INTRODUCTION

AN OVERVIEW OF HELIO- AND ASTEROSEISMOLOGY

Jørgen Christensen-Dalsgaard
Astronomisk Institut, Aarhus Universitet
DK-8000 Aarhus C
Denmark

1. Introduction.

The purpose of the present paper is to give an introduction to the nomenclature, and a few of the results, of helio- and asteroseismology. It is hoped that this may provide a useful background for the more specialized reviews, and the contributed papers, in these proceedings. Other recent, general reviews are, *e.g.*, Deubner & Gough (1984), Leibacher *et al* (1985), and Christensen-Dalsgaard *et al* (1985a).

Section 2 gives a brief overview of the observations of solar and stellar oscillations. In Section 3 I discuss the behaviour of the oscillations, on the basis of both computations and asymptotic theory. Finally Section 4 gives a small taste of the results obtained by means of helioseismology.

2. OBSERVATION OF SOLAR AND STELLAR OSCILLATIONS.

Oscillations with small amplitudes of a spherical body can be separated into normal modes, each of which has a harmonic dependence on time, and depends on the spherical coordinates θ and φ (co-latitude and longitude) as a spherical harmonic. Thus the displacement for a single mode can be written

$$\delta \mathbf{r}(r,\theta,\varphi,t) = \mathrm{Re}\left\{\left[\xi_r(r) Y_\ell^m \mathbf{a}_r + \xi_h(r)\left[\frac{\partial Y_\ell^m}{\partial\theta}\mathbf{a}_\theta + \frac{1}{\sin\theta}\frac{\partial Y_\ell^m}{\partial\varphi}\mathbf{a}_\varphi\right]\right] e^{-i\omega t}\right\} \quad (1)$$

where $\mathbf{a}_r$, $\mathbf{a}_\theta$ and $\mathbf{a}_\varphi$ are unit vectors in the r, θ and φ directions. Here Y_ℓ^m is a spherical harmonic, and the variation of the displacement with the distance r from the centre is determined by the *eigenfunctions* $\xi_r(r)$ and $\xi_h(r)$. The mode is characterized by three wave numbers: n is the *radial order* which, roughly, gives the number of zeros in ξ_r; ℓ is the *degree*, and m, which must be between $-\ell$ and ℓ, is the *azimuthal order* of the mode.

In addition to the *angular frequency* ω, which is used in equation (1), the *cyclic frequency* $\nu = \omega/(2\pi) = 1/P$, is commonly used, particularly in discussions of observed frequencies; here P is the oscillation period.

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The degree is related to the horizontal wavenumber k_h and wavelength λ of the mode at radius r by

$$k_h = \frac{2\pi}{\lambda} = \frac{L}{r}, \tag{2}$$

where $L = \sqrt{\ell(\ell+1)}$. The special case $\ell = 0$ corresponds to *radial* oscillations, where the star expands or contracts spherically. Finally $|m|$ is twice the number of zeros around the equator.

From observations of the variation of the oscillations over the surface of the Sun, ℓ and m can in principle be determined directly. For observations of other stars, on the other hand, on is in general limited to measurements in light integrated over the disk of the star. The resulting cancellation between regions with positive and negative perturbations strongly

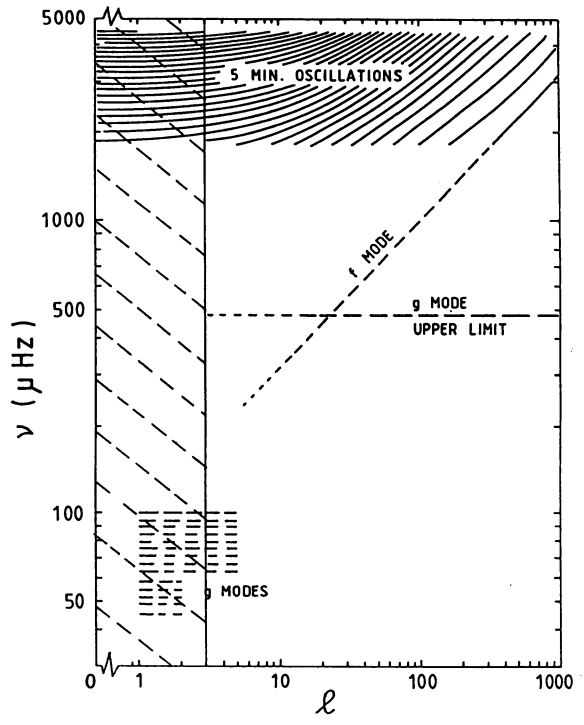

Figure 1. Schematic illustration of the oscillations observed in the Sun. The 5 min oscillations are standing acoustic waves. The *f modes*, which are essentially standing surface gravity waves, have been observed at high degree. The identity of the long period modes has not been definitely established, but they are probably standing gravity waves of low degree. The hatching indicates the region in ℓ that can be observed in light integrated over the disk, as is generally the case for stars.

reduces the sensitivity to modes with $\ell \gtrsim 4$ (*e.g.* Christensen-Dalsgaard & Gough 1982). Such observations therefore only give information about low-degree modes.

