THE EXCESS SECULAR CHANGE IN THE OBLIQUITY OF THE ECLIPTIC AND ITS RELATION TO THE INTERNAL MOTION OF THE EARTH

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Abstract. Criticisms of the senior author's paper (1967) are examined, and it is found that there still seems to be a possibility that the cause of the excess secular change is in the mantle-core coupling. Part II is devoted to an improvement of the previous result (Aoki, 1969) using a model of the elastic mantle and fluid core. In order to give the observed value of excess secular change of the obliquity, a value of the coupling coefficient of the Earth's rotation is 3.5 times larger than Rochester's (1968) value is required. This is not impossible from the geophysical view-point. In fact, if we take into account Hide's argument (1969), then the coupling coefficient will be much larger than the present geophysical value.

1. Introduction

In the previous papers (Aoki, 1967, 1969; hereafter referred to as Paper I and II) one of the present authors has called attention to the implication of an excess secular change in the obliquity of the ecliptic ($-0\rlap{.}''30$/century) to the determination of Oort's constant B and the structure of the Galaxy, and proposed an interpretation of such an excess by a new rotational motion of the Earth due to a frictional coupling between the rigid constituents, the mantle and the core.

It was so large a change in the rotation velocity (390 km/s at the solar neighborhood) of the local standard of rest of the Galaxy, that almost all researchers in the field of stellar dynamics find it difficult to accept our proposal. The problem rests on how to determine the reference framework and the system of proper motions.

There have been three major questions:

(1) relation of the ratio of the velocity dispersions (or axial ratio of velocity ellipsoid) of stellar motions to the ratio of Oort's constants A and B,

(2) accuracy of the ecliptic coordinates compared with the stellar statistics, and

(3) systematic errors in fundamental star catalogs.

In Part I we give our opinion of these criticisms.

Part II of this paper is devoted to a revision of the previous Paper II, since it had in itself a difficulty in the deceleration of the Earth's rotational velocity, which was too large compared with the observations. Part II was produced mainly by the second author (C.K.).

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Revision was made by replacing the rigid body model by a deformable (elastic) mantle and a fluid core, in order to overcome a difficulty met in Paper II. As the obliquity is decreasing, the tidal attractive force will increase the displacement of the equatorial plane and induce an increase in the maximum moment of inertia of the deformable fluid core. Consequently the rotational speed of the fluid core might decrease, which will cause a slow deceleration of the speed of Earth's rotation.

We have, however, a difficult problem in how to formulate the rotational motion of the deformable Earth in a non-conservative system. In the theory of the Earth tides the tangential stress at the boundary between the elastic mantle and the deformable fluid core has, so far, not been formulated. Damping of the Earth's rotational motion is still treated by a phenomenological method (cf. Munk and MacDonald, 1960).

The magnetic coupling, which may be one of the important couplings, between the mantle and the core has been extensively studied by Rochester and Smylie (1965), Rochester (1968), Hide (1967, 1969) and Stacey (1970). The magnetic couplings can transmit energy into the upper mantle with a finite electrical conductivity from the core. We should like to stress that Paper II's frictional coupling implies the magnetic coupling as well as the kinematical viscous coupling. A precise discussion of the magnetic coupling will be studied in future. We shall apply an idea that the tangential stress due to the friction will be transferred into the mantle and affect the motion of the rotation axis of the deformable Earth in space. A treatment by separating the mantle and the core has no merit when discussing the deformable Earth, because of the complicated condition at the core-mantle boundary. Love's number will be used from the results obtained by other authors.

PART I: EXCESS SECULAR CHANGE IN THE OBLIQUITY

2. Ratio of Velocity Dispersions

It is well known that in the rotational system of a galaxy the axial ratio of the velocity ellipsoid of dispersions has a definite relation to the rotational velocity of the system, or (local) Oort constants A and B:

$$\frac{b^2}{a^2} = \frac{-B}{A - B}, \tag{1}$$

where a denotes the velocity dispersion (or rms of the residual velocity) in the radial direction, b the velocity dispersion in the transverse direction, and A and B are Oort's constants.

