Journal of Glaciology, Vol. 26, No. 94, 1980

CALCULATIONS OF AVALANCHE FRICTION COEFFICIENTS FROM FIELD DATA

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ABSTRACT. The friction coefficients needed to solve Voellmy's avalanche-dynamics equations and as input to the numerical, finite-difference computer program AVALNCH are calculated from case studies. The following coefficients of internal friction ν and of surface friction f worked well for program AVALNCH: for midwinter dry snow $\nu = 0.5$ to $0.55\ m^2/s$ and $f = 0.5$ to 0.55; for hard slab $\nu = 0.7$ to $0.8\ m^2/s$ and $f = 0.7$ to 0.8; for fresh, soft slab $\nu = 0.4$ to $0.5\ m^2/s$ and $f = 0.4$ to 0.5. The program predicted run-out distance well for a variety of conditions but performed less well in cases of sharp, adverse grade in the run-out zone. For the Voellmy approach, large design-size avalanches required turbulent friction coefficients ξ of 1 200 to 1 600 m/s^2 and kinetic friction coefficients of 0.15. Two hard-slab avalanches, a slow-moving, wet-slab avalanche, and a soft-slab avalanche that ran through scattered mature timber required ξ of 700 to 800 m/s^2 and μ of $5/V$ when V is velocity in m/s. The coefficient of sliding friction for a hard-slab avalanche that encountered damp snow in the run-out zone was computed directly from movies to be 0.35, 0.43, and 0.32 for three measured sections of the run-out zone.

RÉSUMÉ. *Calculs des coefficients de frottement des avalanches à partir de données de terrain.* A partir de cas étudiés, provenant principalement du Colorado, on a calculé les coefficients de frottement nécessaires pour résoudre les équations de Voellmy et établi le programme numérique aux différences finies AVALNCH. Pour le programme AVALNCH les coefficients de friction interne ν (de 0,5 à 0,55 m^2/s) et de frottement superficiel f de 0,5 à 0,55 sont bons pour la neige sèche du mi-hiver, tandis que les valeurs 0,7 à 0,8 m^2/s et 0,7 à 0,8 s'appliquent aux plaques dures et 0,4 à 0,5 m^2/s et 0,4 à 0,5 pour les plaques douces et neige fraîche. La distance d'arrêt prédite par le programme est correcte dans des conditions variables mais on a connu certaines difficultés dans des cas de pente inverse forte dans la zone d'étalement. Selon l'approche de Voellmy une enveloppe de la zone d'avalanche est obtenue avec une friction turbulente ξ de 1 200 à 1 600 m/s^2 et une friction cinétique de 0,15. Deux avalanches de plaque dure, une avalanche lente de neige mouillée et une avalanche de plaque douce qui a traversé une forêt claire, mure supposent que ξ ait été de 700/800 m/s^2 et μ de $5/V$ où V la vitesse en m/s. Le coefficient de frottement de glissement pour une avalanche de plaque dure qui a rencontré de la neige humide dans la zone de dépôt a été calculé directement à partir d'enregistrement cinématographique comme étant de 0,35 à 0,43 et 0,32 pour les trois sections mesurées de la zone de dépôt.

ZUSAMMENFASSUNG. *Berechnung der Reibungskoeffizienten bei Lawinen aus Feldbeobachtungen.* Aus Fallstudien werden die Reibungskoeffizienten berechnet, die zur Lösung der Lawinendynamik-Gleichung von Voellmy und für das numerische, mit finiten Differenzen arbeitende Computerprogramm AVALNCH benötigt werden. Im Programm AVALANCH erwiesen sich die folgenden Koeffizienten der inneren Reibung ν und der Oberflächenreibung f als zustreffend: für trockenen Mittwinterschnee $\nu = 0,5$–$0,55\ m^2/s$ und $f = 0,5$–$0,55$; für harte Schneebretter $\nu = 0,7$–$0,8\ m^2/s$ und $f = 0,7$–$0,8$; für frische, weiche Schneebretter $\nu = 0,4$–$0,5\ m^2/s$ und $f = 0,4$–$0,5$. Das Programm berechnete richtige Reichweiten für eine Vielfalt von Bedingungen, doch traten Schwierigkeiten im Fall von scharfen Gegenhängen in der Auslaufzone auf. Bei der Voellmy-Näherung erforderten breit angelegte Lawinen Koeffizienten von 1 200 bis 1 600 m/s^2 für die turbulente Reibung ξ und von 0,15 für die kinetische Reibung. Zwei harte Schneebrettlawinen, deren eine feucht und langsam, die andere hingegen weich war und durch schütteren Hochwald abging, erforderten ein ξ von 700 bis 800 m/s^2 und ein μ von $5/V$, wobei V die Geschwindigkeit in m/s bedeutet. Der Koeffizient der gleitenden Reibung für eine harte Schneebrettlawine, die in der Auslaufzone auf feuchten Schnee traf, wurde aus Filmaufnahmen direkt zu 0,35, 0,43, und 0,32 für drei beobachtete Sektionen der Auslaufzone berechnet.

