THE ANGULAR DISTRIBUTION OF THE COSMIC BACKGROUND RADIATION

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ABSTRACT. Crucial cosmological information is provided by the observed angular distribution (isotropy) of the cosmic microwave background radiation. This report treats the current status of searches for anisotropies in this radiation on all angular scales from 180° (the dipole component) to 6". With the exception of the dipole component, only upper limits (at $\sim 10^{-4}$ in $\Delta T/T$) are available, yet these upper limits have played an important role in refining models of the early Universe and of the origin of structure within it.

1. INTRODUCTION

It is an honor to have been asked to review the angular distribution of the cosmic background radiation for this Joint Discussion, and both an honor and a pleasure to have been made so welcome in India.

At the time of its discovery some twenty years ago by Penzias and Wilson (1965), the cosmic background radiation (CBR) was promptly interpreted as a thermal relic of a hot and dense early phase in the history of the Universe (Dicke, Peebles, Roll and Wilkinson 1965). Careful observations of the spectrum of the radiation, such as those reported in this Joint Discussion by George Smoot, confirm that the spectrum of the CBR is Planckian, supporting this view of the origin of the radiation (see also Smoot et al 1985; Meyer and Jura 1984; Peterson, Richards and Timusk 1985).

I want now to ask a question sometimes skipped over in considerations of the cosmic background--what is the origin of the photons we receive when we use radio telescopes to study this radiation? The answer is that these photons originate at a surface of last scattering at a large distance, and hence high redshift, from us. After the epoch of last scattering, the microwave photons travel freely through the Universe without further interaction. In this sense, the Universe is analogous to an oven, in which the thermal photons originate at the walls and are then free to move throughout the volume of the oven. In one important way, however, the Universe as a thermal enclosure difers from an oven; because the Universe is

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expanding, the photons coming from the surface of last scattering reach us highly redshifted.

In this picture, there are two general ways in which angle-dependent variations or anisotropies can be introduced into the radiation. First, if the distribution of matter at the surface of last scattering is inhomogeneous, fluctuations in the observed temperature of the radiation may be introduced on small angular scales. Briefly, denser regions are likely to be hotter than under-dense regions; I shall return to the details of this argument towards the end of my talk. The second possibility for anisotropy is connected to the redshift of the photons that reach us from the surface of last scattering. If the expansion of the Universe is itself anisotropic, then the apparent redshift of the surface of last scattering will be angle dependent, and large angular scale anisotropies will be introduced into the CBR. The same is true if the observer is in motion with respect to the surface of last scattering. In such a case, the ordinary Doppler shift will produce variations in the observed intensity or temperature. A dipole anisotropy pattern will result.

These general ideas have been understood for nearly as long as we have known about the CBR. Have we measured any of these possible varieties of anisotropy in the radiation? Unfortunately, with one exception, the answer is no. The only exception is the dipole anisotropy produced by the velocity of the observer.

I take it as my task at this Joint Discussion to describe the present observational situation, to present the current upper limits on various angular scales, and then to discuss what cosmologists have been able to learn from the very fact that anisotropies have not yet been detected. Although it is disappointing not to be able to report positive results, I believe it is still the case that the tight upper limits on anisotropy on angular scales from arcseconds to tens of degrees constitute some of the most important results of observational cosmology to date. This report is very closely based on a more general review of the CBR soon to be published (Partridge 1986). Indeed, some of this review is taken word for word from that earlier work.

2. LARGE-SCALE ANISOTROPY

Searches for large-scale (say $\gtrsim 5°$ scale) anisotropy have generally been made by specially designed radio telescopes flown above part or all of the earth's atmosphere (on a balloon or in a satellite). In general, the angular resolution of these instruments is low so that discrete sources of radio emission produce negligibly small signals. Great care is taken to reduce systematic errors to a minimum, since the magnitude of the signals sought can be 6-7 orders of magnitude below the thermal temperature of the apparatus itself.