In general the frequency $\nu = \nu_{n\ell m}$ depends on all three wave numbers. However if rotation or other departures from spherical symmetry are ignored, $\nu_{n\ell m}$ does not depend on m. This follows from the fact that in this case there is no preferred axis in the star; since m depends on the choice of coordinate axis, the physics of the oscillations, and hence their frequencies, must be independent of m. For slow rotation a modal description as in equation (1) is still possible, provided that the rotation axis is chosen as coordinate axis. As discussed in Section 4, the dependence of $\nu_{n\ell m}$ on m can then be found from a perturbation analysis; rotation introduces a splitting in ν of order $m\bar{\Omega}/2\pi$, where $\bar{\Omega}$ is an average of the angular rotation frequency.

Figure 1 gives a schematic illustration, neglecting rotation, of the observed modes of solar oscillation in an ℓ-ν diagram, where ν is plotted as a function of ℓ for each value of n. The *5 min oscillations* have frequencies between about 2 and 4 mHz, and extend in ℓ from 0 to about 1000, where the observations are restricted by seeing in the Earth's atmosphere. They have been studied with a variety of techniques; the most detailed observations have so far been made in Doppler velocity (*e.g.* Deubner, Ulrich & Rhodes 1979; Claverie *et al* 1979; Grec, Fossat & Pomerantz 1980; Duvall & Harvey 1983), but they have also been seen in irradiance measurements from the SMM satellite (Woodard & Hudson 1983). These oscillations are identified with standing acoustic waves, or *p modes*, of high radial order or degree. At intermediate frequencies, between 0.1 and 2 mHz, there have been reports, although not substantiated by other observing techniques, of oscillations observed in the solar limb intensity (*e.g.* Hill & Caudell 1985). At even lower frequencies, corresponding to periods of more than 2 hours, there have been observations of a number of modes, both in Doppler velocity (*e.g.* Severny, Kotov & Tsap 1976; Brookes, Isaak & van der Raay 1976; Scherrer & Wilcox 1983) and, somewhat less directly, in intensity (*e.g.* Fröhlich & Delache 1984). Among these, the 160 min oscillation has been studied since 1974, and may have maintained phase throughout this period (*cf.* Henning & Scherrer, these proceedings; Severny, Kotov & Tsap, *ibid.*). The oscillations have been seen in integrated light; thus if they are linear modes of the Sun, their degrees are low. Furthermore oscillations of the Sun with periods in excess of about 100 min must be standing gravity waves, or *g modes* (Christensen-Dalsgaard, Cooper & Gough 1983). However a definite identification of the oscillations has not been possible.

The 5 min oscillations are the only modes where determination of n and ℓ has been made. Consequently helioseismology has up to now principally been based on these modes. A characteristic feature is that their distribution of power as a function of frequency, and the average amplitude per mode, is largely independent of ℓ (Libbrecht *et al* 1986). The distribution for low-degree modes is illustrated by the observed power spectrum shown on Figure 2. The maximum velocity amplitude for a single mode is about 15 cm/sec, corresponding to a amplitude in relative intensity of about 10^{-6}. The mode *lifetimes* appear to be strongly dependent on frequency (Grec *et al* 1980; Isaak 1986; Libbrecht & Zirin 1986), varying from about a day at high frequency (as directly visible in the width of the peaks on Figure 2) to

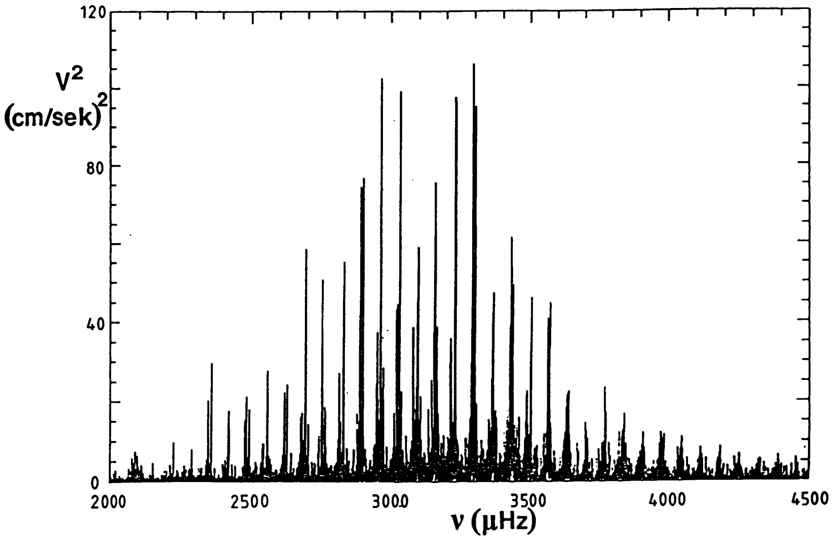

Figure 2. Power spectrum of solar oscillations, obtained from Doppler observations in light integrated over the disk of the Sun. The ordinate is normalized to show velocity power per frequency bin. The data were obtained from two observing stations, on Hawaii and Tenerife, and span 53 days. (See Claverie *et al* 1984).

possibly several months at the lowest frequencies observed. As is also visible on the figure, at low degrees the peaks are approximately uniformly distributed in frequency. This is also predicted by asymptotic theory (*cf.* equation (9) below), and can be used as a signature in the search for solar-like stellar oscillations.

