A COUNTEREXAMPLE TO A CONJECTURE OF D.F. SANDERSON

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In [2, p.511] Sanderson has shown that if every large left ideal of a ring R with identity contains a regular element, and if the regular elements in R satisfy Ore's condition, then the complete (Utumi's) ring of quotients coincides with the classical ring of quotients. He conjectured that the above conditions are also necessary. The following is a counterexample.

Let $R = Z_4[x]$, the polynomial ring in the indeterminate x over the ring of integers modulo 4. The following result, due to L. Small (unpublished), was communicated to me by Professor Lambek, and will appear as an exercise in [1]: if R is a commutative Noetherian ring with identity then the complete and classical quotient rings of R coincide. This, then, is clearly the case for $Z_4[x]$. Also $Z_4[x]$ satisfies Ore's condition (since it is commutative).

Let now $I = 2Z_4[x]$ be the ideal of $Z_4[x]$ generated by the element 2. I is a large ideal of $Z_4[x]$. For, if K is any nonzero ideal of $Z_4[x]$ and $0 \neq k = \sum_{i=0}^{n} a_i x^i \in K$, then either (i) all the nonzero $a_i = 2$, in which case $k \in I \cap K$, or (ii) some nonzero $a_j \neq 2$; in this case $2k \in I \cap K$, and $2k \neq 0$ (since $2a_j \neq 0$). Thus I is a large ideal, but I contains no regular elements, as $2c = 0$ for every $c \in I$.

REFERENCES

1. J. Lambek, Lectures on rings and modules. Blaisdell, New York, (1966).

2. D.F. Sanderson, A generalization of divisibility and injectivity in modules, Can. Math. Bull. 8, (1965), 505-513.

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https://doi.org/10.4153/CMB-1966-063-x Published online by Cambridge University Press