In the past few years, three groups (Fixsen et al 1983; Lubin et al 1983; Strukov and Skulachev 1984) have made such measurements, with results in broad agreement. The details are summarized in

Table I. Here I want to make the point that all three groups detect a clear dipole signature in the intensity of the radiation with a maximum at ∿ 11 hours right ascension and a slightly negative declination. The amplitude of this dipole anisotropy is known to an accuracy of approximately ± 5%; it is $\Delta T/T = 1.2 \times 10^{-3}$. Because the Doppler-induced anisotropy is small, we may use the nonrelativistic Doppler formula to calculate the velocity of the observer (that is, of the earth):

$$v = \frac{\Delta T}{T} c \approx 400 \text{ km/sec.}$$

Of more astrophysical interest is the motion of the center of mass of our galaxy, which may be found from the above by correcting for the (small) annual motion of the earth about the sun and for the motion of the sun about the center of the Galaxy (a topic discussed elsewhere in this volume). When those corrections are made, we discover that the center of mass of our Galaxy is traveling with the velocity of 500-600 km/sec very approximately towards the Virgo cluster.

TABLE I. Large Scale Isotropy

Parameter	Berkeley Lubin *et al* 1983	Princeton Fixsen *et al* 1983	Moscow Strukov and Skulachev 1984
Wavelength, mm	3	12	8
Sky coverage	85%	80%	∿100%
Dipole moment, mK	3.4±0.2	3.1±0.2	2.6±0.1
Solar direction	$11^h.5$ −6°	11^h −10°	$10^h.8 \pm 0^h.3$ −3°±5°
Speed of galaxy, km/sec	comparable	600±50	530
--direction	"	$10^h.4 \pm 0^h.4$ −27°±5°	$10^h.3 \pm 0^h.5$ −32°±5°
Quad. moment, mK	<0.2	<0.19	<0.1

Results of three measurements of the large-scale distribution of the CBR. The dipole moment and direction of galactic motion for the Soviet work are calculated indirectly from their results. The dipole moments are expressed in thermodynamic temperature, as are the 90% confidence upper limits on the quadrupole moment.

Although there were some published reports five to ten years ago of the detection of a quadrupole anisotropy (Fabbri *et al* 1980; Boughn *et al* 1981), it now appears that there is no significant quadrupole anisotropy in the radiation. The best upper limits are those provided recently by the Soviet satellite, $\Delta T/T \lesssim 4 \times 10^{-5}$. I should like to

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make two parenthetical remarks at this point. The first is to call your attention to the exquisite sensitivity of such experiments--the upper limit cited above corresponds to temperature measurements of only ∿ 0.1 mK. The second point is that the absence of a quadrupole moment is consistent with the argument made above that the dipole moment is produced purely by the velocity of the observer. In other words, there is little room for contributions by anisotropic expansion of the Universe itself (see 4 below).

I should note also that attempts to measure the large angular scale distribution of the CBR can also provide upper limits on the anisotropy or inhomogeneity of the radiation on somewhat smaller angular scales, down to the size of the beams of the antennas employed in such work. These angular scales are typically 5°-7° (see, for instance, Fixsen et al 1983). This remark serves to introduce my next topic.

3. SEARCHES FOR SMALL SCALE ANISOTROPIES

For more than 15 years, groups in the Soviet Union, Italy, the United States and Britain have been attempting to find anisotropies or inhomogeneities in the microwave background on angular scales ranging from 90° (the quadrupole scale) down to 6". Measurements have been made at wavelengths from below 1 mm to above 10 cm. There have been some claims of positive detections of anisotropies, but I believe these to be in error, or to be "noise" introduced by the presence of discrete radio sources rather than true cosmological anisotropies. While disappointing, this lack of results has had an important impact on theories of the origin of large scale structure in the Universe, as we shall see. Now let me turn my attention to the newest results (or rather lack thereof), as displayed in figure 1.