1. INTRODUCTION

Avalanche motion is complex, varying from the pure, sliding motion of large blocks to the highly turbulent motion of fully developed powder avalanches. No single existing theory can handle this full spectrum of motion. Instead, several simplified approaches offer reasonably

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good predictions of run-out distance, impact forces, debris location, and debris distribution under most conditions even though the theoretical basis for such predictions falls far short of matching actual avalanche conditions. To assure realistic results from such approaches, it is necessary to calibrate the computation schemes against actual events.

In this study, data from a number of avalanches were used to estimate the friction coefficients needed for two avalanche-dynamics computation procedures—the computer program AVALNCH, and Voellmy's avalanche-dynamics equations. Program AVALNCH (Lang and others, 1979[a], [b]; Lang and Martinelli, 1979), is a numerical, finite-difference computer program based on the Navier–Stokes equations. The Voellmy (1955) equations, based on open channel flow, were developed over 20 years ago and have since been used and slightly modified by subsequent workers (Schaerer, [1975]; Mears, 1976; Leaf and Martinelli, 1977; Buser and Frutiger, 1980).

TABLE I. SUMMARY OF THE PHYSICAL PARAMETERS AND FIELD DATA FOR NINETEEN CASE STUDIES

	Starting zone							*Track*			*Run-out zone*			
Avalanche/Date	*Slope angle* deg	*Slab thickness* m	*Length of slab* m	*Area of slab* m^2	*Volume of snow* m^3	*Density* kg/m^3	*Flow height* m	*Slope angles* deg	*Cross-section (hyd. radius*	*Flow height* m	*Slope angle* deg	*Height of debris* m	*Density of debris* kg/m^3	*Run-out distance* m
Soft-slab avalanches with fractures of about 1 m														
Badger Mine 16 January 1978	40	1.0	—	36 000	36 000	—	—	31/27/22	channel	—	3.5	—	—	130
Spring Gulch 16 January 1978	35	0.8	—	130 000	104 000	—	—	30/26/21	channel	—	13.5	—	—	530
Blikes "a" 16 January 1978	33	1.0	—	180 000	180 000	—	14	25	channel (6 m)	—	14/21/8	2–1	—	810
Chapman Gulch 16 January 1978	32	1.0	—	104 000	104 000	—	15	29/26/23/20	channel (8 m)	—	13/9	—	—	870
Iron Springs 24 January 1978	40/30	1.0	—	87 000	87 000	—	3	22/26	channel	—	12.5	—	—	520
Blikes "b" 13 March 1978	33	1.1	—	78 000	86 000	—	12	25	channel (7.5 m)	—	14/21/8/−11	2–5	—	880
Snodgrass No. 2 10 February 1976	32	1.0	220	31 300	31 300	—	—	23/34/27/14/ 29/20	open	—	4	1.5–2.0	366 280	180
Saddle 10 February 1976	30	1.0	230	44 600	44 600	181 ±37	2.0 ±0.3*	20/28/18	open	—	2.7	1.5	320	210
Design-size avalanches														
Bird Ridge 23 March 1979	44.5	1.5	—	—	—	—	—	34/52/32/21	open	—	0	—	—	600
Nicholson Lake 17 February 1976	40	1.5	—	183 000	275 000	—	—	32/41/15/28	open	8–10	0	1.5	—	290
Dam 8 April 1957	31	2.0	—	—	—	—	—	31/20/12	channel	—	−14	9	—	90
Hard-slab avalanches														
Pallavicini 28 February 1977	35	1.28	180	19 800	25 344	370	1.4	30/23	open	—	10	1–2	370	125
Breckenridge 9 March 1977	37	1.48	30	6 000	8 880	370	—	11/34/22	open	—	13/10	1–2	450	135
Avalanches with unique features														
Floral Park 12 March 1977	38	0.73	—	—	—	220	1.99	35/19	open	2.03	6	0.5–1.0	320	50
Red Lady Basin 8 April 1977	34	1.0	100	—	—	*c.* 350	—	24	open	—	21	0.5–1.5	—	260
Gothic Mtn. No. 4 4 March 1978	34	1.2	850	260 000	260 000	153	—	40/36/24/27	open	*c.* 5.0	10/−3	1–2	—	515
West Guadalupe 20 February 1978	42	1.0	270	—	—	—	—	22/31/17/34	channel	—	−9.6	—	—	320
Stanley 4 March 1977	32	1.5	360	44 100	44 100	300	2.2	30/23/30	channel (2 m)	—	0	—	460	15
Battleship 16 January 1978	32	1.5	—	—	—	—	7.0	29/18/24/26	channel	—	16/−24/−30/ −10	—	—	140

