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Swarming bubbles stir and spread

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A Lagrangian description of bubble swarms has largely eluded both experimental and numerical efforts. Now, in a *tour de force* of deep-learning-enabled optical tracking measurements, Huang *et al.* (2025 *J. Fluid. Mech.* **1014**, R1) have managed to follow the three-dimensional trajectories of 10^5 deforming and overlapping bubbles within a swarm, perhaps for long enough to witness their approach to the diffusive limit. Their results reveal that bubble swarms exhibit a dispersion law strikingly reminiscent of classical Taylor dispersion in isotropic turbulence, but with an earlier, undulatory transition from the ballistic-to-diffusive regime. Huang *et al.* (2025 *J. Fluid Mech.* **1014,** R1), have helped close the loop on our understanding of Lagrangian bubble dispersion – from self-stirring swarms to bubbles in isotropic turbulence.

Key words: turbulent mixing, multiphase and particle-laden flows, isotropic turbulence

1. Introduction

Fluid flows are rarely without dispersed buoyant elements, ranging from rising bubbles to settling droplets and heavy particles. Among these dispersed buoyant bodies, the bubbles are incredibly effective at transporting gases, nutrients and heat – from aeration tanks where bubbly plumes help mix reactants, to the upper ocean where the bubbles redistribute dissolved gases and nutrients (Lohse 2018; Ni 2024; Legendre & Zenit 2025). As they ascend through the liquid, their wake-induced periodic zig-zags and spiralling paths stir up the liquid neighbourhood (Magnaudet & Eames 2000; Ern *et al.* 2012; Mathai *et al.* 2015; Loisy & Naso 2017). When a swarm of such bubbles rise, what ensues is vigorous, turbulent mixing of the liquid (Risso 2017; Lohse 2018; Mathai, Lohse & Sun 2020; Ni 2024; Legendre & Zenit 2025), often referred to as bubble-induced turbulence (BIT) or pseudo-turbulence (Mazzitelli & Lohse 2009; Innocenti *et al.* 2021; Pandey, Mitra & Perlekar 2023). Bubble-induced turbulence is remarkably efficient at transporting momentum, heat and dissolved species, often rivalling even the mixing rates achieved in classical turbulence at a comparable energy input (Alméras *et al.* 2015; Alméras *et al.* 2019; Wang, Mathai & Sun 2019).

Transport and mixing by turbulent fields is typically quantified from an Eulerian viewpoint, and given a long enough time approach Fickian behaviour (Taylor 1915; Richardson 1926; Batchelor 1949). However, a Lagrangian perspective – of following fluid parcels and particles – can help us see further into the mechanisms that underlie this emergent diffusive law. For fluid tracers in isotropic turbulence, Taylor (1922) showed that the mean-squared displacement (MSD) is given by

$$\sigma^2(\varDelta_\tau x) \approx \begin{cases} \sigma^2(u_f)\,\tau^2, & \text{for } \tau \ll T_{L,u_f} \text{ (ballistic)}, \\ 2\,\sigma^2(u_f)\,T_{L,u_f}\,\tau, & \text{for } \tau \gg T_{L,u_f} \text{ (diffusive)}, \end{cases} \tag{1.1}$$

where $\sigma^2(\varDelta_\tau x) \equiv \langle (\varDelta_\tau x - \langle \varDelta_\tau x \rangle)^2 \rangle$. Here, $\varDelta_\tau x$ is the displacement along x direction during a time lag τ, $\langle \cdot \rangle$ denotes an average over trajectories and T_{L,u_f} is the Lagrangian integral time scale of the flow. Further, $\boldsymbol{u}_f$ is the fluid velocity, and the subscript f denotes fluid (liquid) phase quantities, while a subscript b will be reserved for bubble quantities to be discussed later. In essence, for fluid turbulence the correlated movements persist for a finite interval of time, $\tau \ll T_{L,u_f}$; however, for longer interval spans comparable to T_{L,u_f}, the correlations reduce exponentially until giving rise to a ballistic-to-diffusive transition for $\tau \gg T_{L,u_f}$, with a turbulent diffusion coefficient $\approx 2\,\sigma^2(u_f)\,T_{L,u_f}$.

