

# High accuracy measurement of gravitational wave back-reaction in the OJ287 black hole binary

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**Abstract.** Blazar OJ287 exhibits large thermal flares at least twice every 12 years. The times of these flares have been predicted successfully using the model of a quasi-Keplerian eccentric black hole binary where the secondary impacts the accretion disk of the primary, creating the thermal flares. New measurements of the historical light curve have been combined with the observations of the 2015 November/December flare to identify the impact record since year 1886, and to constrain the orbit of the binary. The orbital solution shows that the binary period, now 12.062 years, is decreasing at the rate of 36 days per century. This corresponds to an energy loss to gravitational waves that is  $6.5 \pm 4$  % less than the rate predicted by the standard quadrupolar gravitational wave (GW) emission. We show that the difference is due to higher order gravitational radiation reaction terms that include the dominant order tail contributions.

**Keywords.** black hole physics, gravitational waves, galaxies: BL Lacertae objects: individual (OJ287)

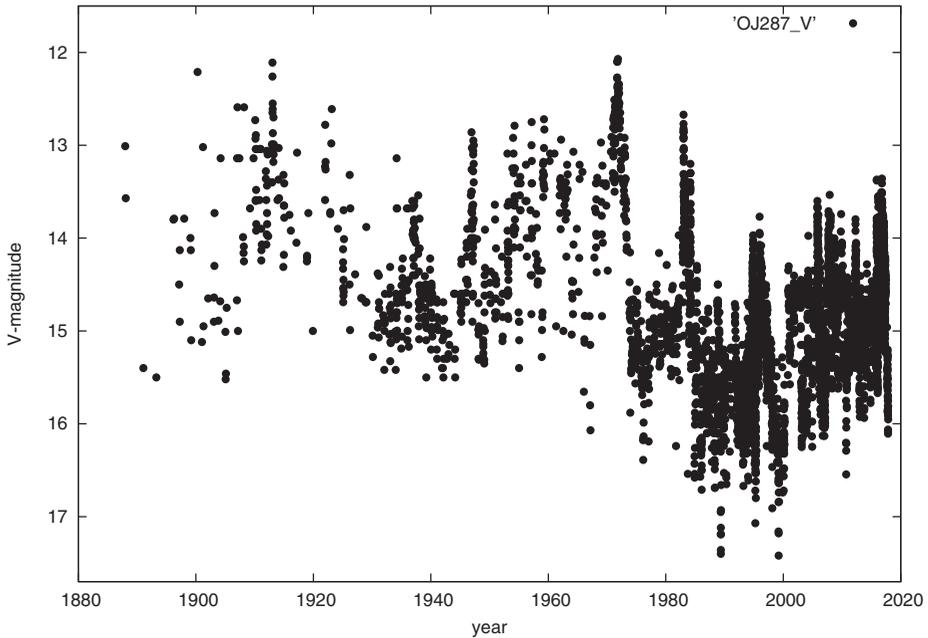
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## 1. Introduction

OJ287 is a potential nHz gravitational wave source that could be observed by the Pulsar Timing Array method in near future when the detection sensitivity has increased by about a factor of three (Liu *et al.* 2012, Babak *et al.* 2016). The nucleus of this galaxy contains a binary black hole system with a 12 yr period and component masses  $1.84 \times 10^{10} M_{\odot}$  and  $1.5 \times 10^8 M_{\odot}$ . Its binary nature was discovered already in 1987 based on huge flares at about 12 yr intervals (Sillanpää *et al.* 1988, see Figure 1). Since then the arrival time of the flares has been predicted successfully in multiple occasions with the rms error of 16 days (See Table 1). In addition, it has been confirmed that the flares belong to the rare category of thermal flares (Valtonen *et al.* 2012, Valtonen *et al.* 2016). Therefore it is likely that the flares arise after a collision of the secondary black hole on the accretion disk of the primary (Ivanov *et al.* 1998, Pihajoki 2016) and it is these collisions that allow us to follow the orbital motion of the secondary around the primary black hole in great detail. The host galaxy of this system has V magnitude about 18 (Takalo *et al.* 1990, Nilsson 2017) which makes it similar to NGC 4889 in intrinsic brightness; the latter galaxy is one of the two bright central galaxies in the Coma cluster of galaxies, and is believed to harbor a supermassive black hole of mass similar to the primary in OJ287 (Graham and Scott 2013).