Schmidt (1967) and Oort (1969) had strong objections to Paper I because the value $B = -24$ km/s/kpc deviates from the Equation (1). However, this relation holds only for the ellipsoidal distribution with rotational symmetry. The actual system of our Galaxy does not have this property rigorously. Theoretically the ellipsoidal theory holds as a limiting case when the dispersions of residual velocity tend to zero. Also

TABLE I

	Aoki (1967)	IAU (1964)
A	+15 km/s/kpc	+15 km/s/kpc
B	−24 km/s/kpc	−10 km/s/kpc
$\frac{-B}{A-B}$	0.62	0.40
$\sqrt{\frac{-B}{A-B}}$	0.78	0.63

the principal axes of the velocity distribution show a vertex deviation when the mass distribution differs from the rotational symmetry. (Kato 1968, 1970). For the case of finite dispersions Miyamoto (1971) in fact showed an example which violates the relation (1) (also see Aoki, 1965; Kato, 1970).

It is easy to understand that the stars with low dispersion are easily affected by the local irregularity of mass distribution, for example, by arm structure; actually they show a vertex deviation. The problem is therefore how we can determine the overall velocity distribution and dispersions which are not affected by the local irregularity, and to construct the rotational velocity ($A-B=\omega$) near the Sun. In order to have a smoothed velocity distribution, we have to pay attention also to the larger velocity stars, for which the relation (1) necessarily does not hold. In this respect, relation (1) cannot serve as a crucial test for the Oort constant B, in our opinion. In other words, there are two ways to get rid of the difficulty: first, to give an actual pattern of the velocity distribution and dispersion with the local irregularity of mass distribution, both from observation and from theory; secondly, to give a standard or smoothed model of the Galaxy which is free from the local irregularity and to give the theoretical relation between velocity dispersions and rotational velocity (or Oort's constants), where the relation is not compatible with (1) in the case of finite velocity dispersion. The above two ways, however, are essentially the same, since the model can be approached observationally by reducing all the local mass irregularity and the effect of such mass distribution on the dispersions and on Oort's constants.

In our opinion it is still an open question whether or not the systematic difference in the value of B existing between McCormic-Cape system and 512 fundamental star system (Fricke, 1967a, b) is real. If the difference is real, this might be due to a local (rotational) motion of the star system near the Sun, since a selection error may cause such a difference.

3. Accuracy of the Ecliptic Coordinates Compared with the Stellar Statistics

The problem started in the days of Newcomb when he determined the precessional constant from the observations (1898). Even now there is not a theoretical method to determine the precessional constant or the dynamical flattening of the Earth. Therefore we have to define the inertial framework from the statistics of stellar motions. In the days of Newcomb the galactic rotation was not known; and he defined the inertial framework so that each star had only proper motion of accidental error type. In other words, his inertial framework was referred to the mean positions of stellar systems as a whole. Unfortunately, his treatment was not correct, because since then systematic motions have been found which lead to a 'galactic rotation'. At present we have the assumption that a systematic motion of fixed stars as a whole is due to the galactic rotation, and the other systematic motions are fictitious and due to precessional motion. But there is an exception in the so-called equinoxial motion. This might come partly from the day-night error or the time system of the old days. Oort gave a comment to us in his private communication (1969) as a part of the criticism to Paper I saying that if one trusts the observations of the sun and planets for the inclination of the ecliptic, then one should logically also trust them for the motion of the equinox. This is a serious problem of the systematic motion of the star system. Actually this term is empirically determined. There is no indication that this is entirely due to errors in the observations of the Sun. However it might also come from the systematic motion of local sysytem. It is still an open question.

Even though it is a serious problem to construct a reference framework, it does not have a direct influence on our problem. This is because the declination system, on which the determination of the rotation around the equinoxial axis depends, may be considered as having rather superior accuracy to the right ascension system, since at least there is no error for the declination system corresponding to the clock error as of the right ascension system in the old days.

