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# An Empirical Estimate of the Labor Response Function for Benefit-Cost Analysis

Donald F. Vitaliano

## Abstract

Since the seminal contribution of Haveman and Krutilla(1968), the subject of the potential drawdown from the pool of unemployed versus diversion of labor from existing employments consequent upon a public investment project has been largely neglected in the BCA literature. The advent of a new BLS series on job vacancies now permits direct estimation of the crucial unemployment-vacancies (U-V) relationship, as compared to the ad hoc sine function using the unemployment rate assumed by Haveman and Krutilla. The probability  $p$  of a worker being drawn from the pool of unemployed is recast as a function of the job vacancy rate (vacancies/labor force) and shows higher values than Haveman and Krutilla at comparable rates of unemployment. At the height of the 2008-09 Great Recession, about half of Stimulus induced jobs were drawn from the pool of unemployed.

**KEYWORDS:** job creation, jobvacancies and unemployment

## Introduction

The ongoing political debate about the effectiveness of government spending in reducing unemployment during a recession is barely reflected in the leading textbooks on benefit-cost analysis (BCA). A survey of Boardman et al. (2006), Zerbe and Bellas (2006), Gramlich (1998), Brent (1996) and Zerbe and Dively (1994), for example, barely finds a mention. The most detailed discussion of hiring unemployed labor appears in Boardman et al. (2006, pp. 99–101), which frames the subject as follows: “Consider, for example, a project that hires 100 workers. How many fewer workers will be unemployed as a result?” (p. 99). The only reference cited is by Haveman (1970), which is based on the seminal work of Haveman and Krutilla (1968). This paper updates this important subject by estimating a labor response function using the new series on job openings begun in 2001 by the U.S. Bureau of Labor Statistics (2012).

It was once common in BCA to assume that involuntarily unemployed labor has a zero opportunity cost (Haveman, 1970, footnote 4). Nowadays it is widely recognized that even enforced ‘leisure’ has value for job search, child care, home care, etc., with the main question being what fraction of the market wage to assign as the value of such foregone leisure, which is denoted as the reservation wage. But preceding this calculation is the question of how many workers newly employed by the project or policy under review would have been otherwise unemployed? Haveman and Krutilla (1968) posited the notion of a *response function* for labor:  $p = f(u)$ , where  $p$  is the probability that unemployment will decrease by one person if the demand for labor increases by one person consequent upon some increment in public expenditure, and  $u$  is the unemployment rate. They did not estimate a response function but rather assumed *a priori* a sine function.<sup>1</sup> In this approach the shadow price of labor for use in BCA is  $(1 - p)W$ , where  $W$  is the money or market wage. If we denote  $W_R$  as the reservation wage, the social opportunity cost (SOC) of labor is expressed as  $SOC = (1 - p)W + pW_R = W - p(W - W_R)$ , which readily captures the relevant alternatives. When  $W_R = 0$ , the SOC reverts to the original Haveman and Krutilla specification. And when  $p = 1$ ,  $SOC = W_R$  and if  $p = 0$ ,  $SOC = W$ .<sup>2</sup> This is symmetrical with the weighted average cost of capital approach to the choice of a discount rate in BCA advanced by Harberger (1972b, p. 126). In both instances, project resources are either diverted from existing employments or from newly

<sup>1</sup> The labor response function assumed by Haveman and Krutilla (op. cit., 135) is  $p = 1 - 0.50 [\sin\{\pi(u - u_{\min})/(u_{\max} - u_{\min}) - \pi/2\} + 1]$ , where  $u_{\max}$  and  $u_{\min}$  are selected maximum (25%) and minimum (2.5%) rates of unemployment. The latter is deemed ‘full-employment’, and  $u$  is the actual unemployment rate. This functional form appears to have been chosen based solely on the fact that it is bounded by 0 and 1.

<sup>2</sup> This way of expressing the SOC of labor is due to an anonymous referee.

converted 'leisure' or consumption units, and the weights are the relative importance of diversion versus new resources. A 'capital response function' analogous to the  $p$  function for labor would provide suitable weights (see Haveman and Krutilla, op.cit., p. 75).

