Canad. Math. Bull. Vol. 54 (3), 2011 pp. 422–429
doi:10.4153/CMB-2011-020-4

Two Conditions on the Structure Jacobi Operator for Real Hypersurfaces in Complex Projective Space

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Abstract. We classify real hypersurfaces in complex projective space whose structure Jacobi operator satisfies two conditions at the same time.

1 Introduction

Let $\mathbb{C}P^m$, $m \geq 2$, be a complex projective space endowed with the metric g of constant holomorphic sectional curvature 4. Let M be a connected real hypersurface of $\mathbb{C}P^m$ without boundary. Let J denote the complex structure of $\mathbb{C}P^m$, N a locally defined unit normal vector field on M and (ϕ, ξ, η, g) the almost contact metric structure induced on M. In particular, $-JN = \xi$ is a tangent vector field to M called the structure vector field on M. We also call $\mathbb{D}$ the maximal holomorphic distribution on M, that is, the distribution on M given by all vectors orthogonal to ξ at any point of M.

The study of real hypersurfaces in non-flat complex space forms is a classical topic in Differential Geometry. The classification of homogeneous real hypersurfaces in $\mathbb{C}P^m$ was obtained by Takagi, see [8], [9], and is given by the following list:

A_1: Geodesic hyperspheres.

A_2: Tubes over totally geodesic complex projective spaces.

B: Tubes over complex quadrics and $\mathbb{R}P^m$.

C: Tubes over the Segre embedding of $\mathbb{C}P^1 \times \mathbb{C}P^n$, where $2n + 1 = m$ and $m \geq 5$.

D: Tubes over the Plucker embedding of the complex Grassmann manifold $G(2, 5)$. In this case $m = 9$.

E: Tubes over the canonical embedding of the Hermitian symmetric space $SO(10)/U(5)$. In this case $m = 15$.

Other examples of real hypersurfaces are ruled real ones, that were introduced by Kimura [4]: take a regular curve γ in $\mathbb{C}P^m$ with tangent vector field X. At each point of γ there is a unique complex projective hyperplane cutting γ so as to be orthogonal not only to X, but also to JX. The union of these hyperplanes is called a ruled real hypersurface. It will be an embedded hypersurface locally although globally it will in general have self-intersections and singularities. Equivalently, a ruled real

Received by the editors September 10, 2008; revised April 15, 2009.
Published electronically March 5, 2011.
The first author is partially supported by MEC-FEDER Grant MTM2007-60731; the second author is supported by R17-2008-001-01001-0 from the National Research Foundation of Korea.
AMS subject classification: **53C15**, 53B25.
Keywords: complex projective space, real hypersurface, structure Jacobi operator, two conditions.

hypersurface is such that $\mathbb{D}$ is integrable or $g(A\mathbb{D},\mathbb{D}) = 0$, where A denotes the shape operator of the immersion.

We will call the Jacobi operator on M with respect to ξ the structure Jacobi operator on M. Then the structure Jacobi operator $R_\xi \in \mathrm{End}(T_pM)$ is given by

$$\big(R_\xi(Y)\big)(p) = \big(R(Y,\xi)\xi\big)(p)$$

for any $Y \in T_pM$, $p \in M$, where R denotes the curvature operator of M in $\mathbb{C}P^m$.

Recently [5] we have classified real hypersurfaces in $\mathbb{C}P^m$ whose structure Jacobi operator satisfies

$$(\nabla_X R_\xi)Y = c\{\eta(Y)\phi AX - g(\phi AX, Y)\xi\} \tag{1.1}$$

for any X, Y tangent to M, where c is a non-zero constant. If we restrict (1.1) to $\mathbb{D}$ we obtain

$$(\nabla_X R_\xi)Y = cg(\phi AX, Y)\xi \tag{1.2}$$

for any $X, Y \in \mathbb{D}$, c being a non-zero constant. We also consider the following condition

$$(R_\xi\phi - \phi R_\xi)X = \omega(X)\xi \tag{1.3}$$

for any $X \in \mathbb{D}$, where ω is an 1-form on M. If we put both conditions together we will prove the following:

Theorem 1.1 *Let M be a real hypersurface in $\mathbb{C}P^m$, $m \geq 3$, satisfying (1.2) and (1.3). Then $c < 0$ and*

(i) *if $c \neq -1$, M is locally congruent to a geodesic hypersphere of radius r such that $\cot^2(r) = -c$,*

(ii) *if $c = -1$, M is locally congruent to a tube of radius $\frac{\pi}{4}$ over a complex submanifold of $\mathbb{C}P^m$.*

Results related to our theorem have been obtained by Ki and the second author [3] for the shape operator of M, and by Baikoussis [1] in the case of the Ricci tensor.