The distribution of power at higher degrees is illustrated on Figure 3. This shows the ridge structure of the oscillations, each ridge corresponding to a definite value of the radial order n. In the original data the ridges can be followed down to $\ell = 1$, thus connecting up with the observations shown on Figure 2. The ridges continue to degrees as high as about 1000 (*e.g.* Deubner *et al* 1979) where the order can be determined directly. This has enabled the identification of the radial order for all observed 5 min modes.

Given the mode identification, we can make a direct comparison between the observed frequencies, and the frequencies of the corresponding modes of solar models. Such a comparison has been made on Figure 3, by superposing the computed frequencies of Model 1 of Christensen-Dalsgaard (1982) (other, socalled *standard models*, give very similar results at this level of precision). Superficially there is good agreement between observations and theory. It might be mentioned that the model predates the observations, so that no attempt has been made to adjust the model parameters to fit the data. Thus the agreement supports the mode identification, and indicates that there are probably no gross errors in the model. A more careful comparison, however,

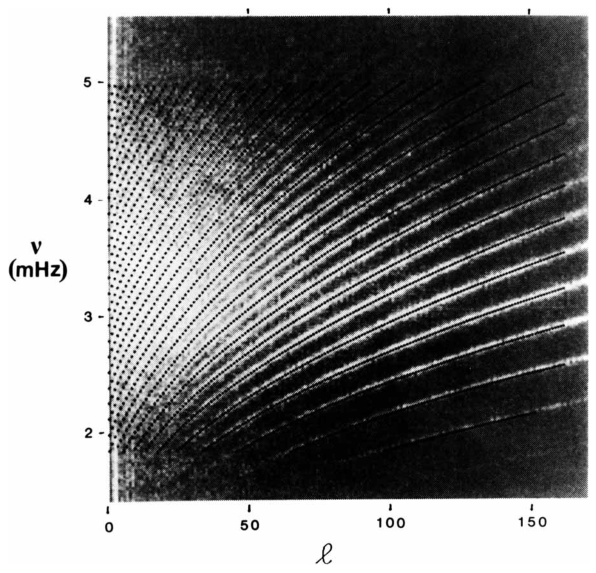

Figure 3. Observed spectrum of solar oscillations, as a function of the degree ℓ and the frequency ν (from Duvall & Harvey 1983). The light ridges are regions of high power, each corresponding to a definite value of the radial order n. Also shown, with the dots, are computed frequencies of a solar model.

reveals differences that far exceed the internal accuracy of the observed frequencies, showing that modifications are required in the model (*e.g.* Christensen-Dalsgaard & Gough 1984).

Observation of *stellar* oscillations at the very low amplitude level seen on the Sun is at present at the limit of feasibility (see Harvey, these proceedings). Possible detections of solar-like oscillations have been made in ε Eri (Noyes *et al* 1984), α Cen A and Procyon (Gelly, Grec & Fossat 1986). In each case indications were found of the pattern of approximately uniformly spaced peaks seen in the solar case on Figure 2. The interpretation of the observations is somewhat problematic, however (*cf.* Däppen, Dziembowski & Sienkiewicz, these proceedings).

Other types of stars also show oscillations with complicated spectra. Thus a number of Ap stars are known to pulsate in several modes (Kurtz 1986). As for the solar 5 min oscillations, these are high-order p modes, and in a few cases the same uniform frequency distribution has been observed. However the amplitudes observed for the Ap stars are approximately three orders of magnitude higher than for the Sun; furthermore there is strong evidence that the excitation of the oscillations is linked to the large-scale magnetic field in these stars. Complex spectra of oscillations have also been detected in white dwarfs (*cf.* Winget, these proceedings). Their periods are also around 5 - 10 min, which, at the very high mean density of white dwarfs, corresponds to g modes of fairly high radial order. The destabilization of the modes in these stars might be controlled by composition discontinuities, caused by gravitational settling. Thus information about the structure of the stars may be contained in the selection of modes which are observed. - Further classes of stars pulsating with low amplitudes will undoubtedly be detected, as the observing techniques are improved, and the extensive survey programmes required to detect such oscillations are undertaken.

The principal problems facing observations of solar and stellar oscillations from the surface of the Earth are the nighttime (or daytime) gaps and the atmospheric noise. The frequency resolution in a given time string is roughly $1/T$, where T is the duration of the observation (*e.g.* Loumos & Deeming 1978). To obtain adequate frequency resolution, observations over many days are required. If such observations are obtained from a single site at intermediate latitudes, gaps in the data are unavoidable; these cause sidebands, which greatly complicate the interpretation of the observations. The gaps can be avoided, in the solar case, by observing from the South Pole during Austral Summer (Grec *et al* 1980; Duvall, Harvey & Pomerantz 1986). Stellar observations from the South Pole during the Winter might be possible, although obviously difficult, but such observations have so far not been attempted. Networks of observing stations are now being set up to allow more extended time series of solar oscillation observations. Limited sets of observations from two sites have been obtained for pulsating Ap stars (Matthews, Kurtz & Wehlau, these proceedings), and more extensive networks for stellar observations are being planned.