## 2. Overview

In recent years a great deal of new data have been added to the optical light curve of OJ287 (Hudec *et al.* 2013, Hudec 2017). They allow the recognition of 25 major flares since 1886 which may arise from disk impacts. Fourteen of them have good enough light curves to identify the start of the thermal flare (listed in Table 1) while two more are positioned on the time axis by dense observational upper limits (1906 and 1945 flares). Many have occurred during the Northern hemisphere summer period when OJ287 was not observed (especially in 1920's and 1930's) and cannot be used in this work. A set of nine flare timings is enough to solve the binary orbit. They determine uniquely the 8 parameters of our BBH central engine model: the two masses, the spin of the primary BH, the major axis, orbital eccentricity and the initial phase of the orbit, plus the two parameters of the standard accretion disk (Valtonen *et al.* 2010). With the addition of the 2015 flare timing we may explore an additional parameter which in this case is the magnitude of the gravitational wave tail term, not included in previous work. Once a solution for the orbit has been found using ten impact timings, we verify that the



**Figure 1.** Historical light curve of OJ287. It has two dominant periodicities, 12 yr and 56 yr which imply the precession rate of  $(12/56) \times 180$  degrees per period in the quasi-Keplerian orbit model. The corresponding mass of the primary is  $\sim 1.8 \times 10^{10} M_{\odot}$ . The light variations arise from the combination of varying Doppler boosting due to the wobble of the primary jet (Valtonen and Pihajoki 2013), from varying accretion rate due to tidal forces on the primary accretion disk (Sundelius *et al.* 1996, Valtonen *et al.* 2009), and from radiation arising from impacts of the secondary on the accretion disk (Lehto and Valtonen 1996).

solution is consistent with all observed flare times in Table 1, as well as the limits based on the observed upper limits. Thus we have altogether 16 constraints that have to be satisfied by a model with 9 parameters. Obviously, in general such problems do not have a solution, but if a solution is found, it is strongly overdetermined and produces very exact values for the parameters. In what follows, we briefly describe our PN-accurate description for BBH dynamics and how we determine the orbital parameters. Finally, we discuss the implications of our findings.

### 3. The Post-Newtonian orbit model

The relative acceleration between the two black holes in their center of mass frame, namely  $\ddot{\mathbf{x}}$ , can be divided in several PN contributions (Mora and Will 2004). Traditionally, the PN approximation provides the equations of motion of a binary as corrections to the Newtonian equations of motion in powers of  $(v/c)^2 \sim GM/(c^2 R)$ , where  $v$ ,  $M$ ,  $R$  and  $c$  are the characteristic orbital velocity, the total mass, the typical orbital separation of the binary, and speed of light, respectively. The relative acceleration and the precessional dynamics of the spin direction  $\mathbf{s}_1$  are described by

$$\begin{aligned} \ddot{\mathbf{x}} \equiv \frac{d^2 \mathbf{x}}{dt^2} = & \ddot{\mathbf{x}}_0 + \ddot{\mathbf{x}}_{1PN} + \ddot{\mathbf{x}}_{2PN} + \ddot{\mathbf{x}}_{2.5PN} \\ & + \ddot{\mathbf{x}}_{SS} + \ddot{\mathbf{x}}_{3PN} + \ddot{\mathbf{x}}_Q + \ddot{\mathbf{x}}_{3.5PN} \\ & + \ddot{\mathbf{x}}_{4PNtail} + \ddot{\mathbf{x}}_{SO} + \ddot{\mathbf{x}}_{4.5PN}, \end{aligned} \quad (3.1)$$

**Table 1.** Overview of quasi-Keplerian binary models of OJ287.

flare/ parameter	1987-model [1]	1995-model [2]	2006-model [3]	2017-model [4]
1913	1913.064	1912.124	1912.984	1912.981
1947	1948.014	1947.304	1947.264	1947.282
1957		1956.004	1957.104	1957.085
1959	1959.664	1959.234	1959.184	1959.212
1964		1963.794	1964.164	1964.226
1971	1971.314	1971.144	1971.104	1971.127
1973		1972.964	1972.954	1972.927
1983	1982.964 [5]	1982.964 [5]	1982.964 [5]	1982.964 [5]
1984		1984.124	1984.124	1984.119
1994	1994.614 [6]	1994.574	1994.604	1994.596 [7]
1995		1995.844 [6]	1995.824	1995.841 [8]
2005	2006.264 [6]	2005.704 [6]	2005.784	2005.744 [9]
2007		2007.724 [6]	2007.674 [6]	2007.691 [8]
2015	2017.914 [6]		2015.960 [6]	2015.875 [10]
primary $10^9 M_\odot$	18.4	17.7	18.25	18.35
secondary $10^9 M_\odot$	0.13	0.14 [11]	0.14 [12]	0.15
precession <i>deg</i>	38.6	33.3	39.1	38.7
spin $\chi$	0	0	0 0.28 [12]	0.381