2 Preliminaries

Throughout this paper, all manifolds, vector fields, *etc.*, will be considered of class C^∞ unless otherwise stated. Let M be a connected real hypersurface in $\mathbb{C}P^m$, $m \geq 2$, without boundary. Let N be a locally defined unit normal vector field on M. Let ∇ be the Levi–Civita connection on M and (J, g) the Kaehlerian structure of $\mathbb{C}P^m$.

For any vector field X tangent to M we write $JX = \phi X + \eta(X)N$, and $-JN = \xi$. Then (ϕ, ξ, η, g) is an almost contact metric structure on M. That is, we have

$$\phi^2 X = -X + \eta(X)\xi, \quad \eta(\xi) = 1, \quad g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y) \tag{2.1}$$

for any tangent vectors X, Y to M. From (2.1) we obtain

$$\phi\xi = 0, \quad \eta(X) = g(X, \xi). \tag{2.2}$$

From the parallelism of J we get

$$(\nabla_X \phi)Y = \eta(Y)AX - g(AX, Y)\xi \tag{2.3}$$

and

$$\nabla_X \xi = \phi AX \tag{2.4}$$

for any X, Y tangent to M, where A denotes the shape operator of the immersion. As the ambient space has holomorphic sectional curvature 4, the equations of Gauss and Codazzi are given, respectively, by

$$\begin{aligned} R(X, Y)Z = {} & g(Y, Z)X - g(X, Z)Y + g(\phi Y, Z)\phi X - g(\phi X, Z)\phi Y \\ & - 2g(\phi X, Y)\phi Z + g(AY, Z)AX - g(AX, Z)AY, \end{aligned} \tag{2.5}$$

and

$$(\nabla_X A)Y - (\nabla_Y A)X = \eta(X)\phi Y - \eta(Y)\phi X - 2g(\phi X, Y)\xi \tag{2.6}$$

for any tangent vectors X, Y, Z to M, where R is the curvature tensor of M.

From the Gauss equation we have

$$R_\xi(X) = X - \eta(X)\xi + \eta(A\xi)AX - \eta(AX)A\xi \tag{2.7}$$

for any X tangent to M.

In the sequel we need the following results:

Theorem 2.1 ([6]) *Let M be a real hypersurface of $\mathbb{C}P^m$, $m \geq 2$. Then the following are equivalent:*

- (i) *M is locally congruent to one of the homogeneous hypersurfaces of class either A_1 or A_2.*
- (ii) *$\phi A = A\phi$.*

We define the type number of M at $p \in M$, $t(p)$, as the rank of the shape operator of M at p. We have:

Theorem 2.2 ([7]) *Let M be a real hypersurface in $\mathbb{C}P^m$, $m \geq 3$, satisfying $t(p) \leq 2$ for any point $p \in M$. Then M is a ruled real hypersurface.*

Theorem 2.3 ([5]) *There exist no real hypersurfaces in $\mathbb{C}P^m$, $m \geq 3$, whose shape operator is given by $A\xi = \alpha\xi + \beta U$, $AU = \beta\xi + \frac{\beta^2 - (c+1)}{\alpha}U$, $AX = -\frac{c+1}{\alpha}X$, for any tangent vector orthogonal to* Span$\{\xi, U\}$, *where U is a unit vector field in $\mathbb{D}$, α and β are non-vanishing smooth functions defined on M and c is a constant.*

3 Some Lemmas

From condition (1.3), for any $Y, Z \in \mathbb{D}$ we get $g\big(R_\xi(\phi Y), Z\big) + g\big(R_\xi(Y), \phi Z\big) = 0$. Differentiating covariantly this equation in the direction of $X \in \mathbb{D}$ we obtain

$$g\Big(\nabla_X\big(R_\xi(\phi Y)\big), Z\Big) + g\big(R_\xi(\phi Y), \nabla_X Z\big) + g\Big(\nabla_X\big(R_\xi(Y)\big), \phi Z\Big) + g\big(R_\xi(Y), \nabla_X \phi Z\big) = 0.$$