Fluctuations in the Earth's atmosphere add noise to the data. This is particularly troublesome at long periods, and may restrict the ability to study g modes from the ground. Also, due to seeing distortion, accurate observations of modes with degree higher than 200 are difficult or impossible.

Space observations (*cf.* Noyes, these proceedings) would eliminate the problems caused by the Earth's atmosphere; with a proper choice of orbit uninterrupted observations are also possible. The satellite SOHO will carry instruments to study solar oscillations to the L_1 point. This will enable the detection of modes with very low amplitudes, including, hopefully, long period g modes, and detailed observations of high-degree modes, which will allow the study of the structure and dynamics close to the solar surface.

3. PROPERTIES OF THE OSCILLATIONS

In contrast to other observed dynamic phenomena on the Sun or other stars, the oscillations are related in a rather straightforward way to the properties of the stellar interiors. This is due to the fact that the oscillations have small amplitudes, and therefore can be treated in linear theory;[1] also the effects of heat loss or gain on the oscillations are negligible except very

[1]Even for normal pulsating stars, like Cepheids, the amplitude is rather low in the parts of the star where the period is determined, and so the period is obtained reasonably accurately from a linear calculation. For stars like the Sun, where the typical relative displacement of the surface in each mode is only of order 10^{-6} to 10^{-8}, linear theory is an excellent approximation.

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near the surface, and so the oscillations can be assumed to be adiabatic with adequate precision.

3.1 Numerical results.

Based on the remarks above, most of the extensive calculations of solar and stellar oscillation frequencies have used the following simplifying assumptions:

1) The oscillations are linear.
2) The oscillations are adiabatic.
3) Rotational distortion of the star can be neglected.
4) Effects of magnetic fields can be neglected.
5) The dynamical effects of convection (the "turbulent pressure") can be neglected.

Under these assumptions, the computation is relatively straightforward. The assumptions can be questioned, however. In particular, calculations have been made that included effects of non-adiabaticity, with a small but non-negligible effect on the frequencies (Christensen-Dalsgaard & Frandsen 1983; Kidman & Cox 1984). Nevertheless the assumptions form a convenient base-line for the study of the complicating effects. Obviously any deviations between the observed and the computed frequencies must lead to reconsideration of the assumptions.

Given a solar model and the standard assumptions, the physics of the oscillations is completely determined. The computation of the frequencies and eigenfunctions reduces to the solution of a boundary value problem for a set of ordinary differential equations. The main difficulties are the number of modes that must be calculated, and the high precision required to match the accuracy of the observed frequencies. Several studies have shown, however, that adequate precision can be achieved.

Computed frequencies for a model of the present Sun are shown on Figure 4. The modes obviously fall in two classes. Those labelled *p modes* have frequencies that increase roughly as $\ell^{1/2}$ for high ℓ. For these modes the main restoring force is pressure, and hence they are predominantly standing acoustic waves. They are responsible for the 5 min oscillations. Towards high frequency they are limited by the *acoustical cut-off frequency* ν_{ac} (*e.g.* Lamb 1909) where the atmosphere ceases to reflect the oscillations. For the Sun ν_{ac} is about 5 mHz. The modes labelled *g modes* have frequencies that tend towards a constant limit at large ℓ. This limit is given by the maximum in the solar interior of the buoyancy frequency N, with

$$N^2 = g\left[\frac{1}{\Gamma_1}\frac{d\ln p}{dr} - \frac{d\ln \rho}{dr}\right], \tag{3}$$

where g is gravity, p pressure, ρ density, and $\Gamma_1 = (\partial\ln p/\partial\ln\rho)_{ad}$. The main restoring force for these modes is buoyancy, and they are predominantly standing gravity waves. They extend to very low frequencies. The *f modes* are generally intermediate in frequency between the p and g modes. At high ℓ their frequency approaches that of a surface gravity wave,