Taylor's analysis provided an elegant, explicit link between Lagrangian dispersion and Eulerian concentration fields. (Note that the discussion here concerns turbulent dispersion (Taylor 1922), which is distinct from the Taylor–Aris dispersion in pipe or channel flows (Taylor 1953)). One could go a step further and ask how the turbulence disperses a vertically drifting particle. For heavy particles that sink through atmospheric turbulence, Csanady (1963) provided us with the answer: a reduced diffusivity, owing to the effect of crossing trajectories (Yudine 1959; Sabban & van Hout 2011). In contrast, the dispersion of buoyant bubbles has remained largely unexplored. For isolated millimetric bubbles in nearly homogeneous and isotropic turbulence (HIT), Mathai *et al.* (2018) provided the first measurements, revealing a ballistic-to-diffusive transition occurring at a much earlier time compared with the liquid. But how does a rising bubble disperse in still liquid? And for a bubble within a bubble swarm (BIT), what kind of dispersion behaviour should we expect? These are precisely what Huang *et al.* (2025) have set forth to study, by exploring the dispersion of an isolated bubble in quiescent fluid and then comparing it with the dispersion of bubbles within BIT (figure 1*a*).

2. Overview

The prospect of studying the Lagrangian properties of buoyant bubbles is not without its challenges. We are faced with the fact that a rising millimetric bubble sweeps far too quickly through the measurement domain before meaningful long-time statistics can accumulate (figure 1*a*). Mathai *et al.* (2018) averted this problem by introducing a downward counter-flow, essentially freezing a few rising bubbles to the laboratory frame for extended durations (figure 1*b*). But if those bubbles sit within a dense bubble swarm (BIT), tracking their movements is in itself a formidable task. Huang *et al.* (2025) have overcome this challenge by employing a deep-learning-enabled three-dimensional Lagrangian bubble tracking technique (Hessenkemper *et al.* 2024). What they achieve is an impressive feat – tracking 10^5 individual bubble trajectories and fluid tracers in BIT at void fractions (α) up to 1.6 % (Huang *et al.* 2025; Ma *et al.* 2025). They then contrasted the dispersion within the swarm (solid lines in figure 2*b*) against two reference

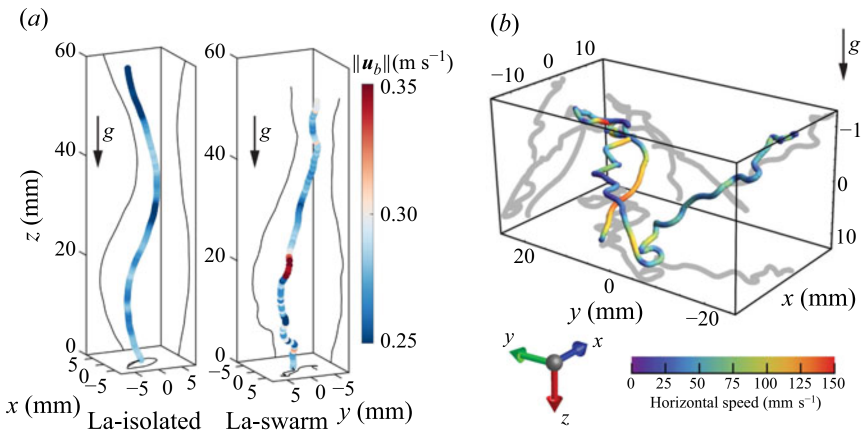

Figure 1. Trajectories of rising bubbles in three different experimental configurations: (*a*) a bubble rising in quiescent liquid (*left*) and in a bubble swarm (*right*) at $\alpha = 1.2\,\%$. (*b*) An isolated bubble rising through nearly homogeneous isotropic turbulence. The trajectories are coloured by bubble velocity magnitude in (*a*). Figures and data in (*a*) and (*b*) were adapted from Mathai *et al.* (2018) and Huang *et al.* (2025), respectively. Here La refers to the larger bubbles and $\boldsymbol{u}_b$ is the instantaneous rise velocity of the bubble.