This yields

$$g\big((\nabla_X R_\xi)\phi Y, Z\big) + g\big(R_\xi(\nabla_X \phi Y), Z\big) + g\big(R_\xi(\phi Y), \nabla_X Z\big) + g\big((\nabla_X R_\xi)Y, \phi Z\big) + g\big(R_\xi(\nabla_X Y), \phi Z\big) + g\big(R_\xi(Y), \nabla_X \phi Z\big) = 0.$$

From (1.2), bearing in mind that $R_\xi(\xi) = 0$, we obtain

$$g\big(R_\xi(\nabla_X \phi Y), Z\big) + g\big(R_\xi(\phi Y), \nabla_X Z\big) + g\big(R_\xi(\nabla_X Y), \phi Z\big) + g\big(R_\xi(Y), \nabla_X \phi Z\big) = 0.$$

As $\nabla_X \phi Z = (\nabla_X \phi)Z + \phi \nabla_X Z = -g(AX, Z)\xi + \phi \nabla_X Z$, we have

$$g\big(R_\xi \phi(\nabla_X Y), Z\big) + g\big(R_\xi(\phi Y), \nabla_X Z\big) - g\big(\phi R_\xi(\nabla_X Y), Z\big) - g\big(\phi R_\xi(Y), \nabla_X Z\big) = 0.$$

Now from (1.3) we obtain $\omega(Z)g(\xi, \nabla_X Y) + \omega(Y)g(\xi, \nabla_X Z) = 0$. That is

$$\omega(Z)g(\phi AX, Y) + \omega(Y)g(\phi AX, Z) = 0, \tag{3.1}$$

for any $X, Y, Z \in \mathbb{D}$. M is called Hopf if ξ is a principal vector field, that is $A\xi = \alpha\xi$ for a certain function α. Let us suppose that M is non-Hopf. Thus we can write, at least locally, $A\xi = \alpha\xi + \beta U$, where U is a unit vector field in $\mathbb{D}$ and β a non-vanishing function defined on M.

If we take $X = U = Y$, $Z = \phi U$ in (3.1) we get

$$-\omega(\phi U)g(AU, \phi U) + \omega(U)g(AU, U) = 0. \tag{3.2}$$

If we take $X = U$, $Y = Z = \phi U$ in (3.1) we have

$$\omega(\phi U)g(AU, U) = 0. \tag{3.3}$$

Lemma 3.1 *If M satisfies* (1.2), (1.3) *and $g(AU, U) \neq 0$, then $\omega(Y) = 0$, for any $Y \in \mathbb{D}$.*

Proof As $g(AU, U) \neq 0$, from (3.3) we get $\omega(\phi U) = 0$. Then, from (3.2), $\omega(U) = 0$.

If in (3.1) we take $Z = \phi U$, we have $\omega(Y)g(AX, U) = 0$, for any $X, Y \in \mathbb{D}$. This means that $\omega(Y)AU$ has no component in $\mathbb{D}$. If there exists $Y \in \mathbb{D}$ such that $\omega(Y) \neq 0$, AU has no component in $\mathbb{D}$, thus $g(AU, U) = 0$ and we arrive at a contradiction. ∎

Lemma 3.2 *With the same conditions as in Lemma* 3.1*, if $g(AU,U) = 0$, either $\omega(Y) = 0$, for any $Y \in \mathbb{D}$, or M is ruled.*

Proof In this case, (3.2) becomes $\omega(\phi U)g(AU,\phi U) = 0$. Taking $Y = U, X = Z = \phi U$ in (3.1) we get

$$-\omega(\phi U)g(A\phi U,\phi U) + \omega(U)g(A\phi U,U) = 0, \tag{3.4}$$

and taking $Y = Z = U, X = \phi U$ in (3.1) we have

$$\omega(U)g(A\phi U,\phi U) = 0. \tag{3.5}$$

If we suppose $\omega(\phi U)\omega(U) \neq 0$, from (3.4) and (3.5) we obtain

$$g(A\phi U,\phi U) = g(AU,\phi U) = 0. \tag{3.6}$$

Take $Z = U$ in (3.1). Then $\omega(U)g(\phi AX,Y)+\omega(Y)g(\phi AX,U) = 0$, for any $X,Y \in \mathbb{D}$. If now $X = U$, we have $\omega(U)g(\phi AU,Y) = 0$, for any $Y \in \mathbb{D}$. This yields $\phi AU{=}0$ and this gives