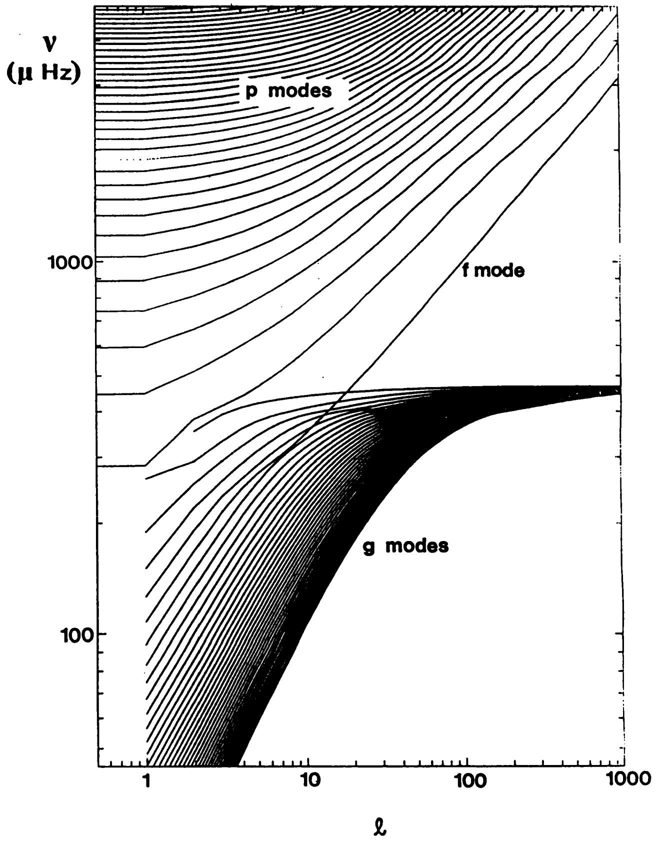

Figure 4. Adiabatic oscillation frequencies for a normal model of the present Sun, as functions of the degree ℓ. For clarity points corresponding to modes with a given radial order have been connected by straight lines. Only g modes with radial order less than 40 have been included.

$$\nu \approx \frac{1}{2\pi}\sqrt{g_s k_h} = \frac{1}{2\pi}\left[\frac{g_s L}{R}\right]^{1/2}, \tag{4}$$

where g_s is surface gravity and R is the radius of the star.

3.2 Asymptotics of the oscillations.

As an aid to understanding the results of the numerical calculations, and to interpret the observations, asymptotic theory has been very useful. This is mainly due to the fact that the oscillations observed on the Sun have high radial order or degree. The p modes can be approximated locally by plane sound waves, with the dispersion relation $k^2 \equiv k_r^2 + k_h^2 = \omega^2/c^2$. Here k_r and k_h are the radial and horizontal components of the wave vector, and c is the adiabatic sound speed. For a mode of oscillation, k_h is given by equation (2). We then obtain

$$k_r^2 = \frac{\omega^2}{c^2} - \frac{L^2}{r^2} \qquad (5)$$

Close to the surface, c is small and hence k_r is large. Here the modes propagate almost vertically. With increasing depth, c increases and k_r decreases (see Figure 5), until the point is reached where $k_r = 0$ and the wave propagates horizontally. The location $r = r_t$ of this *turning point* is determined by

$$\frac{c(r_t)}{r_t} = \frac{\omega}{L} \qquad (6)$$

It corresponds to a point of total internal reflection; for $r < r_t$, $k_r^2 < 0$, and the mode decays exponentially. At the surface the wave is reflected (provided $\omega < \omega_{ac} = 2\pi\nu_{ac}$) by the steep density gradient. Thus the wave propagates in a series of "bounces" between the surface and the turning point. A mode of oscillation is a standing wave, formed as an interference pattern between such bouncing waves.

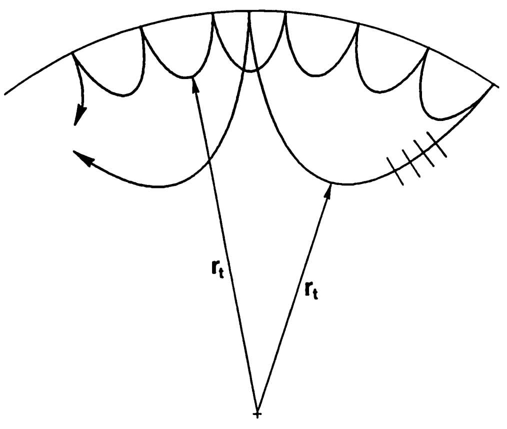

Figure 5. Schematic illustration of the propagation of sound waves in a star. Due to the increase of the sound speed with depth, the deeper parts of the wave fronts move faster. This causes the refraction of the wave described by equation (5). Notice that waves with a smaller wavelength, corresponding to a higher value of the degree ℓ, penetrate less deeply.

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From equation (6) one may calculate r_t as a function of ℓ at given frequency. As shown on Figure 6, r_t increases with increasing ℓ. Modes with highest values of ℓ observed are confined to the outermost fraction of a percent of the solar radius.

The ray description of the p modes may be extended to give an asymptotic dispersion relation for their frequencies (Gough 1984)

$$\int_{r_t}^{R} \left[1 - \left[\frac{Lc}{2\pi\nu r} \right]^2 \right]^{1/2} \frac{dr}{c} \approx \frac{n + \alpha}{2\nu} \tag{7}$$

where α is a quantity (which in general is a function of ν but not ℓ) that depends on conditions near the surface. As r_t is a function of $2\pi\nu/L$, this equation may be written as

$$\frac{n + \alpha}{2\nu} = F\left[\frac{2\pi\nu}{L}\right] \tag{8}$$

where $F(w)$ is defined by equation (7). That solar oscillations satisfy such a relation was first found by Duvall (1982) from observed frequencies.