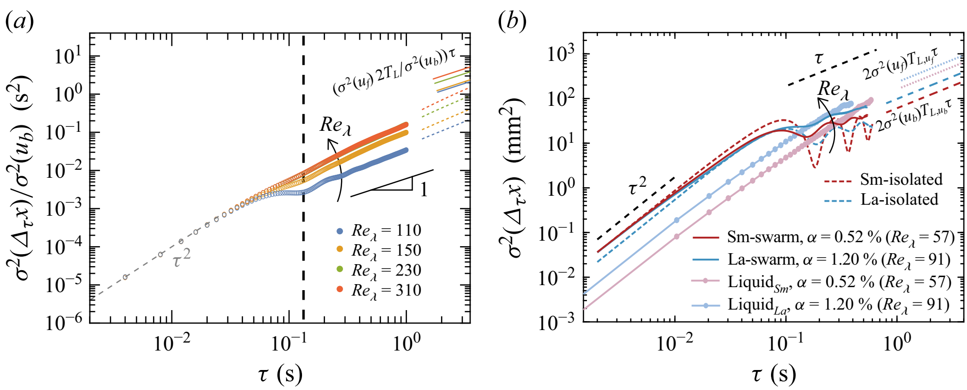

Figure 2. Mean-squared displacement in the horizontal direction for bubbles in HIT (*a*) and that of bubbles and liquid (tracers) in BIT (*b*), as a function of time lag, τ. In (*a*) the MSD is normalised by the standard deviation of the bubble velocity, which collapses the short-time ballistic part. Figures adapted from Mathai *et al.* (2018) and Huang *et al.* (2025). Here Sm refers to the smaller bubbles and Re_λ is the Taylor Reynolds number.

cases: (i) dispersion of the isolated bubbles of diameter, $d_b = 3.5$–4.4 mm, rising in quiescent fluid (dashed lines in figure 2*b*), and (ii) the dispersion of fluid tracers within the bubble swarms (faint solid lines in figure 2*b*).

The central message of Mathai *et al.* (2018) and Huang *et al.* (2025) is clear and resounding: turbulence, whether it is BIT or HIT, progressively blurs out the memory of the path oscillations of rising bubbles (see figure 2*a* and *b*). Vertical MSD of the bubble within a swarm is noticeably larger than it is for an isolated rising bubble, something the authors attribute to the randomising kicks from the BIT (compare figure 1*a* left and right). For the more well mixed of the two swarms that they study ($\alpha = 1.2\,\%$), tracer dispersion transitions away from the ballistic regime much later than the bubbles do, consistent with the deductions of Mathai *et al.* (2018). Tracers disperse slower than the bubbles in the short term and faster in the long run. One might rationalise this: at very short times, the zig-zagging bubble will outpace the fluid tracer, leading to the bubble's faster initial spreading. However, at longer times, the tracers continue to be carried by the correlated turbulent eddies movements; so they catch up and overtake the bubble dispersion. For bubbles, besides the crossing trajectory effect, there is the time scale of inter-bubble passage, $T_{2b} = d_b/(\alpha\ \langle u_b \rangle)$, that further suppresses the long-time dispersion (Alméras *et al.* 2015). Thus, a spherical blob of liquid within the swarm would eventually spread (and mix) much more quickly than a similarly sized cloud of bubbles.

3. Outlook

Could the similarities in the findings of Mathai *et al.* (2018) and Huang *et al.* (2025) hint at a unified picture of Lagrangian dispersion for BIT and HIT? Extended explorations varying the void fraction, bubble Galileo number and the background turbulence (Re_λ) might provide an answer. More generally, the dispersion behaviour of bubble swarms may share some similarities to systems where turbulent fluid motion is induced by vertically drifting (rising or sinking) particles – from rain and hail in the atmosphere to settling sediments and marine snow in the oceans. Surely the bubbly swarms are different from their heavier counterparts in many ways – their path instabilities (Ern *et al.* 2012), their sensitivity to surface conditions and their deformable interfaces. Yet, it is conceivable that some of the differences would fade once the path oscillations are suppressed by the BIT.

We have only begun to scratch the surface of Lagrangian bubbly turbulence. Impressive as they are, the experiments of Huang *et al.* (2025) do not quite reach the asymptotic diffusive limit. As new and improved tools continue to emerge (Hessenkemper *et al.* 2024; Wang *et al.* 2025), some of the experimental barriers are likely to be lifted and longer bubble trajectories will become accessible. It is hoped that this excitement will also spur a wave of fully resolved numerical explorations probing dispersion in bubbly flows. An inviting challenge for the field would be to capture the dispersion laws of bubble swarms – from dilute to dense swarms and for low to high Reynolds number bubbles.

Declaration of interests. The author reports no conflict of interest.

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