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EIGENVALUE APPROXIMATION METHODS FOR QUANTUM LATTICE HAMILTONIANS

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The thesis consists of five major parts or chapters, the first of which is an introduction and review relating the field of investigation to problems in quantum field theory and statistical mechanics. Although the thesis generally deals with eigenvalue bounding techniques, these methods are applied with a view to calculating the critical properties of infinite systems [1]. The approach advocated in the thesis intends the use of approximate methods to calculate the thermodynamics of large, but finite lattices. Finite size scaling [3] can then be used to estimate the bulk system thermodynamics, especially critical phenomena, from the approximate large lattice data.

The approximations used include projected mean fields [5], for both ground and excited states, correlated product wave functions [6], which are extended to kink states, and blocking approximations [2]. Part of this work is motivated by the promise these wave functions have in their application to variationally guided Monte Carlo methods for finite lattices [4]. Also, it is shown that the critical indices predicted by the finite size rescaling formalism, are left invariant by the application of an approximate (blocking) renormalisation group to the finite lattices involved. Moreover, this invariance of the finite size rescaling equations is used to define a sequence transformation suitable for improving

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sequences of critical exponent estimates obtained from Roomanay and Wyld beta functions [7]. The various techniques are tried on the (1+1) dimensional Ising, (1+1) dimensional 3 state Potts, (2+1) dimensional Ising gauge and (2+1) dimensional Ising spin models.

Lower bounding techniques are also addressed. Firstly, a method proposed by Singh [8] is tested numerically on the quantum anharmonic oscillator. In the final chapter, lower bounds to quantum lattice Hamiltonian ground states are reviewed, and a generalisation of the methods to excited states given.

The thesis finishes with the conclusion that little hope is held for further significant progress being made in this area by deterministic eigenvalue calculations. Monte Carlo methods for quantum systems are therefore seen as the most promising path for future investigations. This conclusion, however, does not make the work of the thesis obsolete, since the approximation methods used throughout the thesis are a vital ingredient in improving these new stochastic methods of eigenvalue calculation.

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