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LOCAL CENTRAL $\Lambda(p)$ DUAL OBJECTS

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The dual object Γ of a compact group is called a local central $\Lambda(p)$ set if there is a constant K such that $\|X\|_p < K\,\|X\|_1$ for all irreducible characters X of G. For each $\gamma \in \Gamma$, D_γ is an irreducible representation of G of dimension d_γ. Several authors [1, 2, 3, 4] have observed that Γ is a local central $\Lambda(p)$ set for $p>1$ provided $\sup\{d_\gamma : \gamma \in \Gamma\} < \infty$, and some of them [2, 3] conjectured the converse. Cecchini [1] showed that Γ is not a local central $\Lambda(4)$ set if G is a compact Lie group. Picardello [2] observed that Cecchini's result extends easily to any group G that is not totally disconnected and also showed that Γ is not a local central $\Lambda(4)$ set if G is an infinite product of non-commutative compact groups. This note completes the proof for totally disconnected groups. Hence Γ *is a local central* $\Lambda(p)$ *set for all* $p>1$ *if and only if* $\sup\{d_\gamma : \gamma\in\Gamma\}<\infty$.

Let G be a totally disconnected compact group and suppose that $\sup\{d_\gamma : \gamma \in \Gamma\} = \infty$. For any positive integer N, choose D_γ with $d_\gamma \geq N$. Let $K = \text{Ker}\, D_\gamma$ and X be an irreducible character of maximal degree $M \geq N$ of the finite group $H = G/K$. Then $X\bar{X} = \sum_{j=1}^{s} n_j\theta_j$ where the θ_j are irreducible characters of H and the n_j are positive integers. If e is the identity of H, $\theta_j(e) \leq X(e) = M$. Thus $M^2 = X\bar{X}(e) = \sum_{j=1}^{s} n_j\theta_j(e) \leq \sum_{j=1}^{s} n_j M$ and so $\|X\|_4^4 = \sum_{j=1}^{s} n_j^2 \geq \sum_{j=1}^{s} n_j \geq M \geq N$. Since X extends to an irreducible character X^* of G with the same norms,

$$\|X^*\|_4^4 \geq N \quad \text{while} \quad \|X^*\|_1 \leq \|X^*\|_2 = 1.$$

This shows that Γ is not a local central $\Lambda(4)$ set.

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