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APPROXIMATION BY UNIMODULAR FUNCTIONS: CORRIGENDUM

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Professor J. Detraz has pointed out to me that the proof of part (b) of [**1**, Theorem 1] is incorrect. The proof will be complete, however, once the following proposition is proved.

PROPOSITION. *Let K be a compact subset of the unit circle T of zero Lebesgue measure, and let λ be a real measure on K. If*

$$(*) \qquad \int_K \frac{\sin(\theta - t)}{1 - \cos(\theta - t)} d\lambda(t) \leqq 0$$

for every θ with $e^{i\theta} \notin K$, then $\lambda = 0$.

Proof. Let

$$v(r, \theta) = \int_K P_r(\theta - t) d\lambda(t)$$

be the harmonic extension of λ to the unit disc U; here, $P_r(\theta)$ is the Poisson kernel for $re^{i\theta}$. v may be continued harmonically across every point of T not in K and v vanishes on $T - K$. If $w(r, \theta)$ is the harmonic conjugate of v on U, then w is harmonic across $T - K$ and the assumption $(*)$ is that $w \leqq 0$ on $T - K$.

The analytic function $-w + iv$ is in the Hardy class H^p for $0 < p < 1$ and has positive boundary values a.e. $d\theta$ on T by assumption. Hence, it is a constant (see [**2**]). Thus λ must be zero.

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REFERENCES

1. Stephen Fisher, *Approximation by unimodular functions*, Can. J. Math. *23* (1971), 257–269.
2. J. Newirth and D. J. Newman, *Positive $H^{1/2}$ functions are constants*, Proc. Amer. Math. Soc. *18* (1967), 958.

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