Apart from the *ad hoc* nature of their specification of the response function, Haveman and Krutilla assumed that the 'full-employment' rate of unemployment is an astonishingly low 2.5% (ibid., p. 74). This means that any project undergoing BCA should have its bill for labor services marked down (shadow priced) as long as the national unemployment rate exceeds 2.5%.<sup>3</sup> This view has obviously been rendered out-of-date by the advent of the natural rate of unemployment hypothesis which would put the rate of unemployment consistent with non-accelerating inflation in the range of 5% to 6%. In addition to the lack of an empirical foundation to the Haveman and Krutilla response function, we increasingly hear of a skills mismatch between job openings and unemployed workers (Barlevy, 2011). Thus, a high speed rail project or a green energy initiative may divert most of its labor from existing employments rather than the pool of unemployed if the latter lack the required human capital skills. These considerations call for an up-to-date estimate of this crucial relationship.

### Related Studies

As far as can be determined, Zuidema (1987) is the only attempt to model the labor response function since Haveman and Krutilla. He assumed the following unemployment-vacancy (U-V) function for The Netherlands:

$$U = AV^\alpha, \text{ where } A > 0, \alpha < 0, \quad (1)$$

where  $U$  is the number of persons unemployed and  $V$  the number of job vacancies. Let  $\underline{L}$  represent the (fixed) supply of labor,  $N$  the demand for labor and  $E$  the number of employed persons, so that:

$$V = N - E, \quad (2)$$

and

$$U = \underline{L} - E \quad (3).$$

Substitution of Eq. (1) into Eqs. (2) and (3) yields:

$$N = \underline{L} + (U/A)^{1/\alpha} - U \quad (4).$$

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<sup>3</sup> The full-employment rate of 2.5% is based on the observation that this rate prevailed in 1953 without undue inflationary pressures (op. cit., p. 72).

A marginal increment  $dN$  in the demand for labor results in a decrease of unemployment  $dU$ , so that differentiating Eq. (4) with respect to  $U$  gives:

$$dU/dN = 1/[(1/\alpha)A^{-1/\alpha} U^{1/\alpha - 1} - 1] < 0 \quad (5).$$

The response function, Eq. (5), may be interpreted as the probability  $p$  that a unit increase in labor demand will be drawn from the pool of unemployed ( $0 \leq p \leq 1$ ), i.e.,  $p = -dU/dN$ . Because it is convenient to work with the unemployment rate  $u = (U/\underline{L}) \times 100$  rather than the absolute number of unemployed,  $U$  is replaced in Eq. (5) to yield the desired response function:

$$p = -1/[(1/\alpha)A^{-1/\alpha} (\underline{L}/100)^{1/\alpha - 1} u^{1/\alpha - 1} - 1] \quad (6).$$

Specification, Eq. (6), shows that  $p$  depends upon  $\alpha$  and  $A$ , and may not be bound by 0 and 1. Zuidema takes the values of  $A = e^{24.703}$  and  $\alpha = -1.1369$  (ibid., p. 110) from the Dutch literature and computes  $p$  for alternate values of  $u$  as of 1980. The advantage of Eq. (6) is that it avoids the need to know the number of job openings ( $V$ ), which also seems to motivate Haveman and Krutilla's framing of the question. A comparison of the  $p$ -values computed by Zuidema versus Haveman and Krutilla show much higher  $p$  for any given unemployment rate, which implies a much lower social opportunity cost of labor in The Netherlands. For example, at  $u = 10\%$ ,  $p = 0.27$  in Haveman and Krutilla but  $p = 0.95$  from the Zuidema model (p. 113). From a theoretical perspective both models seem inadequately specified. In particular, neither allows for the influence of wage rates on the labor market. How actively people search for work and their willingness to take a job is likely influenced by the level of wages relative to unemployment insurance benefits, for example (Barlevy, 2011, p. 92). And the economic significance of  $A$  in Eq. (1) is unclear. Is it the constant term from a regression equation or is it meant to capture the influence of covariates?

### Data and Estimation

Recent developments in government data collection allow direct estimation of the U-V function. On the assumption that a public expenditure project hiring, say, 100 workers, increases the number of job vacancies  $V$  by 100, then  $p = -\partial U/\partial V$ , and an appropriate labor response function can be estimated from  $U = F(V, W)$ , where  $W$  is a suitable index of hiring cost.<sup>4</sup> The Bureau of Labor Statistics (2012) began The Job Openings and Labor Turnover Survey (JOLTS) in 2001. This is a

<sup>4</sup> Formally,  $p = -\partial U/\partial N = -\partial U/\partial V \times \partial V/\partial N = -\partial U/\partial V$ , assuming  $\partial V/\partial N = 1$ .