First, let me call your attention to angular scales between roughly ½° and 5°. Observations on these angular scales are difficult, as their relative lack of sensitivity suggests. First there is the simple matter that most conventional radio telescopes are built to work at higher angular resolution. The alternative is to use a horn antenna, such as those used in the large-scale isotropy experiments referred to above; but a horn antenna with a resolution of 1° is prohibitively large. In addition, there is the problem of emission from the earth's atmosphere, a dominant source of noise in many ground-based searches for fluctuations in the CBR. Water vapor is the primary culprit, and water vapor, whether in the form of visible clouds or not, is clumped on scales of a degree or so.

At present, the best limits we have on anisotropies on scales of a few degrees or more are by-products of the searches for larger angular scale anisotropy I have mentioned above.

Three groups have attempted to make direct measurements on these angular scales from the ground. The first such program is that of Pariiskii and his colleagues (Pariiskii, Petrov and Cherkov 1977, and Berlin et al 1983) at 3.9 and 7.6 cm wavelengths. We had hoped that Dr. Pariiskii would be here to discuss his work; unfortunately he

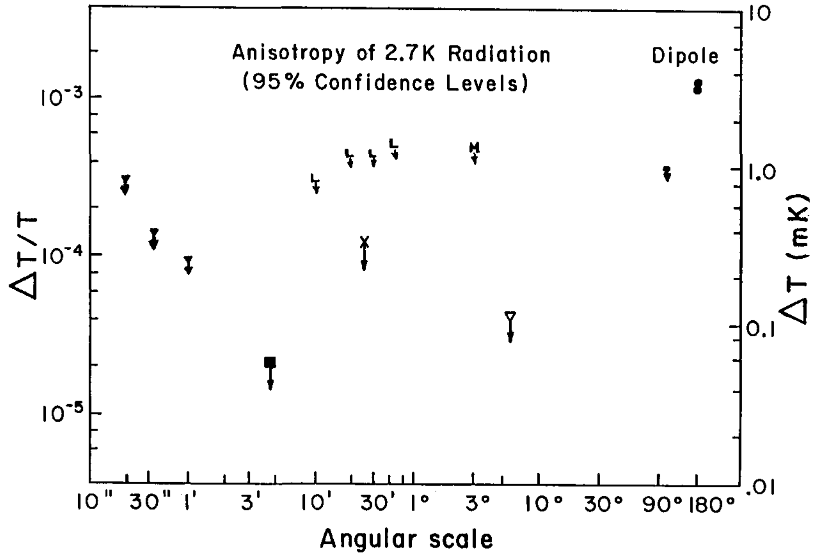

Figure 1. Upper limits on anisotropy in the CBR as a function of angular scale. Only selected values are shown. See text for references.

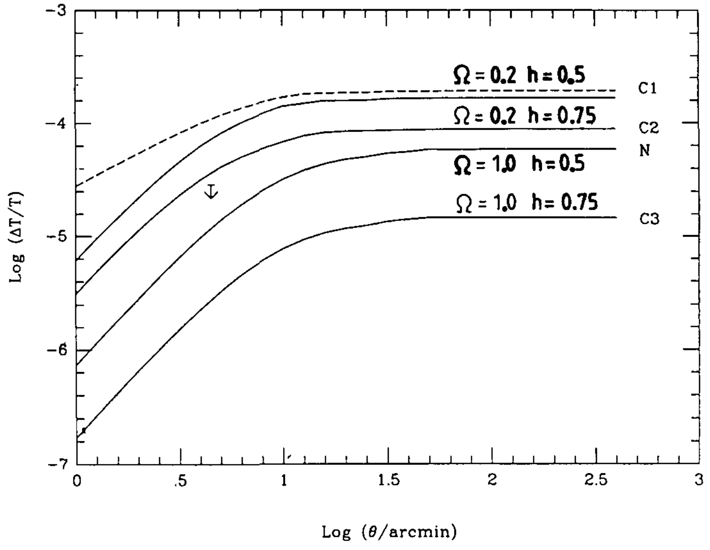

Figure 2. Recent calculations of the level of fluctuation $\Delta T/T$ expected in cosmological models dominated by various kinds of "exotic" particles (Bond and Efstathiou 1984). Theoretical curves for models dominated by cold dark matter (C) and by massive neutrinos (N) are shown for various values of $\Omega \equiv (\rho_o/\rho_c)$ and $h = H_o/100$ Km $sec^{-1}Mpc^{-1}$; the experimental point is the upper limit published by Uson and Wilkinson (1984).

