Suppose $a_1 - b_1$ to be positive and write

$$y = (x + a_1)(x + b_1)^2 + c_1.$$

It follows from the above that the minimum turning value is given by $y = c_1$.

Next, writing $\quad y = (x + a_2)(x + b_2)^2 + c_2,$

we observe that the maximum turning value is given by

$$y = c_2\,;$$

$\therefore$ OX will cut the graph of

$$y = x^3 + qx + r$$

in three different real points if $\dfrac{c_1}{c_2}$ is negative,

and in one real point and two imaginary points if $\dfrac{c_1}{c_2}$ is positive.

There are therefore three different real solutions of the equation, or one real and two imaginary,

according as $\quad \dfrac{c_1}{c_2} \quad$ is negative or positive,

$\therefore$,, ,, $\dfrac{c_1^2}{c_1c_2}$,, ,, ,, ,,

$\therefore$,, ,, c_1c_2 ,, ,, ,, ,,

$\therefore$,, ,, $(r - a_1b^2)(r - a_2b^2)$,, ,, ,, ,,

$\therefore$,, ,, $r^2 + a_1a_2b^4$,, ,, ,, ,, $(\because a_1 + a_2 = 0)$

$\therefore$,, ,, $r^2 - a^2b^4$,, ,, ,, ,,

$\therefore$,, ,, $r^2 - 4b^6$,, ,, ,, ,, by (1′)

$\therefore$,, ,, $r^2 + \frac{4}{27}q^3$,, ,, ,, ,, by (1′) and (2′)

$\therefore$ according as $\quad 4q^3 + 27r^2 \quad$ is negative or positive.

Note on the Problem: To draw through a given point a transversal to (*a*) a given triangle (*b*) a given quadrilateral so that the intercepted segments may have (*a*) a given ratio (*b*) a given cross ratio.

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