REVIEWS

Methoden der Unternehmensforschung im Versicherungswesen. KARL H. WOLFF, Springer-Verlag, Berlin Heidelberg New York, 1966.

This book appeared as volume IV of "Econometrics and Operations Research". It treats some important techniques of operations research in connection with actuarial subjects, all of them belonging to the field of private insurance. As appears from the preface the author's intention was to collect and treat in a coherent way the results of himself and others in this field.

Operations research is a new topic; the special subject of applications to insurance problems is certainly quite new. These investigations date back only a decade. The situation with regard to the, in the actuarial practice, already accepted procedures is still different; at this moment such procedures do hardly exist. To speak of "Anwendungen", as the author of this book does, therefore seems to be slightly optimistic and for the general reader to a certain degree misleading. The situation is rather paradoxical; instead of leading mainly to practical methods, as should be expected from this branch of applied mathematics, operations research here often means "fundamental research". This applies above all to the general insurer's problem to judge and to choose a favourable "risk situation" and is illustrated in this book in particular by the profound and, from a mathematical point of view, beautiful treatment of reinsurance treaties, using utility functions to value the inherent risks.

So far these, perhaps somewhat skeptical, remarks. The actuary who is interested in operations research as such and moreover wants to become acquainted with the new, very interesting studies made in this field by some brilliant actuaries, finds here an excellent, clear introduction to the techniques and a, from a new angle, thorough theoretical treatment of a number of important insurance problems.

"Operations research" is a collective noun for a group of rather different techniques. The methods which play a role in this book are game theory, Monte-Carlo (simulation) techniques, linear programming and optimalisation of utility functions. The book is divided, according to the actuarial problems dealt with, into 6 parts. They are:

Part 1. The determination of actuarial tables and present value factors.
Part 2. The estimation of the mathematical reserve.
Part 3. Interest rate and bonus.
Part 4. Operations research and reinsurance.
Part 5. Collective risk theory and maximum utility.
Part 6. Appendix.

To give the reader a better idea of the contents of this book some additional information follows here on the parts and chapters separately.

Part I. The first chapter introduces game theory (two persons zero sum game) in relation with the problem of determining a mortality table as basis for the reserve calculation. As we may suppose that nature is not interested in making a profit, nature and the actuary are not equivalent partners and therefore the question to be answered has to be slightly modified. Nature may select arbitrarily from a set of admissible tables; then the aim of the actuary is to minimize the possible loss of the company which results from not choosing the "true" mortality table. The solution appears to be rather trivial and depends in a large degree on the assumptions made as regards the loss-function.

This chapter gives a very readable first introduction to game theory. The example of the mortality table illustrates the theory and is of some theoretical interest. However, as the assumptions and remarks made on the loss function are of a highly speculative nature the practical meaning of this approach seems to be rather doubtful.

The third chapter—the title does not mention it explicitly—deals with Monte-Carlo techniques. Simulation methods may be used for estimating present value factors, premiums, etc. The question which technique should be preferred is discussed in detail. From a statistical point of view the method which leads to an estimate with minimum variance is the best one; however "computer-economy" plays a role too and we have to make a compromise. Another and probably more important field of applications is the simulation of claim costs distributions and—in relation with reinsurance—the estimation of probabilities, mean, variance and functions of higher moments of the excess of loss. Here also we may try to do things in an economic way, for instance, by trying to reduce the number of variables the computer should simulate. If the number of contracts, insured persons, is large it is perhaps wise to group the data, e.g. according to insured amounts and age. In connection with this grouping of the data the analogue of a well known theorem on stratified sampling is proved: approximately optimal results are obtained, i.e. estimates with minimum variance, if the number of simulations for each group is proportional to the extent of the group.

This chapter on random variables simulation techniques for actuarial ends is written in an instructive way. As clearly this subject is of practical value the author might also have given some numerical information, for instance as regards computer time needed to obtain reliable estimates for stop-loss reinsurance premiums for certain portfolios etc. In this connection the reviewer wishes to draw once more the reader's attention to a recent study, submitted to the ASTIN-colloquium, held at Arnhem in 1966, by Franckx, d'Hooge and Gennart "Utilisation pratique de la méthode de simulation dans l'assurance "non life" ".