Now, Lieske (1970) pointed out that he could not discriminate the two cases, equator in error or ecliptic in error from the analysis of Eros observations. However, his 95% proof is too rigorous, and it is beyond the usual analysis level. On the other hand as for change in the obliquity, the analysis of Venus by Duncombe (1958) and of Mercury by Clemence (1943) may be considered the best of all, and can hardly be doubted. In any case, it is urgent to decide which case is true from the observations of planets including exterior planets. Particularly, Jupiter is considered to be the best for the determination of ecliptic coordinates, since it has the largest mass in the solar system and its orbit can represent the invariable plane with sufficient accuracy because of its large mass and because of only slight disturbances by the other planets. Such an analysis has not yet been done.

4. Systematic Error in the Declination System of Fundamental Star Catalogs

Fricke pointed out in his paper (1971) that the discrepancy to which we called attention might come from the declination error in the star catalogs, on which the observations of planets necessarily depended for respective epochs. A careful inspection shows that the direction of the error is opposite to that which would reconcile the observed discrepancy. The point is as follows:

Let one suppose, after Fricke, that the determinations of the obliquity by the planet

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observations are referred to the systems of star catalogs at the respective epochs. In this case we consider a situation of stars being near the summer solistice ($\alpha \sim 90°$), for example, whose real proper motions are zero as a whole according to the present system (or FK4) but were considered that they had positive proper motions according to the old system (or GC). This is inferred from a difference of the coefficients of sin α in μ_δ:

$$\text{GC} - \text{FK4} = 0''.23 \sim 0''.20/\text{century} > 0. \tag{2}$$

In the present supposition, planets (including the Sun) are also observed by being referred to a star system, or are affected by the same errors as surrounding stars. The stars had plus (fictitious) proper motions as a whole in this region, and since a planet has a plus declination there, the obliquity therefore was thought (in the old system) to be increasing whereas it has no change at all; contrary to the case under consideration. The situation cannot be altered greatly, or speaking more strictly it cannot change the sign, even if we combine the old data and new data, because the mean proper motion in this region for a duration extending to both epochs can be considered having a positive value.

The arguments in Part I show that there is no definite objection to the previous proposal, although there are so many open questions which should be clarified. There seems to be no reason to withdraw the proposal so far unless there is an essential difficulty in constructing the coupling mechanism between the mantle and the core to give rise to the (observed) quantity $-0''.30$/century of the excess secular change in the obliquity of the ecliptic.

PART II: MOTION OF THE EARTH'S FLUID CORE

5. A Model of the Earth

A model of the fluid core is an incompressible fluid with a uniform density having no magnetic field nor an inner core. We assume that relative displacements to the mean radius of the fluid core are so small and that non-linear coupling due to the advection is negligibly small. Displacements in the mantle are much smaller than those in the fluid core. The re-distribution of the density of the mantle is assumed to consist in part of a change in the moment of inertia and the motion in the mantle is assumed to be negligible, as was adopted by Molodensky (1961).

The reference frame which is rotating relative to space is assumed to coincide with the figure axes. The figure axis of maximum moment of inertia is taken to be the mean axis of rotation of the Earth for the period of the long time interval. We choose it to be the z-axis. The origin of the reference axis is chosen to be the center of gravity of the Earth. A right-handed system is adopted with x and y in the equatorial plane, perpendicular to the z-axis. The coupling with the equatorial components of the angular velocity of the rotation and displacements are omitted.

6. Equation of Motion in the Fluid Core

The tidal disturbing function due to the Moon and the Sun on any point of the Earth is expanded in the spherical harmonics of the two poles, the pole of the rotating frame and that of the ecliptic. The motions of the Moon and the Sun are simply assumed to be circular motions in the ecliptic plane with uniform angular velocity $n_☾$ and $n_\odot$. The tidal disturbing function for the precessional motion is obtained by taking the average over the longitude and we have

$$K = r_b^2\omega^2 \left\{K_1 \frac{xz}{r_b^2} \cos(\nu t - \psi_0) + K_2 \frac{yz}{r_b^2} \sin(\nu t - \psi_0) + K_3 \frac{l^2}{r_b^2}\right\}, \tag{3}$$