* Within 100 m of fracture line.

https://doi.org/10.3189/S0022143000010650 Published online by Cambridge University Press

2. Case studies using program AVALNCH

For each of the 18 avalanches in Table I one or more computer runs were made using program AVALNCH to see what coefficients of internal friction ν and of surface friction f gave the best approximation of the observed run-out distance and debris distribution. For these calculations run-out distance is considered to be the slope distance from the place in the longitudinal profile where slope angle $\leqslant 18°–20°$ to the end of the debris. When good field data were available for leading-edge velocity and/or flow heights, these were also compared to the computations.

The results of this analysis are summarized in Table II. The cell numbers referred to in this table and elsewhere in the paper are units of slope distance on the longitudinal profile of the avalanche path. Cells were 20 m long on all paths except Breckenridge, Floral Park, and Red Lady Basin where they were 10 m.

Table II. Summary of case studies using program AVALNCH

	Run-out distance		*Friction coefficient*			*Debris distribution*		*Maximum debris depths and location*			
								Actual		*Computed*	
Avalanche paths	*Actual*	*Computed*	ν m^2/s	f	*Comments*	*Actual*	*Computed*	m	*Cell number*	m	*Cell number*
	Cell number					*Cell number*					
1. *Soft-slab avalanches*											
(a) Badger Mine	98	98	0.4	0.4		Unknown	80–98	Unknown		2.8	96
(b) Spring Gulch	113	113	0.5	0.4		Unknown	93–113	Unknown		1	106
(c) Blikes "a" 16 January 1978	119	119	0.4	0.4		77–119	102–119	2 1	77–89 97–119	1	112–116
(d) Chapman Gulch	121	122	0.4	0.4		75–121	98–122	Uniform depth		0.8	107–122
(e) Iron Springs	120	121	0.53	0.53		Unknown	94–121	Unknown		1.5	118
(f) Blikes "b" 13 March 1978	125	120	0.4	0.4		77–125	105–120	4–5* 1	85–95 97–125	1.2	116
(g) Snodgrass No. 2	48	46	0.4	0.4		40–48	42–46	2	40–41	1	45
(h) Saddle	52	50	0.4	0.4		42–52	41–50	1.5	Uniformity	2	49
2. *Design-size avalanches*											
(a) Bird Ridge	113	112	0.5	0.5 0.65 0.23	dry snow, upper track damp snow, lower track open water, run-out zone	Unknown	96–112	Unknown		2.1	101–105
(b) Nicholson Lake	82	81	0.4	0.4		68–82	78–81	1.5	Uniformity	1.9	79
(c) Dam	88	88	0.4	0.5 1.2	 mature trees in track	88	88	9	88	9	88
3. *Hard-slab avalanches*											
(a) Pallavicini	46	44	0.7	0.7		40–46	35–44	2	42	0.8	42
(b) Breckenridge	47	47	0.8	0.8 1.0	 boulders in part of track	33–47	40–47	2	34–44	1	42
4. *Avalanches with unique features*											
(a) Floral Park	44	43	0.4	0.7	scattered mature timber in track and run-out zone	34–44	37–43	Uniform depth		0.9	42
(b) Red Lady Basin	71	70	0.55	0.55	high f for wet snow over grass	45–71	56–70	1.5	Unknown	0.4	62–67
	71	71	0.85	0.4	more reasonable f	45–71	56–70	1.5	Unknown	0.4	66
(c) Gothic Mountain	110	109	0.5	0.5 0.7	snow rock pinnacles and boulders	Unknown	89–109	Unknown		4.6	107–108
(d) West Guadalupe	117	103	0.4	0.4 0.1 0.3 0.4	snow cliff in lower track snow below cliff snow in run-out zone	98–117	80–103	deep shallow	98–103 104–117	2.5	102
(e) Battleship	102	97	0.4	0.4		95–102	85–97	deep shallow	95–99 100–102	19†	97

* Due to piling of blocks on top of earlier debris.
† Debris piled up early in adverse grade rather than spreading out as occurred in Nature.