monthly survey of 16,000 business establishments (Clark, Phillips and Stephens, [www.bls.gov/osmr/pdf/st010220.pdf](http://www.bls.gov/osmr/pdf/st010220.pdf)). The monthly series employed here is total non-farm vacancies, seasonally adjusted (BLS series JTS00000000JOL), covering the period January 2001 to November 2011. This series corresponds to the number of job vacancies  $V$  in Eq. (1) because it is the total number of job openings reported. The level of unemployment  $U$  is the seasonally adjusted monthly figure for the population aged 16 and over (BLS series LNS130). To this is added the number of persons aged 16 years and older who state they want a job now but are not in the labor force (BLS series LNS15026639, monthly, seasonally adjusted). Thus, the measure of unemployed persons used here is broader than the conventionally reported monthly figure, but more appropriate to the problem at hand. Consistent with economic theory a wage rate variable is included in the regression, the quarterly employment cost index for all civilian employees (BLS series CIS101). This index number (with 2005 the base year = 100) includes fringe benefits and is thus a comprehensive measure of the cost of hiring and reward for work. Descriptive statistics are shown in Table 1.

The central estimating equation is specified as (ln = natural log):

$$U = \alpha_0 + \alpha_1 (\ln V) + \alpha_2 (\ln W) \quad (7).$$

Eq. (7) was chosen after it became apparent that Zuidema's specification, Eq. (1), suitably transformed, does not fit the time series data. This point is discussed below. This semi-log specification implies  $p = -\partial U / \partial V = -\alpha_1 / V$ . Theory requires  $\alpha_1 < 0$  and  $\alpha_2 > 0$ , namely, that an increase in job openings will tend to reduce unemployment and that higher wage costs increase unemployment, *ceteris paribus*. In addition,  $\partial^2 U / \partial V^2 > 0$  so that successive increments in vacancies reduce unemployment by a lesser amount because those still unemployed are likely to have fewer qualifications. When  $-\alpha_1 = V$ ,  $p = 1$  and  $p \rightarrow 0$  as  $V \rightarrow \infty$ .

**Table 1: Descriptive Statistics (January 2011 to November 2011)**

Variable	Mean	Range
Officially unemployed	9,570,480	6,023,000–15,421,000
Unemployed + Want Job (U)	14,715,500	10,424,000–21,437,000
Job Vacancies (V)	3,638,020	2,112,000–5,082,000
Employment Cost Index (W)	101.39	84.7–115.6

Owing to the time series nature of the data, the econometric technique employed is the Autoregressive-Moving Average (ARMAX,  $p$ ,  $d$ ,  $q$ ) method, with the order of the autoregressive component  $p = 1$ , the order of integration  $d = 0$ , and the order of the moving average  $q = 3$ . The parameters are estimated using nonlinear least squares (LIMDEP, 2007, E12-5). This particular specification was arrived at by trial and error until the pattern of residuals passed both the Box-Pierce and Box-Ljung test for white noise.<sup>5</sup> Table 2 presents the estimation results and related diagnostic statistics.

**Table 2. Estimated Employment Response Function**

(Dependant Variable = Total Unemployed + Want Job)				
Variable	Coefficient	Standard error	b/SE	<i>p</i> -value
phi(1)	0.9126	0.0136	66.872	0.0000
mu ( $\alpha_0$ )	4356.96	1595.90	2.730	0.0063
lnV	<b>-1158.96</b>	190.25	-6.091	0.0000
lnW	1402.22	293.5	4.777	0.0000
theta(1)	0.0030	0.0876	0.035	0.9724
theta(2)	0.2529	0.0849	2.979	0.0029
theta(3)	0.0943	0.0890	1.060	0.2891

Box-Pierce Q statistic = 11.28, degrees of freedom = 12, significance level = 0.5048.

Box-Ljung Q statistic = 12.06, degrees of freedom = 12, significance level = 0.4401.

The diagnostic Q statistics shown in Table 2 permit one to readily accept the hypothesis that the residuals in this model follow a white noise pattern, which is a conventional specification test of ARMAX-type models. Autocorrelation and partial autocorrelation functions for Table 2 regression, as well as a plot of the residuals, is presented in Appendix 1.

