Solar Activity and its Magnetic Origin
Proceedings IAU Symposium No. 233, 2006
V. Bothmer & A. A. Hady, eds.

doi:10.1017/S1743921306001700

On general transformations and variational principles of three-dimensional incompressible gravitating flows in ideal MHD

A. H. Khater[1,2], D. K. Callebaut[2], T. N. Abdelhameed[1] and A. Hady[3]

[1]Department of Mathematics, Faculty of Science, Beni-Suef University , Beni-Suef, Egypt
email: [1] khater_ah@yahoo.com & tarknabil_1312000@yahoo.com

[2]Department Natuurkunde, CGB, University of Antwerp, B-2020 Antwerp, Belgium
email: dirk.callebaut@ua.ac.be

[3]Department of Astronomy, Faculty of Science,Cairo University, Giza, Egypt

Abstract. In this paper, we apply the general theory of Arnold (1965, 1966) and Moffatt *et al.* (1997). We search sufficient conditions for the linear stability of steady three-dimensional incompressible gravitating flows in ideal magnetohydrodynamics (MHD). The results suggest that the solar and the stellar convection zones must be sensitive to the density stratification.

Keywords. magnetohydrodynamics; gravitating fluid; stability; stars: interiors; sun: interior.

1. Introduction

We consider the stability of steady three-dimensional incompressible gravitating flows in MHD, neglecting the resistivity. The stability of such steady states is considered by an appropriate generalization of the Arnold energy techniques. We consider an incompressible, inhomogeneous (with density ρ), perfectly conducting MHD flow contained in a domain D with fixed boundary ∂D. We shall in general suppose that D is bounded, but the theory may be easily modified to deal with the case of an unbounded domain. Let $u(r,t)$ be the velocity field, $h(r,t)$ the magnetic field (in Alfvén velocity units), $p(r,t)$ the gas pressure, $J = \nabla \times h$ the current density in the fluid, $\omega = \nabla \times u$ the vorticity field, and the mass force field $F(r,t)$ with a potential $\Phi(r,t)$ such that $F = -\nabla\Phi$. For self-gravitation we have $\Delta\Phi = 4\pi G\rho$. Then the governing equations in a Boussinesq-like approximation are (Vladimirov (1986), Phillips (1980) and Chandrasekhar (1987)).

$$u_t = u \times \omega + j \times h - \nabla(p + \frac{1}{2}u^2) - \rho\nabla\Phi \tag{1.1}$$

$$Lh = \frac{\partial h}{\partial t} - \nabla \times (u \times h) = 0, \qquad D\rho = 0, \qquad \nabla \cdot u = \nabla \cdot h = 0, \tag{1.2}$$

Here L is a form of Lie derivative.

2. Sufficient conditions to three-dimensional gravitating flows stability.

Let us now consider in more details the expression for $\delta^2 E$.

$$\delta^2 E = \frac{1}{2}\int_D \{h^2 + h\cdot(J \wedge \zeta) + (A\Phi^{''}(A) + 2\Phi^{'}(A)\rho^2)\}dV, \tag{2.1}$$

i) Parallel flow and field

Let Ω be an infinite cylinder of arbitrary cross-section with axis parallel to Oz, and suppose that

$$H = H_0(x, y)e_z. \tag{2.2}$$

After some algebra, (2.1) reduces to

$$\delta^2 E = \frac{1}{2}\int_D \{H_0^2((e_z \cdot \nabla)\zeta)^2 + (A\Phi^{''}(A) + 2\Phi^{'}(A))\rho^2\}dV. \tag{2.3}$$

Proposition 2.1 The state (2.2) is linearly stable to isomagnetovortical (imv) perturbations provided

$$A\Phi^{''}(A) + 2\Phi^{'}(A) \geqslant 0 \,\text{in}\, D. \tag{2.4}$$

ii) Annular basic state

Let D be an annular region between two cylinders C_1, C_2 of arbitrary cross-sections, and let

$$H = -e_z \wedge \nabla B, \tag{2.5}$$

where $B(x, y)$ is the flux-function of H. We shall suppose that $|\nabla B| \neq 0$ in D, i.e H has no neutral points in D.

The expression for $\delta^2 E$ may be reduced to the form

$$\begin{aligned}\delta^2 E = \frac{1}{2}\int_D &[((h + (\zeta \cdot \nu)(J \wedge \nu))^2 + \nabla^2 B(-\nabla^2 B + \frac{\nabla B \cdot \nabla(H^2)}{2H^2}))(\zeta \cdot \nu)^2 \\ &+ (A\Phi^{''}(A) + 2\Phi^{'}(A))\rho^2]dV.\end{aligned} \tag{2.6}$$

Proposition 2.2 The state (2.5) is linearly stable to imv perturbations provided

$$\nabla^2 B \geqslant 0\,, \qquad -\nabla^2 B + \frac{\nabla B \cdot \nabla(H^2)}{2H^2} \geqslant 0\,, \qquad A\Phi^{''}(A) + 2\Phi^{'}(A) \geqslant 0. \tag{2.7}$$

or

$$\nabla^2 B \leqslant 0\,, \qquad -\nabla^2 B + \frac{\nabla B \cdot \nabla(H^2)}{2H^2} \leqslant 0\,, \qquad A\Phi^{''}(A) + 2\Phi^{'}(A) \geqslant 0. \tag{2.8}$$

3. Conclusion

In this paper, we have given sufficient conditions for linear stability of steady three-dimensional gravitating flows in ideal MHD. These stability conditions are obtained by an appropriate generalization of the (Arnold) energy techniques.

References

Arnold. V. I. 1965, *J. Appl. Math. Mech.* 29, 5.
Arnold. V. I. 1966, *Ann. Inst. Fourier,* 16, 316.
Chandrasekhar, S., 1987, *Ellipsoidal Figures of Equilibrium* (Dover)
Phillips, O. M. 1980, *Dynamics of the Upper Ocean,* Cambridge.Moscow.
Vladimirov, V. A. 1986, *J. PMM. USSR,* 50, 559.
Vladimirov, V. A., Moffatt, H. K. & Ilin, K.I. 1997, *J. Plasma Physics,* 57, 89.

https://doi.org/10.1017/S1743921306001700 Published online by Cambridge University Press