*Notes:*

<sup>1</sup>The 1987 tidal model was published in Sillanpää *et al.* (1988). The parameter values for this model are based on the two dominant frequencies of a quasi-Keplerian binary as determined by Valtonen *et al.* 2006 and matched with the tidal calculation by Sundelius *et al.* (1997). <sup>2</sup>The 1995 model was solved from timings of the secondary impacting on the accretion disk of the primary, and the time delay between the impact and an observed flare. The disk impact times come from Sundelius *et al.* (1997) and delay times from Lehto & Valtonen (1996; LV96), respectively. <sup>3</sup>The 2006 model was published in Valtonen (2007). The spin of the primary was assumed zero; a non-zero spin was added in the 2009 model (Valtonen *et al.* 2010). <sup>4</sup>The present 2017 model agrees completely with the observed flare times. <sup>5</sup>All models have been calibrated to the starting time of 1982.964 for the 1983 flare. <sup>6</sup>A prediction. The 2015 prediction uses the 2009 model. <sup>7</sup>The 1994 flare came within a week from the prediction, considering that the first (thermal) part of the flare was not observable due to closeness of OJ287 to the sun. <sup>8</sup>A note in the proofs of LV96 reports the observation of the predicted 1995 flare, exactly as expected. The 2007 flare came within days of the prediction (Valtonen *et al.* 2008). <sup>9</sup>The 2005 flare occurred two weeks late. <sup>10</sup>The 2015 flare was known to be spin sensitive. It occurred 4.5 weeks "too early", indicating an increase over the previous spin value. The rms deviation between observations and predictions has been 16 days in five cases. This contrast with the rms deviation for the predictions of the constant period 1987 model, 444 days. Any constant period model would give a similarly poor fit to observations. <sup>11</sup>Corrected for the Hubble constant  $H=70$  km/s/Mpc. <sup>12</sup>For the 2009 model.

$$\frac{d\mathbf{s}_1}{dt} = (\boldsymbol{\Omega}_{SO} + \boldsymbol{\Omega}_{SS} + \boldsymbol{\Omega}_Q) \times \mathbf{s}_1, \tag{3.2}$$

where we let  $\mathbf{x} = \mathbf{x}_1 - \mathbf{x}_2$  as the center-of-mass relative separation vector between the black holes with masses  $M_1$  and  $M_2$ .  $\ddot{\mathbf{x}}_0$  represents the Newtonian acceleration given by  $\ddot{\mathbf{x}}_0 = -\frac{Gm}{r^3} \mathbf{x}$  where  $m = M_1 + M_2$ ,  $r = |\mathbf{x}|$ . Also we define  $\mathbf{n} \equiv \mathbf{x}/r$ ,  $\dot{\mathbf{x}} = \mathbf{v}$  and  $\eta = M_1 M_2 / m^2$ . We include contributions due to general relativistic spin-orbit, spin-spin and classical spin orbit interactions, denoted by  $\boldsymbol{\Omega}_{SO}$ ,  $\boldsymbol{\Omega}_{SS}$  and  $\boldsymbol{\Omega}_Q$ , respectively in the precessional angular velocity for the primary BH spin.

The PN contributions occurring at the conservative 1PN, 2PN, 3PN and the reactive 2.5PN, 3.5PN and 4.5PN orders, denoted by  $\ddot{\mathbf{x}}_{1PN}$ ,  $\ddot{\mathbf{x}}_{2PN}$ ,  $\ddot{\mathbf{x}}_{3PN}$ ,  $\ddot{\mathbf{x}}_{2.5PN}$ ,  $\ddot{\mathbf{x}}_{3.5PN}$  and  $\ddot{\mathbf{x}}_{4.5PN}$ , respectively, are non-spin by nature. The explicit expressions for these contributions suitable for describing the binary black hole dynamics in the modified harmonic gauge are found in Will and Maitra (2017).