where

$$\begin{aligned} K_1 &= K_2 = \tfrac{3}{2} K_0 \sin\theta \cos\theta\,, \\ K_3 &= -K_0 \left(\tfrac{3}{8}\sin^2\theta - \tfrac{1}{4}\right), \\ K_0 &= \left(\frac{n_\odot}{\omega}\right)^2 + \left(\frac{n_☾}{\omega}\right)^2 \left(\frac{\mu_☾}{1+\mu_☾}\right), \\ \mu_☾ &= M_☾/M_\oplus \\ l^2 &= x^2 + y^2 \end{aligned} \tag{4}$$

and
ω: the mean speed of the Earth's rotation,
r_b: the mean radius of the fluid core,
θ: the inclination of the axis to the ecliptic pole,
ν: the angular velocity of the disturbing body with respect to the rotating frame,
ψ_0: a constant.

In deriving the expression the terms in z^2 are omitted, because there are no coupling modes on the rotational motion of the Earth. The term K_3 affects the change of the rotation of the Earth. We consider linear responses of the tidal potential in the motion of the fluid core, i.e., we take into account the modes of xz, yz and l^2. The potential due to the rotational motion of the deformable Earth is formulated by Molodensky (1961) as follows.

$$V = \phi + K + V_i\,, \tag{5}$$

and

$$\phi = -(\omega_1 xz + \omega_2 yz)\,, \tag{6}$$

V_i: the induced potential due to the redistribution of mass, where ω_i are the components of the angular velocity of the Earth ($i = 1, 2, 3$). The change of the pressure from hydrostatic equilibrium is

$$p = -\eta\varrho\,, \tag{7}$$

and η is the change of the equipotential surface due to displacement, $\mathbf{u}$,

$$\eta = \mathbf{u}\cdot\nabla W\,, \tag{8}$$

where ∇W is the gradient of the geopotential at the equilibrium in the fluid core. We also define the perturbing potential, ψ, as the excess of the pressure per unit mass to the additional potential as follows,

$$\psi = P/\varrho - V. \tag{9}$$

Poisson's equation, which governs the induced potential V_i, can be written by the Laplacian equation with the assumption made in Section 5. We can derive Navier-Stokes' equation for the motion of the fluid core. New dimensionless variables are defined as

$$\begin{aligned}
\mathbf{u} &= r_b \mathbf{q}, \\
\psi &= r_b^2 \omega^2 \Phi, \\
\tau &= 2\pi\omega t, \\
\mathbf{r} &= \mathbf{r}/r_b, \\
\mathbf{k} &= \boldsymbol{\omega}/|\boldsymbol{\omega}| \\
\nabla W &= -\gamma_0^2 (\mathbf{r} + e^2 z \mathbf{l}_z), \\
\gamma_0^2 &= \frac{GM_{\text{core}}}{r_b^2 \omega^2}, \qquad e^2 = \frac{3}{\gamma_0^2 - 1}, \\
\eta &= \mathbf{q} \cdot \nabla W = -\gamma_0^2 \mathbf{q} \cdot (\mathbf{r} + e^2 z \mathbf{l}_z), \\
\phi_0 &= -(k_1 xz + k_2 yz), \\
K &= K_1 xz + K_2 yz + K_3 l^2, \\
\left(\frac{K_j + (V_i)_j}{K_j}\right) &= \begin{array}{l} \chi_L \text{ at the surface of the fluid core}, \\ \chi_M \text{ at the surface of the mantle}, \end{array} \\
& \quad (j = 1, 2, 3),
\end{aligned} \tag{10}$$

where χ_L and χ_M are Love's number k at the boundaries. Navier-Stokes equations are

$$\ddot{\mathbf{q}} + 2\mathbf{k} \times \dot{\mathbf{q}} + \dot{\mathbf{k}} \times \mathbf{r} = -\nabla\Phi + E\nabla^2 \dot{\mathbf{q}} \tag{11}$$

