SACKS, G. E., *Degrees of Unsolvability* (Annals of Mathematics Studies, No. 55, Princeton University Press, London: Oxford University Press, 1963), ix+174 pp., 28s.

The second sentence of this book is " We say f and g have the same degree of recursive unsolvability if f is recursive in g and g is recursive in f". This sentence gives a good indication of the prerequisites that a reader of this book should have: a fair knowledge of recursive functions and an insensitivity to omissions of the word " that ". The number of misprints appears to be very small, except for several trivial errors in the first four pages and a spelling mistake in the page headings to Section **5**.

This book gives a connected account of certain results concerning functions of the natural numbers. More specifically, the results described relate to the partial order structure of the set of sets of functions with the same degree of recursive unsolvability. This topic is about ten years old. The book obviously will be essential reading for those hoping to work in the subject, but anyone with the necessary background will be able to read it profitably.

R. M. DICKER

LOÈVE, MICHEL, *Probability Theory* (D. van Nostrand Co., London, 3rd ed., 1963), xvi+685 pp., 115s.

The third edition of this book is very similar to the second edition published in 1960. The only major change is in Section 36 on martingale times.

It is divided into five parts. Part One is on " Notions of Measure Theory " needed in the rest of the book. Part Two is on probability distributions and their characteristic functions. Part Three is on limit theorems for independent random variables. Part Four is on conditional probability, martingales, ergodic theorems and second order theory. Part Five is on stochastic processes in continuous time. There is also an Introductory Part on random events which includes an account of Markov chains.

The emphasis throughout is on rigour and mathematical generality. Loève only considers real-valued random variables and stochastic processes but he never makes the assumption, dear to applied probability theorists, that random variables are either integer-valued or continuous. This makes the book rather heavy, but it is good to have one fully rigorous book on probability theory. Its main weakness is that it is not well motivated. Most mathematicians are attracted to probability theory because of its applications. But Loève concentrates almost exclusively on existence, convergence and uniqueness theorems.

M. W. BIRCH

HEYTING, A., *Axiomatic Projective Geometry* (Noordhoff-Groningen; North-Holland Publishing Co., Amsterdam, 1963.), xii+148 pp., 36s.

It is well known that Desargues' theorem on triangles in perspective, although it holds in the familiar real and complex projective geometries, cannot be deduced from the axioms of incidence alone in two dimensions. The purpose of this book is to investigate in detail the logical relations between Desargues' proposition, possibly weakened by the introduction of additional incidences, Pappus' proposition and similar configurations. (It is necessary, in a systematic study of this type, to distinguish between a *proposition* concerning the elements of an axiomatic theory and a *theorem*, or valid proposition, which is deducible from the axioms.) An aspect of especial interest is the relationship between these propositions and the properties of the algebraic systems obtained by introducing coordinates into the geometry.

After a brief preliminary chapter containing remarks on the axiomatic method, analytic geometry and vector spaces over a division ring, the various geometrical

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propositions are introduced in Chapter 2. Next, algebraic systems called ternary fields are used to provide coordinates in the plane and a study is made of the algebraic consequences of the geometrical situations. The remaining chapters deal with the extension to three dimensions, with the fundamental proposition of projective geometry and with the concept of order.

The book is generously provided with figures which, whilst not essential, are of considerable help to the reader in following the exacting logical arguments. There are very few references and no bibliography, the reader being referred for these to G. Pickert's comprehensive monograph " Projektive Ebenen " (Springer, Berlin, 1955). This is perhaps a slight defect in a book which is otherwise compact and self-contained. A small related complaint is that the name " Moulton plane " might have been attached to the model, denoted by M, used to establish the independence of Desargues' proposition from the axioms of incidence.

The geometrical propositions treated in the book are strictly confined to those relevant to its logical purpose. Thus conics are not mentioned (except for a sentence to that effect on p. 61). It cannot therefore be regarded as a textbook for a traditional course in Projective Geometry. This remark, however, is in no way a criticism: those who are interested in its important theme will find it very useful.

D. MONK

QUINE, WILLARD VAN ORMAN, *Set Theory and its Logic* (Harvard University Press, London: Oxford University Press, 1964), xv+359 pp., 48s.

This account of set theory and its foundations is sound, lucid and well produced. The author has a lively style and strikes the right balance in presenting his material, some of which is original. Few words are wasted, unnecessary details being omitted, yet important points are always clearly explained and are often presented in a new way. It is therefore perhaps the most concise and readable general survey of axiomatic set theory at present available.

The book is unique in first introducing the basic topics of abstract set theory within a framework which does not commit the author to any particular one of the usual axiomatic systems. This neutrality is preserved by adopting only very weak comprehension axioms (axioms asserting the existence of particular classes or sets) and, where stronger existence assumptions are needed, incorporating them as explicit hypotheses for particular theorems. Use of the axiom of choice is made explicit in the same way.

This approach has two advantages. It gives the reader a sound, unprejudiced basis for an appreciation and comparison of the main axiomatic systems of set theory and also emphasises the central problem of such theories—that of accepting comprehension axioms which are strong enough for all mathematical needs but which are weak enough to prevent the derivation of the well-known paradoxes which follow from the naïve comprehension schema

$$(\exists y)(\forall x)[x \in y \Leftrightarrow P(x)]$$

where $P(x)$ is any condition which can be formulated in the system without using the variable " y ". An interesting by-product is the author's treatment of the natural numbers and mathematical induction which, unlike the classical treatments, does not require the existence of infinite sets.

Part I of the book contains the rigorous development of elementary set theory and the arithmetic of natural numbers, the use of a notation for class abstracts (the author calls them " virtual classes " since the existence of a corresponding real class is not implied) making for conciseness and clarity of expression. Part II covers the following topics: Rational and Real numbers, constructed in such a way that the reals contain

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