### Policy Implications

Based on the estimated employment response function, it is easy to predict the value of  $p$ . During July 2009 the number of job vacancies reached its minimum of 2112 (in thousands) during the 2008–2009 Great Recession. At that point, stimulus spending that creates, say, 100 jobs, would draw  $p = 1159/2112 = 0.55$  of the workers from the pool of unemployed. Thus, somewhat more than half of

<sup>5</sup> The model is  $y(t) = \mu + bx + \text{phi}(1)y(t - 1) \dots \text{phi}(p)y(t - p) + e(t) + \text{theta}(1)e(t - 1) \dots \text{theta}(q)e(t - q)$ .  $y(t) = [(1 - L)^d] Y(t)$  (differences); and  $t$  indexes time. Greene (2000, p. 762) notes that “The process of finding the appropriate specification is essentially trial and error”.

the workers employed by this hypothetical project would have been drawn from the ranks of the unemployed, broadly defined. At the same time, the conventional unemployment rate was 9.5% and the broader unemployment rate used here was 13.3%. By way of contrast, the Haveman and Krutilla  $p = 0.46$  at 13% unemployment and the Zuidema  $p = 0.97$ . The maximum number of job vacancies during the period January 2001 to November 2011 is 5082, which implies  $p = 0.228$ , and at the mean number of job vacancies  $p = 0.32$ . Table 3 displays these  $p$ -values and standard errors computed using the Wald function procedure for analyzing linear and nonlinear functions of parameters in LIMDEP (2007, R11-12).

**Table 3: Probability of Reduced Unemployment Due to Job Vacancy**

Vacancies (V) in 000s	Probability ( $p$ )	Standard error
Mean (3638)	0.318	0.052
Minimum (2112)	0.548	0.090
Maximum (5082)	0.228	0.037

The hypothesis of zero difference between the Great Recession  $p = 0.548$  and Haveman and Krutilla's  $p = 0.46$  is not rejected.<sup>6</sup>

In addition to the regression reported in Table 2, the model is also estimated using the narrower 'official' definition of unemployment as the dependent variable. The crucial  $\alpha_1$  coefficient on vacancies is -1093.43 (standard error = 152.14), which is very close to the estimate with the preferred broad definition of unemployment used in Table 2. Full results of this alternate estimating equation are presented in Appendix Table A2.

Before concluding this section it is worthwhile to briefly consider the estimation of Zuidema's unemployment response function equation, Eq. (1). The best estimate is  $A = e^1$  and  $\alpha = -0.06$ , which when inserted into Eq. (6) yields  $p = 1$  for all values of unemployment between 2% and 15%, for example.<sup>7</sup> This is obviously an inadequate representation of the U.S. economy.

A simple way to summarize the analysis is to compute the  $p$ -value for each month during the observation period and regress that against the corresponding vacancy rate (V/labor force), using least squares:

$$p = 0.67 - 14.11 (\text{vacancy rate}), R^2 = 0.95, \quad (8)$$

(t = 92.0)    (t = -47.6)

<sup>6</sup> The t-test statistic is  $(0.548 - 0.46)/0.09 = 0.970$ .

<sup>7</sup> Various values of ARMAX (p, d, q) were tried, but it was not possible to find a specification that generated white noise residuals. However, all gave similar parameter estimates.

During the time period covered by the data the vacancy rate varied from 0.0137 to 0.035. Thus, a benefit-cost analyst looking to shadow price labor might estimate the number of job vacancies or the vacancy rate during the life of the project and compute  $p$  and the shadow wage as  $(1 - p)W + pW_R$ . In this regard, Haveman and Farrow (2011) summarize how labor inputs might be presented in a BCA under conditions of high unemployment. They note that any surplus accruing to workers because they are paid more than the reservation wage counts as a project benefit, but that this does not alter the value of the SOC of labor. This formulation appears attributable to Harberger (1972a, p. 166). To illustrate, consider their example of a proposed project with \$110 in output benefits,  $W = \$100$  and  $W_R = \$80$ . With the implied  $p = 1$  net benefits are \$30, which includes \$20 of labor surplus. The \$100 in wages is a transfer from taxpayers to workers who in turn sacrifice \$80 of 'leisure', which sum remains the relevant SOC of labor. Using the  $p = 0.55$  estimated above during the Great Recession, the shadow wage is \$89 in this example. Prior to Harberger (1972a), the social opportunity cost of labor (SOC) was primarily viewed as foregone output, thus leading to the conclusion that involuntarily unemployed labor has a zero opportunity cost. However, owing to differences in the attractiveness and location of jobs, e.g., workplace hazards or amenities, commuting and childcare expenses, urban versus rural housing and food costs, Harberger proposed the supply price of labor as the relevant opportunity cost for BCA. This is the sophisticated counterpart to the commonplace observation that it is nearly impossible to find people willing to work for nothing.

## Conclusion

Using data unavailable to earlier researchers and the appropriate econometric method, it is estimated that 'stimulus' spending during the 2008–2009 recession is likely to have reduced unemployment by approximately 0.55 of the project labor requirements. This is statistically equivalent to the 0.46 predicted by Haveman and Krutilla (1968), but far below the implicit assumption of politicians that government spending expands employment on a one-for-one basis. A compact statement is that the proportion  $p$  of newly hired workers drawn from the pool of unemployed is  $p = 0.67-14$  (vacancy rate), expressed in decimal units.