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could not come. Lasenby and Davies (1983) have also set upper limits on fluctuations in the microwave background at a wavelength of 6 cm: on 30 arcmin scales the 95% confidence limit is $\Delta T/T \lesssim 4.6 \times 10^{-4}$, and the result at 60 arcmin scales is a factor of 5 less sensitive. In addition, a group of Italians and Norwegians of which I was also a part attempted observations on angular scales of 3° and above at a wavelength of 3 cm (Mandolesi et al 1986). In addition to "discovering" cumulus clouds, we were able to set 95% confidence limits of $5\text{-}7 \times 10^{-4}$ in $\Delta T/T$ on scales of 3°-10°.

Now let us turn to the observational results for the smallest angular scales so far observed, 6"-60". To make observations on such small angular scales at microwave wavelengths requires the use not of a single radio antenna, but an array of antennas, for which the angular resolution is determined by size of the array. There is a substantial price to be paid however: the efficiency of arrays is small, and our limits on fluctuations in the microwave background are therefore not as sensitive as may be achieved by using single, filled-aperture, telescopes. Basically, most photons hit the ground between the telescopes of the array, not the collecting surfaces themselves. Nevertheless, two groups (Fomalont et al 1984a; and Knoke et al 1984 and Martin, Partridge and Ratner 1986) have used the Very Large Array in New Mexico to set limits on the amplitude of fluctuations in the microwave background on angular scales of 6"-60". Those results are summarized in Table II; the wavelength employed for all the observations was 6 cm. Later in this volume, Dr. Kellerman will present better upper limits at 18"-60" scales recently determined by him and his colleagues.

TABLE II. Limits on Very Small-Scale Anisotropy of the CBR

Angular Scale	Fomalont et al (1984)	Knoke et al (1984)	Martin et al (1986)
6"	--	3.2	--
12"	--	1.7	--
18"	1.0	1.2	0.3
30"	0.8	--	0.13
60"	0.5	--	0.10

Upper limits on $\Delta T/T$, in parts per 10^3, are given at 2σ confidence. All observations were made at $\lambda = 6$ cm using the Very Large Array. For recent, more stringent upper limits, see the report by K. I. Kellermann in this volume.

Even these results are roughly 3-10 times less sensitive than the limits established at larger angular scales of 1-10 arcmin. On the other hand, aperture-synthesis observations do offer one substantial advantage: they provide us a real two-dimensional map of the microwave background. A final side benefit is that observational programs of this sort also provide information about very weak discrete radio sources (that is, they extend the radio source counts to smaller fluxes; see Fomalont et al 1984b and Martin et al 1986).

By far the most interesting observational limit on fluctuations in the microwave background is the new result of Uson and Wilkinson (1984 and 1985) on an intermediate angular scale of a few arcmin.

These observations were carried out at a 40 meter telescope and an effective wavelength of 1.5 cm. The angular resolution achieved was 1.5 arcmin, but upper limits were also obtained at somewhat larger angular scales. Indeed, the more fundamental angular scale is the beam switch scale, which was 4.5 arcmin for their work. At 1.5 arcmin, their observations limit fluctuations in the microwave background to $\Delta T/T \lesssim 2.5 \times 10^{-5}$ at the 95% confidence level, and the limit at an angular scale of 4.5 arcmin is equally stringent. I should note, however, that some questions have also been raised about the statistical analysis of these data. Even if there is a slight revision of this particular result, the microwave background is now known to be remarkably isotropic on small angular scales. A quick physical analogy may drive the point home. If we were to try to manufacture a billiard ball as smooth as the CBR, we would have to control its dimensions to a few wavelengths of light.

