

Differentiable manifolds modelled on locally convex spaces

Truong Công Nghệ

We construct differentiable manifolds modelled on locally convex spaces using Yamamuro's theory of Γ -differentiation [4], [5], [6], manifolds which we term as Γ -manifolds and $B\Gamma$ -manifolds. We show how to extend to the $B\Gamma$ -manifolds the standard properties of Banach manifolds: the Smale Density Theorem [1] as well as the Transversality Theory [1], [2], [3]. As first applications, we give several simple results about genericity of smooth maps using our Γ -technique instead of the usual standard Banach techniques.

The thesis is divided into five chapters. In the first chapter we prove two local results on Γ -differentiation, namely the Γ -omega lemma and the $B\Gamma$ -differentiability of the evaluation map.

Chapter 2 is devoted to definitions and examples of Γ - and $B\Gamma$ -manifolds as well as Γ - and $B\Gamma$ -bundles. We prove that the space $C^\infty(X, Y)$ of C^∞ maps from a compact C^∞ -manifold X into a finite-dimensional C^∞ manifold Y is a Γ -manifold of class C_Γ^∞ . Hence the spaces $\text{diff}^\infty(X)$ and $\text{emb}^\infty(X, Y)$ introduced in [2], [3] are C_Γ^∞ -manifolds. We also give several simple examples of $B\Gamma$ -manifolds.

From Chapter 3 onward, we restrict our attention to the subclass of $B\Gamma$ -manifolds and use the full strength of the Inverse Mapping Theorem [5], [6]. Chapter 3 contains a generalisation of the Smale Density Theorem [1] to $B\Gamma$ -manifolds followed by a brief discussion of the notion of $B\Gamma$ -maps between Γ -manifolds. This notion cannot be defined in any natural

Received 29 September 1977. Thesis submitted to the Australian National University, September 1977. Degree approved, February 1978. Supervisor: Dr Sadayuki Yamamuro.

fashion; however, it yields many interesting results.

The standard transversality theory [1], [2], [3] is generalised to the $B\Gamma$ -context in Chapter 4 under the name of $B\Gamma$ -transversality. We show that all the standard transversal theorems [1] remain valid: the $B\Gamma$ -Transversal Density Theorem and the $B\Gamma$ -Transversal Isotopy Theorem.

Some applications of our $B\Gamma$ -Transversal Density Theorem appear in Chapter 5 where we give simple "generic" results for local smooth maps which parallel the usual ones [2], [3].

References

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