$$AU = \beta\xi. \tag{3.7}$$

Choosing $Z = \phi U$ in (3.1) we obtain $\omega(\phi U)g(\phi AX,Y)+\omega(Y)g(AX,U) = 0$, for any $X,Y \in \mathbb{D}$. From (3.7), $g(AX,U) = 0$, for any $X \in \mathbb{D}$. Therefore, the above equation yields $g(\phi AX,Y) = 0$, for any $X,Y \in \mathbb{D}$. As $\mathbb{D}$ is ϕ-invariant this yields that M is ruled.

If we now suppose $\omega(U) \neq 0$, but $\omega(\phi U) = 0$, take $Z = Y = U$ in (3.1). We obtain $g(\phi AX,U) = 0$, for any $X \in \mathbb{D}$. Thus $A\phi U = 0$. Taking $Y \in \mathbb{D}_U = \operatorname{Span}\{\xi,U,\phi U\}^{\perp}$, $Z = U$ in (3.1) we get $\omega(U)g(\phi AX,Y)+\omega(Y)g(\phi AX,U) = 0$, for any $X \in \mathbb{D}$. As $A\phi U = 0$, this yields $g(\phi AX,Y) = 0$, for any $X \in \mathbb{D}$, $Y \in \mathbb{D}_U$. Thus $A\phi Y = 0$, for any $Y \in \mathbb{D}_U$. Thus the type number at any point is at most 2, and from Theorem 2.2, M is ruled.

Now if $\omega(U) = 0$, $\omega(\phi U) \neq 0$, from (3.2), (3.3) and (3.4) we have $g(A\phi U,\phi U) = g(AU,\phi U) = 0$. Taking $Y = Z = \phi U$ in (3.1) we obtain $g(AX,U) = 0$, for any $X \in \mathbb{D}$. Thus $AU = \beta\xi$. If in (3.1) we take $Z = U, Y = \phi U$, we have $g(\phi AX,U) = 0$, for any $X \in \mathbb{D}$. Therefore, $A\phi U = 0$. Take now $Y \in \mathbb{D}_U$, $Z = \phi U$. From (3.1), $\omega(\phi U)A\phi Y - \beta\omega(Y)\xi$ has no component in $\mathbb{D}$. Then from any $X \in \mathbb{D}$ we obtain $g(A\phi Y,X) = 0$. As $\mathbb{D}_U$ is ϕ-invariant this means $AY = 0$ for any $Y \in \mathbb{D}_U$ and M is ruled.

Finally, we consider the case $\omega(U) = \omega(\phi U) = 0$. Taking $Z = U$ in (3.1) we have $\omega(Y)g(\phi AX,U) = 0$, and taking $Z = \phi U$, we get $\omega(Y)g(AX,U) = 0$, for any $X,Y \in \mathbb{D}$. If there exists $Y \in \mathbb{D}_U$ such that $\omega(Y) \neq 0$, we should have $AU = \beta\xi$, $A\phi U = 0$ and $\mathbb{D}_U$ is A-invariant. If in (3.1) we take $Y = Z \in \mathbb{D}_U$ such that $\omega(Y) \neq 0$, we get $g(\phi AX,Y) = 0$ for any $X \in \mathbb{D}_U$, thus $A\phi Y = 0$. Now from (3.1) we obtain $g(\phi AX,Z) = 0$ for any $X,Z \in \mathbb{D}_U$, and M must be ruled. ∎

4 The Non-Hopf Case

From Lemmas 3.1 and 3.2, suppose that M is ruled. Then $A\xi = \alpha\xi + \beta U$, $AU = \beta\xi$, $AZ = 0$, for any $Z \in \operatorname{Span}\{\xi, U\}^{\perp}$. Thus $R_\xi(\phi U) = \phi U$ and $R_\xi(U) = U + \alpha AU - \beta A\xi = (1-\beta^2)U$. Thus $\phi R_\xi(\phi U) = -U$, $R_\xi(\phi^2 U) = -R_\xi(U) = (\beta^2 - 1)U$. If M satisfies (1.3), $\phi R_\xi(\phi U) - R_\xi(\phi^2 U) = -\beta^2 U = \omega(\phi U)\xi$. This yields $\beta = 0$, which is impossible.