To summarize, 20 years after its discovery, the microwave background is not only consistent with a pure blackbody spectrum at 2.7-2.8 K but is, I believe, completely featureless on all angular scales except the dipole scale, which, as we have seen, can easily be explained as a consequence of the velocity of the Earth. I hope now to show that these tight upper limits on $\Delta T/T$ are of great importance to modern cosmology.

4. WHAT HAVE WE LEARNED FROM THESE OBSERVATIONS?

Let us begin with the one positive result we have--the measurement of the dipole anisotropy in the CBR. I shall adopt the conventional view that the dipole moment results from the motion of the observer. When the observed velocity is corrected for the motion of the sun around the center of our Galaxy, we obtain a value for the velocity of the center of mass of our Galaxy with respect to matter at large distances. This value in turn is being actively exploited to tell us about the large-scale distribution of matter in the Universe, since the velocity of the Galaxy seems to be at least in part produced by the gravitational acceleration caused by a near-by clump of matter (perhaps the Virgo cluster, with an overdensity of ~ 2). If gravity is in fact the cause of the observed velocity, we may go further and use the dipole anisotropy to set constraints on a vital parameter in cosmology, the mean mass density ρ_o. Recall that if the density of the Universe is less than a critical value, ρ_c, normally taken to be several times 10^{-30} gm/cm^3, the Universe will continue to expand

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forever: on the other hand, if ρ_o is greater than ρ_c, the Universe will recollapse gravitationally. Now what is the connection between the observed velocity of our Galaxy and ρ_o? The underlying physical idea is that a large overdense clump will produce a gravitational acceleration which increases with both the overdensity factor $\bar{\delta}$ and with ρ_o (see, e.g., Davis and Peebles 1983, and Davis 1985). In turn, the induced velocity v depends on the gravitational acceleration. If we make the most straightforward assumptions and treat the clump as a sphere and allow for no non-linear effects, we find

$$v/H_o r = \frac{1}{3} \bar{\delta} \, (\rho_o/\rho_c)^{0.6}$$

where H_o is Hubble's constant, and r is the distance to the clump. Hence our accurate knowledge of v would lead directly to ρ_o if we knew the overdensity $\bar{\delta}$. As a first approximation we may assume that $\bar{\delta}$ can be determined merely by counting galaxies: $\bar{\delta} = \Delta N/N$ where ΔN is the excess number of galaxies observed in a clump. For the Virgo cluster, as noted above, $\Delta N/N \sim 2$. With this value inserted in the equation, we find $\rho_o/\rho_c \approx 0.3 \pm 0.2$. Using galaxy counts to determine $\bar{\delta}$, however, requires the assumption that all the matter in the Universe is distributed in the same way as the luminous matter in galaxies. Recent advances in particle physics and in theories of the early Universe teach us to be cautious on this point--many forms of "dark matter" may exist, including relict particles such as neutrinos or axions (see the splendid invited discourse by Dr. Rubin). In many cases, the "dark matter" is expected to be more uniformly distributed than the luminous (baryonic) matter. In such cases, Galaxy counts overestimate $\bar{\delta}$. If the true value of $\bar{\delta}$ is less, ρ_o can be greater. A value of ρ_o as large as the critical value can by no means be excluded. Clearly more work on the quantity $\bar{\delta}$ and on the underlying theory are needed before we can exploit the full power of this idea.

The lack of an observed quadrupole anisotropy, and the fact that the dipole anisotropy is relatively small and has a natural explanation, both tell us that the large-scale expansion of the Universe is isotropic (and shear-free) to a remarkable degree (Collins and Hawking 1973; Barrow *et al* 1984). Although the isotropic (Friedman-Robertson-Walker) models are a subset of measure zero of possible cosmological solutions to Einstein's equations, it appears that they provide a remarkably good model for the Universe as we observe it. Why this should be so is an intriguing mystery.

Next, let me focus attention on the observed isotropy of the radiation on angular scales of a few degrees. As I noted earlier, this is a difficult angular scale from an observational point of view. It is also, however, an angular scale that may be of particular cosmological interest.

