_p T + gZ \qquad \text{(A.7)}$$

is conserved during adiabatic processes. The budget for s in the environment is given by

$$\frac{\partial}{\partial t}\left[(1 - \sigma_p)\rho \tilde{s}\right] = - \overline{\nabla \cdot (\rho \underset{\sim}{v} s)} - E\tilde{s} + \sum_{d.p.} DiS_{Di} - \frac{\partial}{\partial z}(\tilde{M} \tilde{s}) + \tilde{Q}_R , \qquad \text{(A.8)}$$

where $\sum_{d.p.}$ refers to a sum over detraining plumes and S_{Di} is the static energy of the i^{th} detraining plume; Q_R represents radiative heating in the environment. Using equ. (A.1), (A.2) and (A.6) we may easily transform (A.8) to the following:

$$(1 - \sigma_p) \rho \frac{\partial \tilde{s}}{\partial t} = \left(- \overline{\nabla \cdot (\rho \underset{\sim}{v} s)} + \tilde{s} \; \overline{\nabla \cdot (\rho \underset{\sim}{v})}\right) +$$

$$+ \sum_{d.p.} D_i (s_{Di} - \tilde{s}) + Mp \frac{\partial \tilde{s}}{\partial z} + \tilde{Q}r \, . \qquad \text{(A.9)}$$

We will now assume the following to be adequate approximations:

$$\overline{\nabla \cdot \rho \underset{\sim}{v}} \sim \nabla \cdot (\overline{\rho \underset{\sim}{v}}) , \qquad \text{(A.10a)}$$

$$\overline{\nabla \cdot \rho \underset{\sim}{v} s} \sim \overline{v} \cdot (\rho \, \overline{\underset{\sim}{v}} \, \overline{s}) \, . \qquad \text{(A.10b)}$$

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Also, for $\sigma_p << 1$, it is readily shown that $(1 - \sigma_p) \sim 1$,

$$\bar{s} \sim \tilde{s},$$

and $$\tilde{Qr} \sim \overline{Qr}\ .$$

Equ. (A.9) then becomes

$$\rho \frac{\partial \bar{s}}{\partial t} + \rho \, \bar{\underset{\sim}{v}} \cdot \nabla \bar{s} + \overline{\rho w} \frac{\partial \bar{s}}{\partial z} = Mp \frac{\partial \bar{s}}{\partial z} + \sum_{d.p.} Di(s_{Di} - \tilde{s}) + \overline{Qr}\ . \qquad (A.11)$$

Let us finally assume each plume detrains at precisely that level where its static energy equals that of the environment (i.e., where it looses buoyancy: this is consistent with the known instability of decelerating jets). Then

$$\sum_{d.p.} Di(S_{Si} - \tilde{s}) = 0$$

and (A.11) becomes

$$\rho \frac{\partial \bar{s}}{\partial t} + \rho \, \bar{\underset{\sim}{v}} \cdot \nabla \bar{s} + \overline{\rho w} \frac{\partial \bar{s}}{\partial z} = Mp \frac{\partial \bar{s}}{\partial z} + \overline{Qr}\ . \qquad (A.12)$$

We see from (A.12) that the primary effect of convective plumes on the environment is to introduce a heating term $Mp \frac{\partial \bar{s}}{\partial z}$. This term is easily interpreted: a portion of $\overline{\rho w}$ (i.e., Mp) which rises in plumes does not give rise to adiabatic cooling in the environment - and appears, therefore, as a heating term. If we ignore $\overline{\rho w}$, Mp must be compensated by equal subsidence which does, in fact, lead to compressional heating in the environment. A crucial point which may be made from (A.12) is that if convective plumes are to supply heat which is then carried off by radiation, $\frac{\partial \bar{s}}{\partial z}$ must be positive!

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