For small ℓ, equation (7), with L replaced by $\ell + \frac{1}{2}$, reduces to

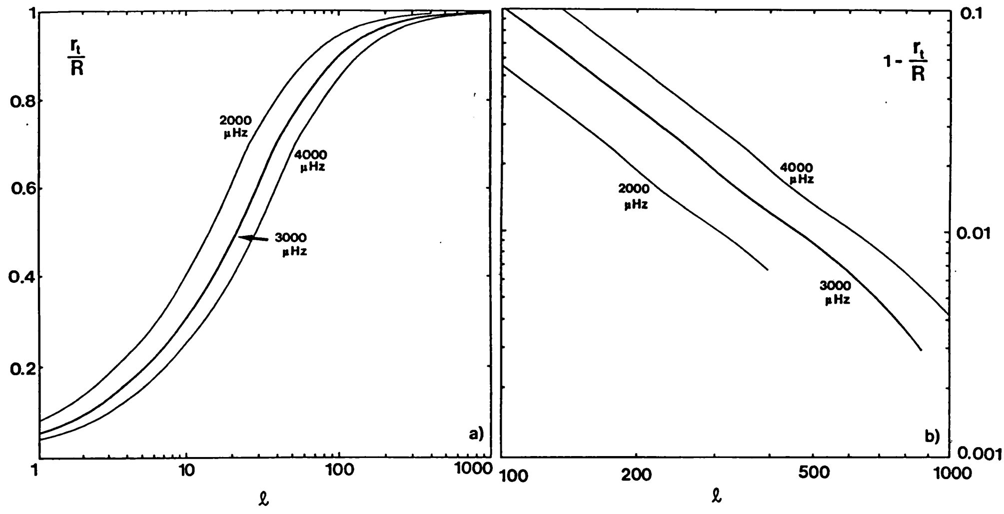

Figure 6. The turning point radius r_t (a) and the penetration depth $R - r_t$ (b), in units of the solar radius R, as a function of degree ℓ for three values of the frequency ν. This has been calculated from equation (6) for a normal model of the present Sun. On b) the curves terminate at the degree where the frequency equals the f mode frequency; for p modes the degree is below this value at a given frequency (see also Figure 4).

$$\nu \sim (n + \frac{\ell}{2} + \frac{1}{4} + \alpha)\Delta\nu \quad (9)$$

where

$$\Delta\nu = \left[2\int_0^R \frac{dr}{c}\right]^{-1} \quad (10)$$

is the inverse of twice the sound travel time between the centre and the surface (*e.g.* Tassoul 1980). Thus there is approximately a uniform spacing $\Delta\nu$ between modes of same degree, but different order. Equation (9) also predicts the approximate equality $\nu_{n\ell} \approx \nu_{n-1,\ell+2}$. This frequency pattern has been observed for the solar 5 min modes of low degree (*cf.* Figure 2), and may be used in the search for stellar oscillations of solar type.

The *deviations* from this simple relation have considerable diagnostic potential. Thus the separation $\delta\nu_{n\ell} = \nu_{n\ell} - \nu_{n-1,\ell+2}$ is predominantly determined by conditions in the solar core (*e.g.* Provost 1984; Gough 1986). Physically this may be understood from the fact that only near the centre is k_h comparable with k_r. Elsewhere the wave vector is almost vertical, and the dynamics of the oscillations is largely independent of their horizontal structure, *i.e.* of ℓ. More generally the behaviour of modes with almost the same frequency, but different ℓ, are very similar except near and below the turning point of the mode with the highest ℓ.

High-order, low-degree g modes satisfy an asymptotic relation analogous to equation (9):

$$\frac{1}{\nu_{n\ell}} = P_{n\ell} \approx \frac{P_0}{L}(n + \frac{\ell}{2} + \beta) \quad (11)$$

(*e.g.* Tassoul 1980), where

$$P_0 = 2\pi^2\left[\int_0^{r_c} N\frac{dr}{r}\right]^{-1}, \quad (12)$$

r_c being the radius at the base of the convection zone. Thus these modes are approximately uniformly spaced in *period*, with a spacing that decreases with increasing ℓ as L^{-1}.

3.3 *Effects of rotation.*

Rotation induces a splitting of the frequencies,

$$\nu_{n\ell m} = \nu_{n\ell 0} + m\Delta\nu_{n\ell m} \quad (13)$$

where $\Delta\nu_{n\ell m}$ is approximately a weighted average over the Sun of the rotation frequency $\Omega(r,\theta)/2\pi$; in general Ω is a function of both r and θ.

In the special case where Ω is independent of θ, $\Delta\nu_{n\ell m}$ is independent of m,

$$\Delta\nu_{n\ell m} = \beta_{n\ell}\int_0^R K_{n\ell}(r)\frac{\Omega}{2\pi}dr \quad (14)$$

where $\beta_{n\ell}$ is a normalization constant which, for the 5 min modes, is close to 1. The kernels $K_{n\ell}$ are determined from the eigenfunctions ξ_r and ξ_h. Some examples are shown on Figure 7. It follows from asymptotic theory