Here for the first time we must introduce an assumption about the distance, expressed in redshift terms, to the surface of last scattering mentioned early in this report. Often, the epoch of last scattering is taken to be the epoch of recombination of the primeval plasma*. At recombination, free electrons disappear and Thompson scattering of the CBR photons ceases (see Peebles 1971). The redshift corresponding to this epoch is $\sim 10^3$. With this assumption, the angle θ_h = 1°-3° is the angular scale corresponding to the largest regions of the Universe that were causally connected at the epoch of last scattering. Thus observations on scales $\gtrsim \theta_h$ span regions larger than the causal horizon at the time the radiation began its journey to us. Hence classical Big Bang cosmology can offer no explanation for the homogeneity on these scales or larger. On the other hand, the new inflationary models do (see, for instance, Steinhardt 1985). In these new variants of the classical Big Bang cosmology, the Universe undergoes a period of exponential expansion well before the radiation last scatters from matter. The expansion factor is so great that the entire visible Universe "grew from" a region small enough to have been causally connected at a still earlier time. Thus inflation in effect magnifies small, causally homogeneous, regions. It therefore includes a mechanism for producing the large-scale isotropy we see, and the observational results in turn provide important support for the inflationary scenario.

The most severe observational constraints we have on anisotropy in the CBR are on angular scales of a few minutes of arc. These constraints are also among the most useful in shaping cosmological theory.

As I noted earlier, if matter is inhomogeneously distributed on the surface of last scattering, fluctuations in the CBR are expected. The relationship between the fractional density perturbation $\Delta\rho/\rho$ and $\Delta T/T$ depends on the redshift of the surface of last scattering, on the nature of the matter perturbations and their mass, and even on the nature of the particles making up the mass density of the Universe. The last point deserves further comment. Until recently, it has been assumed that virtually the entire density of the Universe is made up of baryonic matter. New theories developed on the interface of cosmology and particle physics call that into question by suggesting a substantial or even dominant contribution to ρ_o of low mass particles such as axions or neutrinos having mass (see the Invited Discourse of Vera Rubin). In cosmological models containing both baryons and some form of these "exotic" particles, it may happen that the inhomogeneity in the baryon component at the surface of last scattering is very small so that the CBR fluctuations are also small. Indeed, there exists a very wide range of theoretical possibilities once we include non-baryonic matter. The range is far

*Alternative models allow for a subsequent epoch of re-ionization shifting the surface of last scattering closer, say to $10 < z < 100$ (see below).

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too wide to discuss in detail here (but see a forthcoming review by Bond and Efstathiou 1986). Here I want to note certain universal features in the predictions of cosmic background fluctuations, and to comment on the confrontation between these predictions and the observations I have discussed.

I will at first restrict my attention to the case where the surface of last scattering is at recombination, that is at $z \sim 1000$. Since the recombination of the primeval plasma occurs rapidly at a well-defined redshift, we may make quite detailed calculations of the present temperature fluctuations in the CBR. All such calculations (e.g., Silk and Vittorio 1984; Bond and Efstathiou 1984) show a characteristic dependence on angular scale reflected in Figure 2. The rapid falloff at small angular scales does not necessarily imply smaller density inhomogeneities; rather, it reflects the fact that smaller inhomogeneities are transparent, so we see an averaging effect, reducing the detectable $\Delta T/T$. To fix the angular scale, 10' corresponds approximately to 10^{15} solar masses, although this relationship depends a bit on the cosmological model assumed.