Thus we must suppose that $\omega(X) = 0$, for any $X \in \mathbb{D}$. Now (1.3) becomes $R_\xi\phi = \phi R_\xi$. Therefore $R_\xi(\phi U) = \phi U + \alpha A\phi U = \phi R_\xi(U) = \phi(U + \alpha AU - \beta A\xi)$. So we have

$$\alpha A\phi U = \alpha\phi AU - \beta^2\phi U. \tag{4.1}$$

From (4.1) it is clear that $\alpha \neq 0$. Moreover, if $X \in \mathbb{D}_U$, $R_\xi(\phi X) = \phi X + \alpha A\phi X = \phi R_\xi(X) = \phi(X + \alpha AX)$. As $\alpha \neq 0$, we get

$$A\phi X = \phi AX \tag{4.2}$$

for any $X \in \mathbb{D}_U$. Let $X, Y \in \mathbb{D}$. From (1.2), $g\big((\nabla_X R_\xi)Y, \xi\big) = cg(\phi AX, Y) = g\big(Y, (\nabla_X R_\xi)\xi\big) = -g\big(Y, R_\xi(\phi AX)\big)$. If we develop this equation we obtain

$$(c+1)g(\phi AX, Y) = -\alpha g(Y, A\phi AX) + \beta^2 g(U, \phi AX)g(U, Y) \tag{4.3}$$

for any $X, Y \in \mathbb{D}$. Thus $(c+1)\phi AX + \alpha A\phi AX - \beta^2 g(U, \phi AX)U$ has no component in $\mathbb{D}$. Therefore

$$(c+1)\phi AX + \alpha A\phi AX - \beta^2 g(U, \phi AX)U = -\alpha\beta g(A\phi U, X)\xi \tag{4.4}$$

for any $X \in \mathbb{D}$.

From (4.3) we also obtain that $(c+1)A\phi Y + \alpha A\phi AY - \beta^2 g(U, Y)A\phi U$ has no component in $\mathbb{D}$, for any $Y \in \mathbb{D}$. Thus

$$(c+1)A\phi X + \alpha A\phi AX - \beta^2 g(U, X)A\phi U = (c+1)\beta g(\phi X, U)\xi - \alpha\beta g(A\phi U, X)\xi \tag{4.5}$$

for any $X \in \mathbb{D}$.

From (4.2), (4.4) and (4.5) we obtain $\beta^2 g(U, \phi AX)U = 0$. This means that $g(AX, \phi U) = 0$ for any $X \in \mathbb{D}_U$. From (4.1), for any $X \in \mathbb{D}_U$ we have $g(\phi AU, X) = 0$. This yields $g(AU, X) = 0$, for any $X \in \mathbb{D}_U$. Therefore, $\mathbb{D}_U$ is A-invariant, and from (4.2) the eigenspaces of the restriction of A to $\mathbb{D}_U$ are holomorphic, which means that they are invariant by ϕ.

First suppose that $c = -1$. From (4.4) and (4.5), we have now

$$\alpha A\phi AX - \beta^2 g(U, \phi AX)U = -\alpha\beta g(A\phi U, X)\xi \quad \text{and}$$
$$\alpha A\phi AX - \beta^2 g(U, X)A\phi U = -\alpha\beta g(A\phi U, X)\xi$$

for any $X \in \mathbb{D}$. If we take $X = \phi U$ we have $g(A\phi U, \phi U) = 0$. From (4.1) we obtain $\alpha g(A\phi U, U) = \alpha g(\phi AU, U) = -\alpha g(A\phi U, U)$. This gives $g(AU, \phi U) = 0$. Thus

$$A\phi U = 0. \tag{4.6}$$

https://doi.org/10.4153/CMB-2011-020-4 Published online by Cambridge University Press

Again from (4.1), $\alpha\phi AU = \beta^2\phi U$. Applying ϕ to this equality we get

$$AU = \beta\xi + \frac{\beta^2}{\alpha}U. \tag{4.7}$$

From (4.3), for any $X \in \mathbb{D}_U$, $A\phi AX = 0 = \phi A^2 X$. If we suppose that $AX = \lambda X$, $\lambda = 0$. Thus the type number $t(p) \leq 2$ at any point of M. Thus M should be ruled. Then, from [7], we know that $A\xi = \alpha\xi + \beta U$, $AU = \beta\xi$. This and (4.7) give a contradiction. So we must suppose that $c \neq -1$.