where $E = \nu/\omega r_b^2$ is Ekman's number, ν is the kinematical viscosity (cm^2/s), and quantities with dots stand for the derivatives with respect to the dimensionless time. Ekman's number E is very small in the Earth's core. We can obtain an approximation to the internal flow without taking account of the viscosity, the equation of ψ, and the relative angular momentum of the fluid core referred to the rotating frame. Those equations are used to describe the motion of the deformable fluid core in terms of the disturbing potential Φ_{ij}, and the relative angular velocity k_i, where Φ_{ij} are defined by

$$\Phi = \Phi_{21} xz + \Phi_{22} yz + \Phi_{20} l^2. \tag{12}$$

7. Boundary Flow

We shall omit the secondary effect of the viscous boundary on the redistribution of angular momentum of the internal flow, and both the normal component of the boundary flow and the stress of the boundary flow in the thin boundary layer. The boundary flow is taken into consideration to satisfy the boundary condition of non-slip at the core-mantle boundary and to produce the tangential stress which brings the coupling torque. We assume the form of the boundary flow to be a mode of toroidal field as follows,

$$\begin{aligned} \dot{\mathbf{q}} &= \nabla \times (\chi \nabla W) \quad \text{for non-axially symmetric} \\ &= \nabla \times (\chi_3 \mathbf{k}) \quad \text{for axially symmetric} \end{aligned} \tag{13}$$

where

$$\chi = x f_1 + y f_2 , \qquad \chi_3 = l^2 f_3 .$$

The sign of $\dot{\mathbf{q}}$ is chosen to be positive, if it coincides with that of the rotational velocity. A special mode is chosen for the form of decrease with the distance from the center. The non-axially symmetric mode is taken to be a diurnal periodic variation in the rotating frame, the axially symmetric mode is very close to the steady flow. Contributions of the boundary flow to the relative angular momentum are negligible.

8. Equations of the Rotational Motion of the Earth

The angular momentum relative to the rotating frame is caused by the change of the moment of inertia, C_{ij} in the whole Earth,

$$\Delta C_{ij} = \int (x_k x_k \delta_{ij} - x_i x_j)\, dm \tag{14}$$

and the relative motion in the fluid core,

$$\delta \mathbf{L} = \int_{\text{core}} \mathbf{r} \times \dot{\mathbf{q}}\, \mathrm{d}m , \tag{15}$$

based on the assumption of the very small displacement in the mantle and of the very thin boundary layer.

Torques on the rotational motion of the Earth consist of two parts, the precessional torque $\mathbf{N}_{\text{prec}}$, over the deformable Earth and the frictional torque $\mathbf{N}_f$.

$$\mathbf{N}_f = \int_{\text{Earth}} \mathbf{r} \times \nabla \cdot \mathbf{T}\, \mathrm{d}m , \tag{16}$$

where

$$\tilde{\mathbf{T}} = T_{ij} = E\varrho^2 \omega \{\partial \dot{q}_j / \partial x_i + \partial \dot{q}_i / \partial x_j\} . \tag{17}$$

The torque $\mathbf{N}_f$ reduces to the following by the aid of the assumption of the small displacement in the mantle made in Section 5.

$$\mathbf{N}_f = \int_{\text{core}} \mathbf{r} \times \nabla \cdot \mathbf{T}\, \mathrm{d}m = - \int_{\text{core surface}} \mathbf{n} \times \mathbf{r} \cdot \mathbf{T}\, \mathrm{d}S , \tag{18}$$

where $\mathbf{n}$ is the normal unit vector.

We obtain approximate equations to the rotational motion of the fluid core as

$$\frac{d\mathbf{L}}{dt} + \boldsymbol{\omega} \times \mathbf{L} = \mathbf{N}, \tag{19}$$

and

$$\mathbf{L} = \begin{Bmatrix} A\omega_1 + \Delta C_{13}\omega_3 + \delta L_1 \\ A\omega_2 + \Delta C_{23}\omega_3 + \delta L_2 \\ C\omega_3 + \Delta C_{33}\omega_3 + \delta L_3 \end{Bmatrix} \tag{20}$$

$$\mathbf{N} = \mathbf{N}_{\text{prec}} + \mathbf{N}_f. \tag{21}$$

The equations of the relative angular velocity are written in the form of the external disturbing function K with the aid of the perturbing potential function derived in Section 6.