What about the vertical scale? How do we normalize this plot? A crucial step in this process is the recognition that the present, inhomogeneous, distribution of matter on large scales in the Universe must have arisen from smaller amplitude perturbations in the density at earlier times. A well-known result first shown by Lifschitz 40 years ago is that the relative density fluctuation $\Delta\rho/\rho$ grows as $(1 + z)^{-1}$. On mass scales of very roughly 10^{15} $M_\odot$, $\Delta\rho/\rho$ today is of order unity. Hence it must have been of order 10^{-3} at the redshift of last scattering. One then must associate a value of $\Delta T/T$ with this value of $\Delta\rho/\rho$. Only in a single case, for massive ($\gtrsim 10^{15}$ $M_\odot$) adiabatic perturbations in pure baryonic matter, is the connection straightforward: $\Delta T/T = 1/3\ \Delta\rho/\rho \gtrsim 3 \times 10^{-4}$. A great deal of theoretical work has gone into calculating the connection between temperature fluctuations and density perturbations for more complicated models, particularly those that contain some form of dark matter. Figure 2 illustrates some of the numerical results, particularly for models which contain either massive neutrinos or cold dark matter particles as the predominant mass component of the Universe (Bond and Efstathiou 1984). The measurement of Uson and Wilkinson (1984) appears to exclude many but not all of the models. On the basis of these results alone, it would appear that a model dominated by massive neutrinos is acceptable (although this model has other theoretical problems), and so too is a high density model with cold dark matter providing most of the density of the Universe.

For an observer, a situation like this is of particular interest--the observations are already useful in constraining the theories, and relatively modest improvements in the sensitivity of the observational results could make them much more valuable still. I am confident that the results shown in Figure 2 will be improved by factors of 2 or 3, probably in two ways--by making the measurements more sensitive, and by making measurements of comparable sensitivity but at larger angular scale.

A number of workers have pointed out, however, that the conclusions drawn above are strongly based on the assumption we made that the redshift of last scattering is at $z = 1000$. If the matter contents of the Universe are reionized at a later epoch (smaller redshift), free electrons will again be available and Thomson scattering will again affect the CBR. The surface of last scattering would thus shift to lower redshift.* The primary consequence of such subsequent scattering is that all information about primeval anisotropies is scattered away. Hence we lose vital cosmological information. On the other hand, it is fair to point out that whatever process causes the reionization can hardly be absolutely homogeneous. It follows that this new surface of last scattering may itself be inhomogeneous, producing temperature fluctuations. Detailed predictions of temperature fluctuations in one such scenario have been made. This is the explosive scenario of Ikeuchi (1981) and Ostriker and Cowie (1981). One of the interesting features of this work (Vishniac and Ostriker 1985) and an earlier reionization scenario of Hogan (1980, 1982) is that substantial fluctuations on arcsecond scales are predicted (unlike the earlier case for primeval fluctuations). The aperture synthesis upper limits on $\Delta T/T$, while not as tight as larger angular scale measurements, are able to constrain these reionization models better. For instance, Hogan's specific models are ruled out (Hogan 1984; Knoke et al 1984). Some possible scenarios in explosive models for galaxy formation also run into trouble with the observations. Vishniac and Ostriker (1985) have made detailed predictions of the amplitude and angular scale of microwave anisotropies expected from explosive galaxy formation. They find $\Delta T/T \sim 2 \times 10^{-6}\ \theta^{-1}\ (1 + z_f)^{7/4}\ (\rho_o/\rho_c)^{-\frac{1}{2}}$, where z_f is the redshift corresponding to the epoch of galaxy formation, and θ is in arcminutes. With $(\rho_o/\rho_c) = 1$, the limits on $\Delta T/T$ discussed earlier require $z_f \lesssim 8$. Obviously, tighter limits on $\Delta T/T$ on sub-arcminute scales would provide the best test of such a model; the results presented by Dr. Kellerman in his report in this volume limit z_f to $\lesssim 6$.

More recent and more general calculations by Ostriker and Vishniac (1986) suggest that most scenarios for galaxy formation will produce CBR fluctuations on arcminute scales at just about the level the interferometric searches are now reaching. The fact that both our work and Kellermann and his colleagues' work is consistent with real CBR fluctuations at the level of $\Delta T/T \sim 10^{-4}$ is intriguing. It would be nice to think that, after nearly twenty years of effort, we are finally seeing structure in the cosmic background.

*Reionization can shift the redshift of last scattering only if it occurs at redshifts larger than about 15; even complete reionization at epochs later than that corresponding to $z = 15$ will produce optical depths below unity in Thomson scattering.

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As I have been at pains to emphasize, however, even the long succession of upper limits has played a crucial role in guiding modern cosmological theory.

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