The eight soft-slab avalanches (Group 1, Table II) all ran in fresh, soft, dry snow. The first four of these ran the same day in adjacent paths on the same ridge. Battleship avalanche, 9 km south-east of these four paths, also ran on the same day. Slab thickness was about 1 m for all eight of the soft-slab avalanches, yet three of them (Blikes "a", Chapman Gulch, and Blikes "b") had run-out distances greater than 800 m. In general, friction coefficients $\nu = 0.4$ to 0.5 m²/s and $f = 0.4$ to 0.5 gave good estimates of run-out distances but tended to show debris concentrated more toward the end of the run-out zone than was actually observed.

The two hard-slab avalanches (Group 3, Table II) also showed consistent results. Coefficients of $\nu = 0.7$ to 0.8 m²/s and $f = 0.7$ to 0.8 represent conditions well. Again, run-out distances were closely approximated by the computations, but debris tended to be confined more toward the end of the run-out zone than it should have been, and maximum debris depths were definitely underestimated.

The three design-size avalanches (Group 2, Table II), were all large for their paths. For this reason, they are important for developing design criteria for avalanche zoning and for designing structural controls in the lower track and run-out zone. These avalanches also furnished useful data on some unusual conditions. For example, the Bird Ridge avalanche started in cold, dry, wind-toughened snow, encountered damp or wet snow in the track, and ran out 600 m over the unfrozen tidal water of Turnagain Arm. The Dam avalanche, which was very large for its path, traveled down a gully, through a dense stand of mature coniferous trees, and crossed a stream channel before piling debris 9 m deep on the highway 60 m up a 15° adverse grade. Program AVALNCH handled all these conditions well, especially for the Dam avalanche where actual conditions were duplicated very accurately using $\nu = 0.4$ m²/s and $f = 0.5$.

In each of the last five avalanches in Table II there were some exceptional field data available for comparison with computed results. At Floral Park, snow plastered on the up-hill side of trees in the track and the run-out zone gave good evidence of flow heights. At Red Lady Basin the avalanche was a slow-moving (estimated < 10 m/s), wet slab that ran over wet grass but stopped on a steep (21°) slope. The Gothic Mountain No. 4 avalanche ran through rock pinnacles and boulders in the starting zone. At West Guadalupe the avalanche fell over a cliff in the lower track and came to rest 320 m up an adverse grade of 9.6°. Battleship came down a moderately steep track (26°), crossed a narrow stream, and ran up a steep (23°), adverse grade for a slope distance of 140 m.

Program AVALNCH duplicated run-out distance well for all the avalanches in Group 4 (Table II) except Battleship and West Guadalupe. In these two cases the program underestimated run-out distance by 5 and 12% respectively—probably because of the steep, adverse grade in the run-out zone. The measured flow heights of about 2 m in the track and run-out zone at Floral Park were estimated by program AVALNCH to be only 0.3–0.6 m. This underestimation is the result of the partial fluidization of fast-moving, soft-slab snow. This type of snow has a greater flow height than the height of the dense flow regime (core material) predicted by program AVALNCH because the program assumes incompressibility.

TABLE III. INTERNAL FRICTION ν AND SURFACE FRICTION f COEFFICIENTS FOR USE WITH PROGRAM AVALNCH

Snow and/or terrain conditions	ν m²/s	
Fresh, soft, dry snow	0.4 to 0.5	0.4 to 0.5
Hard slab	0.7 to 0.8	0.7 to 0.8
Mid-winter snow with some wind or age toughening	0.55	
Damp or wet snow in track		0.65 to 0.7
Scattered mature timber in track		0.7
Dense mature timber in track		1.2
Open water surface in run-out zone		0.2

For the slow-moving, wet-slab avalanche at Red Lady Basin, two sets of coefficients, $\nu = 0.55$ m²/s, $f = 0.55$ and $\nu = 0.85$ m²/s, $f = 0.4$, give acceptable results. In the first case, the leading-edge velocity was computed to be $\leqslant 10$ m/s, but the f value seems too high for wet snow sliding over grass. In the second case, leading-edge velocity was $\leqslant 8$ m/s, and the f value seems more reasonable. Table III is a summary of ν and f values suggested for use in program AVALNCH for a variety of snow and terrain conditions. These values should be used with caution until more case studies can be made and more experience can be gained in the use of the program.