Part II, the estimation of reserves, has as subject linear programming in relation with, partly well known, approximative methods for calculating the mathematical reserve. Often the calculation technique implies the grouping of the insured sums to certain characteristics, such as the age of the insured and the number of future premium payments etc. in combination with the introduction in advance, of certain magnitudes, among which the premiums, depending on the same characteristics. Multiplying the grouped, non negative, insured sums with these magnitudes and adding the products there result a few linear relations of which the values are known in advance. As the reserve is also a linear relation in these insured sums, we may apply linear programming to obtain an interval $(\underline{V}, \bar{V})$ of possible values for the reserve. Of course linear programming gives no new information in case the reserve is calculated accurately. If however the method implies additional linear estimates, for instance for the mean of certain age variables (t-method of Jecklin) we obtain a point-estimate for the reserve too. Taking into account all linear relations and estimates of which the values are available, we may determine ($\underline{V}$, $\overline{V}$) in order to get an idea of the possible range of the calculated estimate of the reserve, V^*. As Wolff remarks there is, at least theoretically, no guarantee that V^* lies in ($\underline{V}$, $\overline{V}$); in which situation we may perhaps say that V^* is "inconsistent". If V^* is a consistent estimate, $\sigma = \frac{\underline{V} + \overline{V} - 2V^*}{2V^*}$ may be interpreted as a measure for the reliability of the actual applied calculation technique. Different methods lead of course to different values of σ, it appears that the interpolation method of W. Pöttker gives the most accurate results. The question as whether the method gives a too low or too high value is not answered as yet. This problem is attacked in the last chapter. The approach is the same as used for the choice of the mortality table in part I. Game theoretical considerations in relation with the application of linear programming in the manner sketched, lead to a method to minimize the possible loss. In practice the weak point is again the difficulty to determine the loss-function A (V, V^*), V^* the calculated and V the real reserve.

This part of the book gives an excellent description of the linear programming technique and gives, from a theoretical point of view, an interesting, new look, on the problem of reserve calculations.

The next part deals with questions of interest rate, dividend and bonus policy and is based largely on a publication of Benjamin. The problems are tackled again with game theory and linear programming. In the last two paragraphs the optimal finance plan is studied. Optimal here means maximum constant dividend payments. As regards the annual receipts of the company there exist a number of linear restrictions. The dividend payments, expressed as a linear relation of these receipts, can be maximized by means of L.P. It may be interesting to note that the treatment of the problem of the dividend payments here is a deterministic one. In part V, this topic is again dealt with, but now in the frame of collective risk theory where the capital flow of the insurance company is considered to be a random process.

In part IV theoretical questions around reinsurance are discussed. This section leans heavily on the articles, published in recent years by Borch. Readers familiar with his ideas find here a unified treatment. Borch uses the concept of utility function. We may consider the position of only one of a group of companies or the risk situation of the group as a whole. In general the situation is optimal if the utility of the risk situation of each of the insurers cannot be improved without worsening it for the other participants. Here we have two important theorems.

a. The stop-loss contract gives, for a rather general class of reinsurance contracts—i.e. transformations of the claim distributions—and for a given fixed net reinsurance premium, minimum variance for the remaining insured amount. In the—of course highly unrealistic—situation that the insurer has only to pay the net premium, we may assume that the risk situation of the insurer is most favourable (optimal). At this stage we strictly don't need intricate utility function concepts.

b. The other theorem, which essentially relates to a closed group of in the reinsurance market participating companies, says that, if there exist certain linear relations between the utility function values of the companies, proportional reinsurance is optimal.

Wolff gives first a detailed discussion of the situation for two companies and for a special type of utility function. It is furthermore shown that in case the utility function is merely a decreasing function of the variance of the insured amount, proportional reinsurance is optimal. Between the proportion factors for the reinsured amounts then exists the simple relation $t_1 + t_2 = 1$.

The reflections given on the so called Pareto-optimum are very interesting. Let $z(x, y)$ be the amount which the company with claim costs x has to pay to the second company with costs y. The Pareto-optimum then is defined as the set of functions z for which there is no function z which improves at least the risk situation of one of the companies without worsening it for the other participating companies.

It appears that in case the safety loading of the reinsurance premium is proportional to the standard deviation of the reinsured amount, the reinsurance treaty, apart from some isolated points, cannot belong to the Pareto-optimum. As it seems reasonable to introduce such a safety loading it appears that the conditions of the reinsurance market are not quite in accordance with the conditions as discussed for the optimal reinsurance risk situation. Hence, after all we may conclude that the problem of choosing the optimal policy remains a very difficult one which can only partly be solved by theoretical means, in particular, methods of operations research. In the last section of this chapter, Wolff treats more general types of utility functions and the case of more than two companies.

Part V. The in- and outflow of money of the company can be looked upon to be a random process and in consequence the capital of the company as a time dependent random variable. Essentially the random process which is studied here in detail is a superposition of

a. a, in the absence of claims, deterministic process for the development of the capital

b. a compound Poisson-process, consisting of a Poisson-process for the number of claims and a continuous distribution for the individual claim costs.

The discounted value of the dividend payments can now be considered to be a random variable too. General expressions for the distribution of this random variable are derived. The actual management problem is again a decision problem, the company has also to reckon with the probability of ruin. The introduction of utility functions yields—at least theoretically—a solution for this problem.

The final part, added as appendix, treats some rather diverse subjects such as optimal experience rating and directives for the insurance agent activity.

The information given here as regards the contents of this book has of course to be incomplete and also rather subjective.

In conclusion the reviewer thinks this book to be a valuable one as it is the first one in this field and is moreover written in a clear style. Although the methods presented are not all of immediate practical value they throw a new light on some important actuarial questions. Finally, we like to mention that the book contains a list with many useful references.

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