In deriving Equation (18), we did not discuss in detail the torsional stress in the mantle. We should like to consider the magnetic stress rather than the toroidal stress in the mantle which has a low electrical conductivity.

9. Motion of the Rotating Axis in Space and Changes of the Rotational Speed

The angular velocity of the rotating system with respect to the fixed axes in space is expressed in terms of Eulerian angles ϕ, θ and ψ, which are shown in Paper II, expression (A2). Eulerian angles are defined by the rotation around the ecliptic pole ϕ, the rotation around the axis of the rotating pole ψ and the inclination of the z-axis to the ecliptic pole θ in the right-handed system.

A linear approximation to $\dot{\phi}$ and $\dot{\theta}$ brings the differential equation to the second order. The solution for the precessional angular velocity, $\dot{\phi}$ coincides with that of the rigid Earth and the secular variation of the obliquity, $\dot{\theta}$ is derived by the same mechanism as explained in Paper II, i.e. $\dot{\theta}$ is caused by the loss of the angular momentum which is induced by the differential precessional torque on the fluid core. Adopting Rochester's (1968) value of the magnetic torque between the mantle and the core for the rotation ($\lambda_{\parallel}$ in Paper II, appendix), which corresponds to $\nu = 2.1 \times 10^9$ cm^2 s^{-1} for our model, we obtain a value about $^1/_{12\cdot 6}$ times the observed value of $\dot{\theta} = -0''.32$/century. In our model the damping time of the rotation of the Earth is proportional to the viscosity to the power $-\frac{1}{2}$ with the consideration of the gradient of the boundary flow. In other words, in order to give the observed value of $\dot{\theta}$, a coupling coefficient ($\lambda_{\parallel}$) is required which is 3.5 times larger than Rochester's value. Hide (1969) pointed out that the geographical features at the mantle-core boundary increase the magnitude of the magnetic coupling by about an order of magnitude larger than the value obtained by Rochester and Smylie (1965), even if the height of a typical 'bump' at the mantle-core boundary is significantly less than 4 km. This means that $\dot{\theta}$ may increase by two orders of magnitude greater than the results obtained with the use of Rochester's (1968) value.

The secular deceleration of the rate of the Earth's rotation with the decrease of the obliquity may be explained as the result of a decrease of the maximum moment of inertia of the fluid core. We obtain $\Delta\omega/\omega = -5.8 \times 10^{-14}$/century, which is much smaller than the observed value of the deceleration of the rate of the Earth's rotation -10^{-8}/century.

Both $\dot{\phi}$ and $\dot{\theta}$ show the free oscillation with the angular velocity. The ratio of the angular velocity to the rotational angular velocity is obtained as ($\frac{1}{3}$) (centrifugal force/gravitational force) at the surface of the fluid core. The period of the free oscillation is about 520 sidereal days. It may be an interesting problem to study in detail whether the free oscillation could be excited or not by the planetary motions, because the free oscillation relates to the nearly diurnal free oscillation of the axis of rotation referred to the rotating frame.

10. Conclusion

Discussions in Part I show that there remains a possibility to look for a pertinent mechanism to give the excess secular change in the obliquity of the ecliptic. In Part II the frictional loss of the tangential stress into the mantle from the deformable fluid core at the mantle-core boundary is considered. The result is not unsatisfactory. The numerical value of the change of the obliquity derived with use of the intensity of the magnetic coupling obtained by Rochester (1968) is about 1/12.6 of the observed value. This value may be amplified several times, if we take into consideration Hide's (1969) arguments.

The secular deceleration of the rate of the Earth's rotation due to the change of the obliquity may occur with the increase of the maximum moment of inertia of the deformable fluid core. The numerical result, however, shows that it is very much smaller than the observed value. Therefore, we may consider that the main cause of the deceleration of the Earth's rotation should be looked for in the other phenomena, for example, in the ocean tide or the body tide, not in the boundary layer discussed here.

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