3. Case studies using Voellmy's approach

Run-out distances were calculated using Voellmy's approach (Voellmy, 1955; Leaf and Martinelli, 1977) for most of the avalanches that ran on non-channeled (or open) tracks and for one-channeled avalanche. The procedure outlined by Mears (1976) was used for the channeled path. The following equations were used for the paths with open tracks:

$$V^2 = \xi h'(\sin \Psi - \mu \cos \Psi), \tag{1}$$

$$\frac{V_2}{V_1} = \left[\frac{\sin \Psi_2}{\sin \Psi_1}\right]^{\frac{1}{3}} \quad \text{and} \quad \frac{h_2}{h_1} = \left[\frac{\sin \Psi_1}{\sin \Psi_2}\right]^{\frac{1}{3}}, \tag{2}$$

$$s = \frac{V_{LT}^2}{2g[\mu_{LT} \cos \beta - \sin \beta] + V_{LT}^2/\xi(h_{LT} + V_{LT}^2/4g)}. \tag{3}$$

Where V is the maximum velocity in the starting zone; ξ is the coefficient of turbulent friction (different values can be used in different parts of the path); h' is flow height in the starting zone; Ψ is slope angle in the starting zone; μ is the coefficient of kinetic or sliding friction (can be varied for different parts of the path); s is run-out distance from the break in grade at the top of the run-out zone to the end of the debris; g is gravitational acceleration (taken as 10 m/s²); β is the slope angle of the run-out zone (minus values indicate adverse grade). Subscripts 1 and 2 refer to sections of the track and LT to the lower track.

3.1. *Non-channeled avalanches*

For avalanches with non-channeled tracks, Equation (1) was used to compute velocity in the starting zone. Equations (2) give velocity and flow height for uniform sections of the track working down from the starting zone. Equation (3) was used for run-out distance based on the lower-track velocity and flow height from Equations (2). The same ξ and μ values used to compute velocity were also used to compute run-out distance. (Average ξ and μ's were used to facilitate comparisons of coefficients for the various events. Some workers feel μ in the run-out zone should be larger than in the track.) Flow height in the lower track (h_{LT}) as computed by the second of Equations (2) was used in most cases. For the Nicholson Lake avalanche, the measured flow height in the lower track was also used. Although the measured height in the lower track was three times that computed using the second of Equations (2), this made no significant difference to the run-out distance. In most cases a range of μ values was used, including the Schaerer ([1975]) convention of $\mu = 5/V$ (where V is in m/s). Several combinations of ξ and μ could often be found to satisfy the field observations.

For the two open-slope avalanches that ran in soft, fresh snow (Saddle and Snodgrass No. 2), ξ values of 1 000 to 1 200 m/s², combined with μ values of 0.10 to 0.15, gave good fits to the field data (Table IV). At Snodgrass No. 2, $\xi = 700$ m/s² and $\mu = 0.10$ were also satisfactory. The values that gave the best fit at Saddle ($\xi = 1\,000$; $\mu = 0.10$) overestimated run-out on Snodgrass No. 2 by almost a factor of two even though the two avalanches actually started from the same fracture line. This most likely reflects the more uneven nature of the terrain at Snodgrass No. 2.

TABLE IV. SUMMARY OF VOELLMY FRICTION COEFFICIENTS THAT GIVE GOOD FIT TO FIELD OBSERVATIONS (OPEN TRACKS ONLY)

Avalanche	*Flow height* m	*Run-out distance* m	*Run-out slope* deg	*Friction coefficient* ξ m/s²	μ
Saddle	1.0	210	2.7	1 000	0.10
(soft slab)				≈1 600	0.15
Snodgrass No. 2	1.0	180	4.0	≈1 200	0.15
(soft slab)				≈700	0.10
Nicholson Lake	1.5	340	0	≈1 700	$5/V$
(design size)				1 800	0.15
				≈1 200	0.10
Bird Ridge	1.5	600	0	2 000+	0.10
(design size)	2.0			2 000	$5/V$
	2.0			≈1 500	0.10
Breckenridge	1.5	135	11	≈700	$5/V$
(hard slab)					
Pallavicini	1.5	125	10	≈700	$5/V$
(hard slab)					
Floral Park	1.24	50	6	≈700	$5/V$
(soft slab through trees)				800	0.3
Red Lady Basin	1.0	260	21	≈700	0.15
(wet-slab, slow moving)				≈700	0.25
				800	$5/V$
				≈900	0.40

The two hard-slab avalanches (Breckenridge and Pallavicini) fit the Voellmy approach better than any of the others. Both conform well to $\xi \approx 700$ m/s² and $\mu = 5/V$.