The expression for the radiation reaction terms may be written as

$$\begin{aligned} \ddot{\mathbf{x}}_{2.5PN} &= \frac{8}{5} \frac{G^2 m^2 \eta}{c^5 r^3} \{A_{2.5} \dot{r} \mathbf{n} - B_{2.5} \mathbf{v}\} \\ \ddot{\mathbf{x}}_{3.5PN} &= -\frac{8}{5} \frac{G^2 m^2 \eta}{c^7 r^3} \{A_{3.5} \dot{r} \mathbf{n} - B_{3.5} \mathbf{v}\} \\ \ddot{\mathbf{x}}_{4.5PN} &= \frac{8}{5} \frac{G^2 m^2 \eta}{c^5 r^3} \{A_{4.5} \dot{r} \mathbf{n} - B_{4.5} \mathbf{v}\} \end{aligned}$$

where, for example,  $A_{2.5} = 3v^2 + \frac{17}{3} \frac{Gm}{r}$  and  $B_{2.5} = v^2 + 3\frac{Gm}{r}$  while the corresponding expressions for the 3.5 and 4.5 PN orders are more complicated (see Will and Maitra 2017). We find that when integrated over a quasi-Keplerian orbit the contribution from the A-coefficients are nearly symmetric but opposite in sign with respect to the pericenter. Therefore, they do not contribute much to the orbital averaged quantities. On the other hand, the B-coefficients, even though also symmetric with respect to the pericenter, make contributions of the same sign, and are thus important to us. The dominant order hereditary tail contributions to the reactive orbital dynamics are introduced by assuming that they are directly proportional to the ‘‘Newtonian’’ radiation term:

$$\ddot{\mathbf{x}}_{4PNtails} = radfac \ddot{\mathbf{x}}_{2.5PN}$$

This is mainly due to the absence for closed form expressions for the tail contributions to energy and angular momentum fluxes of an eccentric binary (Gopakumar *et al.* 1997). The value of *radfac* is unknown and it is determined as one of the parameters of the orbit solution and there are on-going investigations to constrain it from theoretical considerations for our BH binary.

The leading order spin-orbit contributions to  $\ddot{\mathbf{x}}$  appear at the 1.5PN order (Barker & O’Connell 1979) while the next higher order is at the 2.5 PN level (Will & Maitra 2017). These terms are included in  $\ddot{\mathbf{x}}_{SO}$ , and they are linear in the Kerr parameter  $\chi$ . The Kerr parameter and the unit vector  $\mathbf{s}_1$  define the spin of the primary black hole by the relation  $\mathbf{S}_1 = Gm_1^2 \chi \mathbf{s}_1 / c$ . The Kerr parameter  $\chi$  is allowed to take values between 0 and 1 in GR. The terms  $\mathbf{x}_{SS}$  and  $\mathbf{x}_Q$  include the leading order general relativistic spin-spin interactions and certain classical spin-orbit interactions that involve the quadrupole moment of the primary BH and they appear at the 3PN order for slowly rotating BHs (Will & Maitra 2017). The equations of motion includes certain reactive 4PN spin-orbit contribution, while the  $\ddot{\mathbf{x}}_{4PNtail}$  term models the above mentioned leading order gravitational wave tail contributions.

**Table 2.** Parameters of the orbit solution.

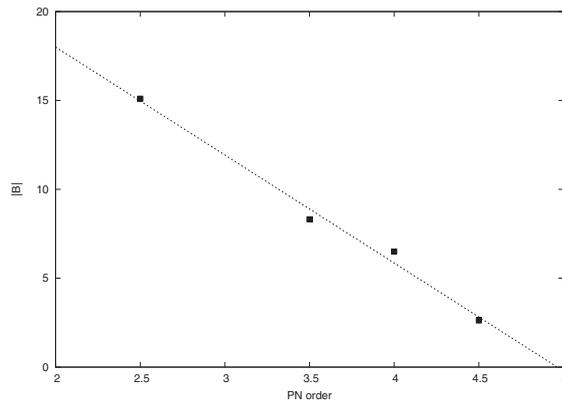
Parameter		unit	error
$M_1$	18348	$10^6 M_\odot$	$\pm 101$
$M_2$	150.13	$10^6 M_\odot$	$\pm 0.43$
$\chi_1$	0.381		$\pm 0.0025$
$h$	0.845		$\pm 0.002$
$t_d$	0.7734		$\pm 0.0015$
$\Delta\Phi$	38.726	deg	$\pm 0.012$
$\Theta_0$	55.57	deg	$\pm 0.21$
$e_0$	0.657		$\pm 0.001$
$P_{2017}$	12.062	yr	$\pm 0.001$
$\Delta P$	36.2	d/100 yr	$\pm 0.25$
$radfac$	1.304		$\pm 0.01$
$\Delta 2.5PN$	-0.065		$\pm 0.04$
$\alpha_g$	0.1		$\pm 0.05$