**Appendix 1**

**Table A1: Time Series Identification for Errors (E)** (From Table 2 Regression)

Box-Pierce statistic = 11.2832    Box-Ljung statistic = 12.0696

Degrees of freedom = 12                      Degrees of freedom = 12

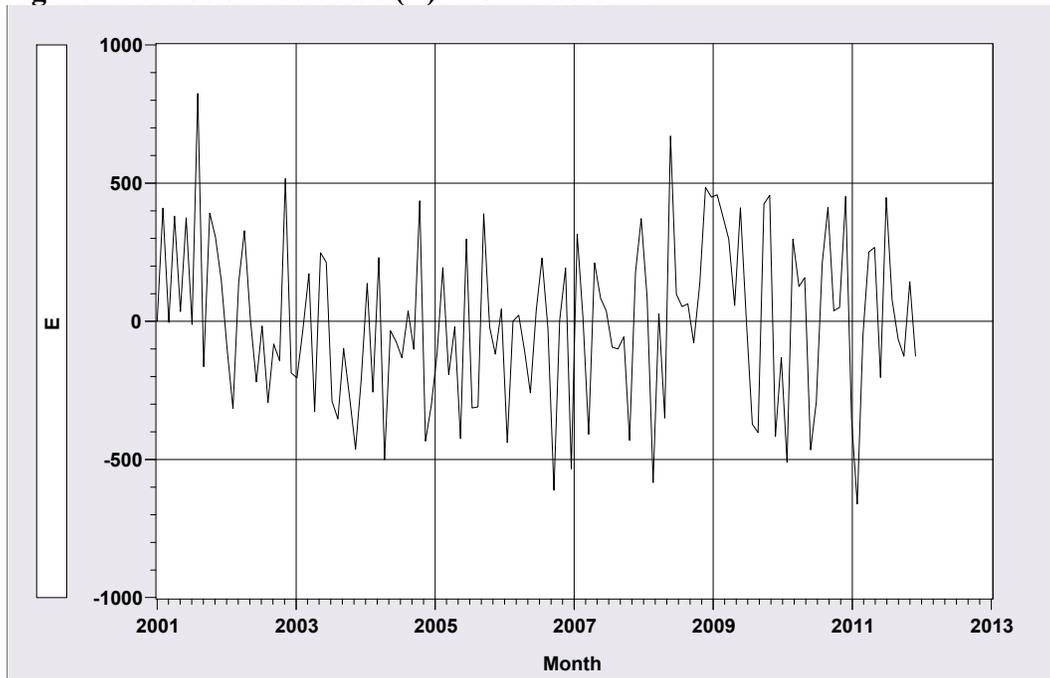
Significance level = **0.5048**                  Significance level = **0.4401**

\* = > |coefficient| > 2/sqrt(N) or > 95% significant.

PACF is computed using Yule-Walker equations.

Lag	Autocorrelation function	Box/Prc	Partial autocorrelation
1	0.001                        *	0.00	0.001                        *
2	-0.028                    *	0.11	-0.028                    *
3	-0.013                    *	0.13	-0.013                    *
4	0.175*                      **	4.16	0.176*                      **
5	0.043                        *	4.40	0.042                        *
6	0.119                        *	6.25	0.135                        *
7	0.118                        *	8.08	0.135                        *
8	0.063                        *	8.60	0.052                      *
9	0-.079                    *	9.42	-0.086                    *
10	0.007                        *	9.43	-0.035                    *
11	0.081                        *	10.30	0.022                        *
12	-0.087                    *	11.28	-0.169                    **

**Figure A1: Plot of Residuals (E) From Table A1**



**Table A2: Alternate Estimated Employment Response Function**  
(Dependant Variable: Officially Unemployed)

Variable	Coefficient	Standard error	b/SE	p-value
phi(1)	0.9044	0.0127	71.170	0.0000
mu ( $\alpha_0$ )	4882.52	1218.24	4.008	0.0001
lnV	<b>-1093.43</b>	152.14	-7.187	0.0000
lnW	1071.35	277.69	3.858	0.0000
theta(1)	0.1478	0.0857	1.724	0.0846
theta(2)	0.2306	0.0865	2.664	0.0077

Box-Pierce Q statistic = 16.30, degrees of freedom = 12, significance level = 0.1775.

Box-Ljung Q statistic = 17.46, degrees of freedom = 12, significance level = 0.1329.

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