The Floral Park avalanche was also very tractable with $\xi \approx 700$ m/s² and $\mu = 5/V$, provided the measured value of 1.24 m was used for flow height in the starting zone h'. This h' value, when adjusted to the lower track by the second of Equations (2), closely duplicated the measured flow height of 2 m in the lower track and run-out zone. The one apparently inconsistent value computed for this avalanche is the high value of about 0.3 for μ. This value gave a good fit for the hard-slab cases, but is much higher than the best-fit values for the other soft-slab cases. A lower μ value was expected based on the field description of the snow; however, flow through the trees on this slope probably contributed to the higher value.

The steep run-out zone (21°) of the Red Lady Basin avalanche offers some problems. Any μ value smaller than 0.38 does not permit the avalanche to stop. Values that high, however, seem unreasonable for wet snow moving over smooth, wet grass. Using different μ's for the starting zone and run-out zone may provide better results in this case. Good results were obtained with values of $700 < \xi < 800$ m/s² for the entire path, $0.15 < \mu < 0.25$ for the starting zone, and 0.4 for the run-out zone. Although this gives a good estimate of run-out distance, the lower track velocity of about 15 m/s is half again more than the field estimate of 10 m/s or less.

A more logical approach for estimating values for this type of avalanche is to assume the motion was sliding rather than flowing. This assumption enables one to disregard friction work caused by internal flow deformation. For sliding motion, the net slope-parallel force F acting on the block (Fig. 1) is $F = \mu mg \cos \beta - mg \sin \beta$. Since the block is decelerating in the run-out zone, the net force is acting up the slope. The friction work done by the block moving through distance s is Fs. From the work-energy theorem, disregarding flow work, dissipation of kinetic energy is equal to the friction work over the distance s, thus $1/2mV_0^2 = (\mu mg \cos \beta - mg \sin \beta)s$. Hence

$$\mu = \frac{V_0^2}{2gs \cos \beta} + \tan \beta, \qquad (4)$$

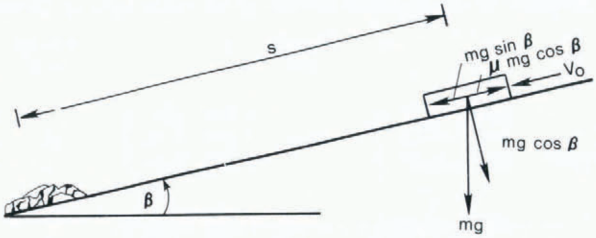

Fig. 1. Sliding block run-out model.

where V_0 is velocity at the up-hill end of the section of the avalanche path in question, s is travel distance (along the slope), g is the acceleration due to gravity (9.8 m/s²), μ is the coefficient of sliding friction, and β the slope angle in the run-out zone.

Substituting an assumed velocity of 10 m/s and the measured run-out distance s of 260 m in Equation (4) gives a μ value of 0.405. The above approach may be a more realistic calculating procedure for low-velocity avalanches that move primarily as sliding blocks.

Two of the design avalanches (Nicholson Lake and Bird Ridge) had very steep starting zones, steep lower tracks, flat run-out zones, and average starting zone-slab thicknesses of 1.5 to 2 m. Both had very large snow-dust clouds and Nicholson Lake had air blast in the far end of its run-out zone. The midwinter slide at Nicholson Lake, where $h' = 1.5$ m, required $\xi = 1\,800$ m/s² and $\mu = 0.15$; $\xi = 1\,800$ m/s² and $\mu = 5/V$ or $\xi = 1\,200$ m/s² and $\mu = 0.1$ to match field run-out conditions.

The spring avalanche at Bird Ridge, which had damp snow in the lowest 300 m of its path, required $\xi > 2\,000$ m/s² with $\mu = 0.1$ and $h' = 1.5$ m or $\xi \approx 1\,500$ m/s² with $\mu = 0.1$ and $h' = 2$ m to produce a 600 m run-out distance.