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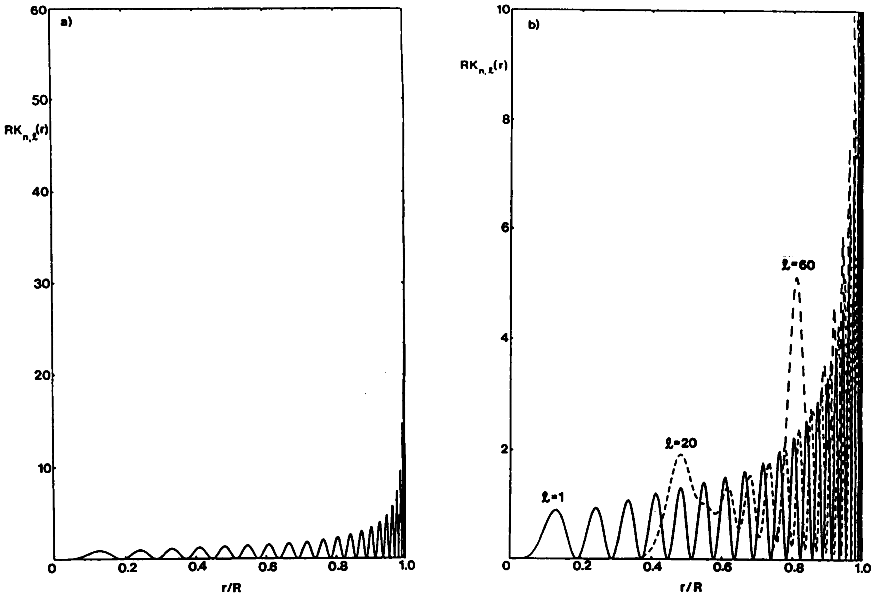

Figure 7. Kernels $K_{n\ell}$ for the frequency splitting caused by spherically symmetric rotation (*cf.* equation (14)). On a) is plotted $RK_{n\ell}(r)$ for a mode with $\ell = 1$, $n = 22$ and $\nu = 3233\,\mu$Hz. The maximum value of $RK_{n\ell}(r)$ is 57. On b) is shown the same mode, on an expanded scale, together with the modes $\ell = 20$, $n = 17$, $\nu = 3367\,\mu$Hz (- - - - - - - - - -), and $\ell = 60$, $n = 10$ and $\nu = 3231\,\mu$Hz (— — — — —).

that the envelope of the kernel is roughly proportional to c^{-1}; thus the weighting is heavily concentrated towards the solar surface. The kernel is vanishingly small below the internal turning point r_t.

In the more general case of latitudinally dependent rotation one may show that, approximately,

$$\Delta\nu_{n\ell m} \approx \frac{\int_{r_t}^{R}\int_0^{\pi} \frac{\Omega(r,\theta)}{2\pi}\frac{1}{c(r)}\left[P_\ell^m(\cos\theta)\right]^2 \sin\theta\, d\theta\, dr}{\int_{r_t}^{R}\int_0^{\pi} \frac{1}{c(r)}\left[P_\ell^m(\cos\theta)\right]^2 \sin\theta\, d\theta\, dr}. \qquad (15)$$

Here the weighting with $1/c$, as in $K_{n\ell}$, corresponds to Ω affecting $\Delta\nu_{n\ell m}$ in proportion to the time spent by the mode in a given region of the Sun. In addition Ω is weighted by the square of the Legendre function, which gives the latitudinal distribution of the mode. Modes with fairly low m are distributed over all latitudes, whereas sectoral modes, with $|m| \approx \ell$, are concentrated in a band near the equator of latitude width roughly $2\sqrt{2/\ell}$ radians. Observations of the splittings of the latter modes therefore provide

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measurements of the *equatorial* rotation inside the Sun. - In general, observation of $\Delta\nu_{n\ell m}$ at all m should allow determination of the variation of Ω with latitude.

4. SOME HELIOSEISMIC RESULTS.

A detailed review of the results of helioseismology is outside the scope of the present paper. Instead I present a few selected results on three key aspects of the solar interior: the structure of the solar core, the sound speed in the Sun and the solar internal rotation.

The solar core.

As discussed in section 3.2, for small ℓ the difference $\delta\nu_{n\ell} = \nu_{n\ell} - \nu_{n-1,\ell+2}$ is mainly determined in the solar core. Thus it is particularly well suited to test effects of modifications to the models aimed at reducing the neutrino flux. Two such modifications are partial mixing due to turbulent diffusion (Schatzman *et al* 1981), and contribution from weakly interacting massive particles (WIMPs) to the transport of energy in the core (*e.g.* Gilliland *et al* 1986). Table 1 (from Faulkner, Gough & Vahia 1986; see also Däppen, Gilliland & Christensen-Dalsgaard 1986) shows a summary of the results. The frequency separation is represented by the average $\delta\nu_0$ over n of $\delta\nu_{n0}$.

Table 1

	$\delta\nu_0$
Observations	9.2 ± 0.6 μHz
Normal models	10.0 ± 0.4 μHz
Mixed model	13.4 μHz
Model with WIMPs	9.2 μHz

The value and standard deviation for the normal models have been obtained by averaging different calculations; thus the deviation is a measure of the uncertainty in current theoretical estimates. It is evident that both the normal models and the model with WIMPs are consistent with the observations, the latter being the closer. On the other hand the mixed model seems to be ruled out by the oscillation data.

The sound speed in the solar interior.