From (4.4) and (4.5) we obtain $(c+1)\phi AX - \beta^2 g(U, \phi AX)U = (c+1)A\phi X - \beta^2 g(U, X)A\phi U - (c+1)\beta g(\phi X, U)\xi$, for any $X \in \mathbb{D}$. Taking $X = U$ in the above equation we have $(c+1)\phi AU - \beta^2 g(U, \phi AU)U = (c+1)A\phi U - \beta^2 g(U, X)A\phi U$. Taking its scalar product with ϕU we get

$$(c+1)g(AU, U) = (c+1-\beta^2)g(A\phi U, \phi U). \tag{4.8}$$

If we take the scalar product of (4.1) and ϕU we have

$$\alpha g(A\phi U, \phi U) = \alpha g(AU, U) - \beta^2. \tag{4.9}$$

Moreover, from (4.1) we have $g(AU, \phi U) = 0$. From (4.8) and (4.9) we get $g(A\phi U, \phi U) = -\frac{c+1}{\alpha}$ and $g(AU, U) = \frac{\beta^2-(c+1)}{\alpha}$. That means $AU = \beta\xi + \frac{\beta^2-(c+1)}{\alpha}U$, $A\phi U = -\frac{c+1}{\alpha}\phi U$. From (4.2) and (4.3), for any $X \in \mathbb{D}_U$ such that $AX = \lambda X$, $\lambda(c+1+\lambda\alpha) = 0$. Thus either $\lambda = 0$ or $\lambda = -\frac{c+1}{\alpha}$. From Theorem 2.3 at least there exists $X \in \mathbb{D}_U$ such that $AX = \lambda X$ with $\lambda \neq -\frac{c+1}{\alpha}$. Thus there exists $X \in \mathbb{D}_U$ such that $AX = 0$. The proof of the main theorem in [5] yields that this is not posssible.

Thus M must be Hopf.

5 The Hopf Case

Suppose that $A\xi = \alpha\xi$. You can easily see that now $R_\xi\phi = \phi R_\xi$, Then, if $X \in \mathbb{D}$ satisfies $AX = \lambda X$, $\alpha\lambda\phi X = \alpha A\phi X$. Thus either $\alpha = 0$ or $A\phi X = \lambda\phi X$. From Theorem 2.1, the second possibility yields M is of type either A_1 or A_2.

If $\alpha = 0$, then

$$\begin{aligned} g\big((\nabla_X R_\xi)Y, \xi\big) &= g\big(Y, (\nabla_X R_\xi)\xi\big) = -g\big(Y, R_\xi(\phi AX)\big) \\ &= -g\big(R_\xi(Y), \phi AX\big) = -g(Y, \phi AX) = cg(\phi AX, Y). \end{aligned}$$

Thus $c = -1$, and M is locally congruent to a tube of radius $\frac{\pi}{4}$ over a complex submanifold of $\mathbb{C}P^m$ (see [2], [8]). It is easy to see that these real hypersurfaces satisfy both (1.2) and (1.3).

If we consider a geodesic hypersphere of radius r, $0 < r < \frac{\pi}{2}$, we can write $A\xi = 2\cot(2r)\xi$, $AX = \cot(r)X$, for any $X \in \mathbb{D}$. Now $(\nabla_X R_\xi)Y = \big(\cot^2(r) - 1\big)\nabla_X Y - g(\phi AX, Y)\xi - 2\cot(2r)A\nabla_X Y - 4\cot^2(2r)g(\phi AX, Y)\xi$. In order to satisfy (1.2), taking the scalar product of the above equation and ξ we must have $\cot^2(r) = -c$. Then

$g\big((\nabla_X R_\xi)Y, W\big) = 0$ for any $W \in \mathbb{D}$. This means that geodesic hyperspheres appearing in our theorem satisfy both (1.2) and (1.3).

If we consider a type A_2 real hypersurface, we can write $A\xi = 2\cot(2r)\xi$, and there exist $X, W \in \mathbb{D}$ such that $AX = \cot(r)X$, $AW = -\tan(r)W$. If we repeat the above reasoning we have $-\cot^2(r) = c = -\tan^2(r)$. Thus $c = -1$, $r = \frac{\pi}{4}$, and this finishes the proof. ■

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