There are 12 degrees of freedom in the  $\ddot{\mathbf{x}}_{4.5PN}$  term (Gopakumar *et al.* 1997). However, these arbitrary parameters completely drop out of the orbit averaged equations for the orbital elements (Will & Maitra 2017). This allows us to make a free choice of these parameters without significantly affecting the derived orbit, a fact that we have verified by experimenting with different combinations of those parameters. The spin of the primary black hole precesses due to the leading order general relativistic spin-orbit coupling as described by equation (3.2). The precessional equation for the unit spin vector  $\mathbf{s}_1$  enters the binary dynamics essentially at the 2PN order.

#### 4. Solving the orbit

The orbit search algorithm starts from a trial orbit, and if the adopted outburst times do not fit, it automatically adjusts itself until a good model orbit is found. There are no solutions in general, but if one is found, it is uniquely defined by the nine parameters of the model. The details of the algorithm are given in Valtonen (2007).

We find that a solution exists if only the first ‘‘Newtonian’’ radiation reaction term (PN2.5) is used. Adding the next level correction (at 3.5PN) leads to a loss of solution, and it is not helped by adding the 4.5PN level. Therefore it is necessary to incorporate the effects of 4PN tail contributions. Unfortunately, there exists no closed form expression for such tail contributions to the orbital dynamics of an eccentric binary. This is why we invoke an ambiguity parameter  $radfac$  as a coefficient of our additional ‘‘Newtonian’’ radiation reaction term and use it as a free parameter in the orbit solution. The fifth column in Table 1 describes the best model with the current data. Most important of the nine independent parameters of the model are the two masses  $M_1$  and  $M_2$ , the spin of the primary  $\chi$ , and the precession rate of the major axis per period  $\Delta\Phi$ . Further, we also determine the eccentricity of the orbit  $e_0$ , and its orientation at a given epoch  $\Theta_0$ . The two parameters of the standard accretion disk in the calculation are the disk thickness  $h$  and the delay time between an impact and a flare  $t_d$ , both given in units of Lehto and Valtonen (1996). It is possible to translate them to the more familiar disk parameters  $\alpha_g$  (viscosity) and  $\dot{m}$  (mass accretion rate). In addition, we determine the ambiguity parameter related to the 4PN term  $radfac$ . The other properties of interest are the present (redshifted) period of the orbit  $P_{2017}$  and the rate of decrease of the period  $\Delta P$ . From the ambiguity parameter  $radfac$  we also derive the correction to the ‘‘Newtonian’’ gravitational wave radiation reaction arising from higher order radiation



**Figure 2.** The absolute value of the B component of gravitational radiation term at the pericenter of the binary orbit, and at different Post Newtonian orders. The sign of the 3.5PN term is opposite of the other terms. Therefore the terms above 2.5PN nearly cancel each other at the pericenter.

terms  $\Delta 2.5PN$ . The mean values of these parameters and their standard deviations for 97 orbital solutions (found from 1000 trials) are listed in Table 2.

In Figure 2 the absolute values of the radiation reaction terms and the tail term are compared with each other at the pericenter of the binary orbit. We note that there is a monotonic decrease of the terms with increasing PN order. A linear regression suggests that GW emission induced 5PN contributions are negligible in the OJ287 problem, and thus we have the required level of accuracy in use.

## 5. Implications

A strongly over-determined orbital solution implies that its parameters are known very accurately, at better than one percent level for the masses and the primary BH spin. The orbit solution shows that the period of the orbit, now 12.062 year, has decreased at the rate of 36 days per century. This corresponds to an energy loss to gravitational waves that is  $6.5 \pm 4\%$  less than the rate predicted by the standard quadrupolar GW emission. We have shown that the difference is due to higher order gravitational radiation reaction contributions to BBH dynamics that includes the dominant order tail contributions to GW emission. The orbital shrinkage rate agrees within error limits with the rate calculated by Damour, Gopakumar and Iyer (2004). At present, we are using the system to test General Relativity in certain strong field regime unexplored so far. This involves constraining a hypothetical fifth force that arises as a modification of General Relativity in some models of dark matter, dark energy and unification theory. Additionally, we confirm that monitoring of the next flare, predicted to peak on July 31, 2019, at noon GMT, should allow us to test, for the first time, the celebrated black hole no-hair theorem for a massive black hole at the 10% level (Valtonen *et al.* 2011).

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