The asymptotic relation in equation (8) is a relation between observed quantities, and so the function F can be determined observationally. Given F, equation (7) provides an integral equation for the sound speed c as a function of r. This can be inverted analytically (Gough 1984) to yield the sound speed as a function of r. The results of such an analysis (Christensen-Dalsgaard *et al* 1985b) are illustrated on Figure 8. This shows the difference between the inverted sound speed for the a set of observed solar oscillation frequencies, and the sound speed obtained by inverting theoretical frequencies for the same modes in a solar model. This procedure gives an

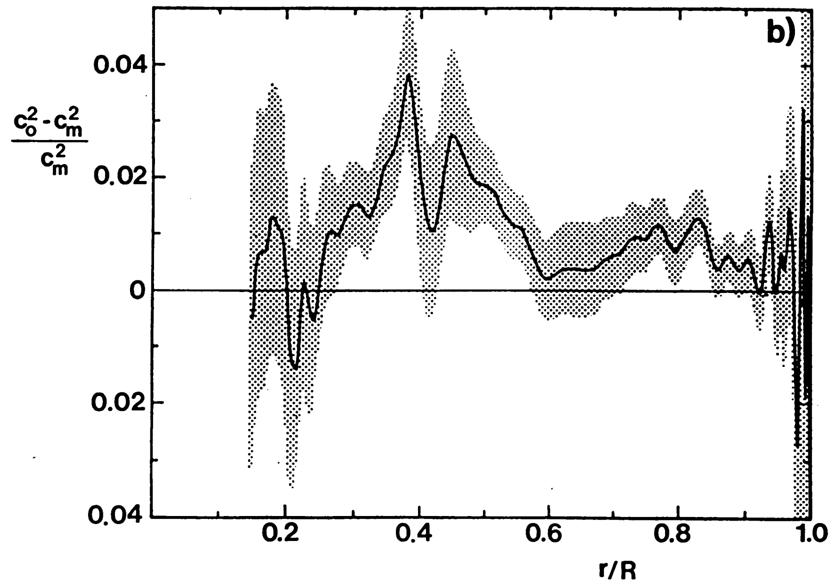

Figure 8. The relative difference between squared sound speed c_o^2 obtained by inverting observed frequencies of solar oscillation, and the corresponding c_m^2 obtained by inverting computed frequencies of the solar model. The shaded area estimates the effects of the errors in the observed data.

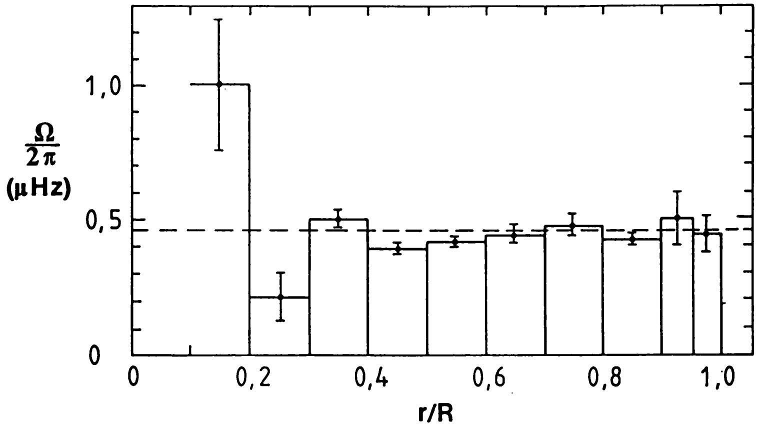

Figure 9. The equatorial rotation frequency $\Omega/2\pi$, obtained by inverting the observed splittings of sectoral modes. The inversion was carried out by representing Ω as a piecewise constant function, and determining the constant values by fitting equation (14) to the splittings. The error bars indicate the observational uncertainties.

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estimate of the actual difference between the sound speed in the Sun and in the model. The differences in c^2 are below about 4 percent in the part of the Sun, where the inversion method succeeds. Between $r/R = 0.3$ and 0.6, c^2 in the Sun appears to exceed that of the model by a few per cent. This corresponds to a similar difference in the temperature. The cause for this difference is at present unknown; however a relatively modest (of order 20 per cent) error in the opacity, with a suitable temperature dependence, might well be sufficient to account for it.

The solar internal rotation.

Duvall & Harvey (1984) measured of the rotational splitting for sectoral modes, with $|m| = \ell$. As discussed in Section 3.3, this is mainly determined by the equatorial rotation rate. The results were inverted by approximating Ω by a piecewise constant function, obtaining the values by a least-squares fitting to the observed splittings (Duvall *et al* 1984). The inferred Ω, shown on Figure 9, is generally below the surface equatorial value, although the core may be rotating more rapidly. A consequence of these results is that the rotational flattening of the Sun, which can be calculated from $\Omega(r)$, probably causes no significant perturbation in the gravitational field outside the Sun. Thus measurements of Mercury's orbit agree with the predictions of General Relativity (Narlikar & Rana 1985).

These brief remarks in no way do justice to the importance of the results, or to the effort required in obtaining them. As is documented elsewhere in these proceedings, they all represent very active areas of research, as far as both the observational and the theoretical aspects are concerned.

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