Sensitivity testing at $\xi = 1\,800$ m/s² and $\mu = 0.15$ for the Nicholson Lake avalanche showed ξ for the run-out zone and flow height in the lower track h_{LT} make little difference in run-out distance when other things are held constant. Small changes in μ and β, however, make great differences in run-out distance. For example, a change in β from 5° to −2° shortens run-out distance by 500 m. At $\xi = 1\,400$ m/s² and $h' = 1.5$ m a change in μ from 0.15 to 0.1 lengthened the predicted run-out distance by 150 m.

3.2. *Channeled avalanche*

Chapman Gulch was the one channeled avalanche analyzed by the Voellmy approach (as modified by Mears, 1976). Starting-zone velocities were estimated from Equation (1). The combinations of ξ and μ values needed to produce starting-zone velocities of 15, 25, and 30 m/s are given in Table V. From these velocities, the length of the starting zone, and the volume of snow released, three estimates of the discharge of snow from the starting zone were computed. Assuming continuity of discharge down the track, values for μ and ξ in the track (Table V) are obtained by the simultaneous solution of the equations for velocity and discharge. Based on the computed track velocities, the measured run-out distance s (870 m on a 12° slope), and assumed values of ξ (500, 1 000, and 2 000 m/s²), μ values for the run-out zone can be computed (Table V). The cluster of μ values around tan β (0.212 56) sharply emphasizes the physically unacceptable fact that as μ approaches tan β, run-out distance as computed by Equation (3) becomes independent of velocity.

3.3. *Summary*

In four of the nine cases analyzed by the Voellmy method a turbulent coefficient ξ between 700 and 800 m/s² with kinetic friction, $\mu = 5/V$ gave good duplication of observed run-out

TABZE V. SUMMARY OF COEFFICIENTS NEEDED TO FIT FIELD DATA AT CHAPMAN GULCH

	ξ	μ	*Velocity* m/s	ξ	μ	*Velocity* m/s	ξ	μ	*Velocity* m/s
Starting zone:									
$\Psi = 32°$	900+	$5/V$		1 800	$5/V$		2 000+	$5/V$	
Slope length = 448 m	500+	0.15	≈15	1 500	0.15	25	2 200	0.15	30
Snow volume = 104 000 m³	500	0.10		1 500	0.10		2 000	0.15	
Track:									
$\Psi = 25°$	285	0.45		490	0.44		700	0.44	
$R = 8$ m			5.8			9.7	1 140	0.45	11.6
Area = 600 m²	740	0.46		790	0.45		2 945	0.46	
Run-out zone:									
$\beta = 12°$	500	0.213–0.215		500	0.215–0.220		500	0.22–0.23	
$s = 870$ m	1 000	0.213–0.215		1 000	0.215–0.220		1 000	0.22–0.23	
	2 000	0.213–0.215		2 000	0.215–0.220		2 000	0.22–0.23	

distances (Table IV). These cases include the two hard-slab avalanches, a soft slab that ran through an open stand of mature trees, and a slow-moving, wet slab. Two of the design avalanches required ξ values between 1 200 and 2 000 m/s² even with μ values as low as 0.10 to 0.15. The long-running, soft slabs of modest size required ξ between 1 200 and 1 600 m/s² for μ of 0.15 or ξ of 700 to 1 000 m/s² for μ of 0.10. Had μ been set at 0.2 to 0.25 for the run-out zone, as is often done in practice, even higher ξ values would have been required.

The extreme sensitivity of the run-out equation to slope angle β and μ in the run-out zone creates uncertainty in the use of this approach, especially as μ approaches tan β.

Fig. 2. Slushman avalanche flowing through the upper part of the run-out zone. (*Photograph by R. G. Oakberg, Montana State University.*)

https://doi.org/10.3189/S0022143000010650 Published online by Cambridge University Press

4. Additional dynamics data

Direct computations were also made of the coefficient of sliding friction μ and the leading-edge velocity V in the lower part of the run-out zone from movies of the Slushman avalanche in the Bridger Range of southern Montana. This hard-slab avalanche ran in March 1978. It consisted of relatively dry, dense snow that encountered wet snow in the run-out zone. Debris movement in the areas under study showed negligible internal agitation.

Five trees along the edge of the avalanche path were used as markers (Fig. 2). The distances and slope angles between marker trees were measured in the field. Travel time for the leading edge of the avalanche was determined by identifying the movie frames when the leading edge passed the markers and estimating the lapsed time based on filming speed. Because of the parallax problem when viewing the film, several observers made independent estimates of when the avalanche passed the markers and an average was used to establish travel time between markers (Table VI). Velocities are based on the measured distances divided by these average travel times.

Table VI. Travel time and velocity between marker trees, Slushman avalanche

Tree number	*Station* m	*Slope distance* m	*Average slope angle* deg	*Time* s	*Velocity* m/s
1	0.00				
		128.10	19.4	8.58	14.93
2	128.10				
		76.25	19.4	5.28	14.44
3	204.35				
		42.70	16.4	4.91	8.70
4	247.05				
		24.40	18.0	2.96	8.24
5	271.45				

The reduction in velocity, starting at about tree three, is of interest. Although there is only a small reduction in slope angle, velocity drops by about 40%. On film this reduction in velocity appears to take place about where the avalanche flow becomes laterally confined in a more channeled part of the run-out zone (Fig. 2). Ordinarily, in open-channel flow a lateral constriction would be expected to produce an increase in velocity as a reflection of greater flow height. In the case of sliding motion, however, there is no velocity gradient, hence no increase in velocity when flow height increases. In addition, the constriction produces more friction, thus reducing velocity.

The coefficient of sliding friction μ can be calculated by equating the change in kinetic energy over some given reach of length s to the frictional work done within this same reach. Therefore, the difference of kinetic energy equals the sliding work or

$$\tfrac{1}{2}m(V_0^2-V_1^2) = smg(\mu\cos\beta-\sin\beta), \tag{5}$$

where V_0 is the velocity at the beginning of the control reach, V_1 is the velocity at the end of the reach, and m is mass. Other terms were defined in Section 4.1. Solution of Equation (5) for the friction coefficient gives

$$\mu = \frac{V_0^2-V_1^2}{2gs\cos\beta}+\tan\beta, \tag{6}$$

over three measured reaches (1–2 to 2–3; 2–3 to 3–4; and 3–4 to 4–5), μ equals 0.35, 0.43, and 0.32 respectively (Table VII).

https://doi.org/10.3189/S0022143000010650 Published online by Cambridge University Press

TABLE VII. CHANGES IN VELOCITY AND COEFFICIENTS OF FRICTION, SLUSHMAN AVALANCHE

Mid-point of the interval	*Station* m	*Velocity* V m/s	*Slope angle* β deg	*Slope distance* s m	*Friction coefficient* μ
1–2	64.05	14.93			
			19.1	102.18	0.35
2–3	166.23	14.44			
			17.4	59.47	0.43
3–4	225.70	8.70			
			16.9	33.35	0.32
4–5	259.25	8.24			

These friction values are close to those estimated by the Schaerer convention ([1975]) o assuming $\mu = 5/V$ for $10\text{ m/s} < V < 50\text{ m/s}$. For example $5/14.93 = 0.335$ and $5/14.44 = 0.346$. These two also are close to the assumed value of 0.3 used by LaChapelle and Lang (1980) in their analysis of this same avalanche.

5. CONCLUSIONS

With the proper selection of friction coefficients, program AVALNCH gives good estimates of run-out distance for a wide variety of snow and terrain conditions. There were not enough data to get a good evaluation of the program's ability to predict leading-edge velocity and flow height in the track, or debris distribution in the run-out zone. There appears to be a tendency, however, to under-predict flow heights, debris dispersal, and run-out distance in cases of steep, adverse grade in the run-out zone. The failure to predict proper debris dispersal may be due, in part, to snow entrainment during flow, which program AVALNCH can handle provided sufficient data are available. The rather narrow range of values for the friction coefficients that suffice for most snow conditions, together with the wide range possible to describe unusual conditions combined with the simplicity of the input data, make the program easy to use and invites experimentation.

Voellmy turbulence coefficients ξ of 1 500 m/s^2 or greater, and kinetic friction μ values of 0.15 or lower, were needed to duplicate some of the field data. If μ in the run-out zone is set at 0.2 to 0.25, as is common practice with some workers, even higher ξ values are needed to match observed conditions. These are much higher ξ values than were suggested by Voellmy (1955), but are close to those given by Schaerer ([1975]) for flow over a deep, dense snow cover. The extreme sensitivity of the run-out equation to kinetic friction μ and slope angle β specifically when μ approaches tan β, greatly limits the usefulness of this equation.

The lack of unique solutions for velocity and run-out distance is a major shortcoming of both the Voellmy approach and program AVALNCH. Both techniques also require experience and judgment in the selection of proper coefficients for the conditions encountered. Additional case studies are badly needed to improve objectivity.

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