

MOTION OF A CHARGED PARTICLE AROUND A BLACK HOLE PERMEATED BY MAGNETIC FIELD AND ITS CHAOTIC CHARACTERS

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Abstract. Motion of a charged particle around a black hole immersed in magnetic field is calculated. It is shown that this motion has a chaotic property depending on initial parameters.

1. Introduction

Chaotic phenomena in conservative Hamiltonian system have been found in classical mechanics, for example, in galactic dynamics (Contopoulos 1987). Relativistic equations of the motion of a particle in the curved space-time due to a black hole are separable in variables for particular metrics of Schwarzschild and Kerr (Misner et al. 1973). In that case orbits are classified in the diagram of effective potential depending on the total energy and angular momentum of a particle. Compared with classical cases, in general relativistic one we expect trapping of the particle by the central black hole under some condition. We can estimate these conditions for the motion in the Schwarzschild and Kerr metrics.

On the other hand we have a similar Hamiltonian system for the motion of a charged particle in the curved space-time with magnetic field, but in this case separation of variable has not been found, and therefore this system may show nonintegrability (Prassana and Varma 1977). So in this case orbital motion becomes complicate depending on initial parameters. We may be able to expect some chaotic behaviours of motion as found out in the quantum states of a hydrogen-like atom in a magnetic field (Hasegawa and Takami 1991).

2. Equations

As one of the properties of black hole, it does not have a magnetic field, but we imagine a magnetic field around the black hole and it has been considered that energetic phenomena (such as the jet beams) in the active galactic nuclei relate with this field.

We calculate some particle's orbits in the off-equatorial plane near central hole immersed in a dipole type magnetic field, of which effect appears in the time component of vector potential \mathbf{A} . The magnetic field around the black hole is assumed to be generated by a current flowing in the accretion disk situated at the distance R from the central black hole. Then the components of vector potential \mathbf{A} are

$$A_t = \frac{-3a\mu}{2\gamma^2\Sigma} [[r(r - M) + (a^2 - Mr) \cos^2 \theta] \times \frac{1}{2\gamma} \log \left(\frac{r - M - \gamma}{r - M - \gamma} \right) - (r - M \cos^2 \theta)]$$

and

$$A_\phi = \frac{-3\mu \sin^2 \theta}{4\gamma^2\Sigma} [(r - M)a^2 \cos^2 \theta + r(r^2 + Mr + 2a^2) - [r(r^3 - 2Ma^2 + a^2r) + \Delta a^2 \cos^2 \theta] \times \frac{1}{2\gamma} \log \left(\frac{r - M + \gamma}{r - M - \gamma} \right)]$$

where μ is a dipole moment and γ is given by $\sqrt{(M^2 - a^2)}$ (Petterson 1975). Taking account of this contribution, we have the Lagrangian

$$L = \frac{1}{2} \left[- \left(1 - \frac{2Mr}{\Sigma} \right) \dot{t}^2 - \frac{4Mar}{\Sigma} \sin^2 \theta \dot{t} \dot{\phi} + \frac{A}{\Sigma} \sin^2 \theta \dot{\phi}^2 + \frac{\Sigma}{\Delta} \dot{r}^2 + \Sigma \dot{\theta}^2 + qA_t \dot{t} + aA_\phi \dot{\phi} \right]$$

If the motion is limited within the $\theta - r$ plane, the θ - and r - components of the equation of motion become

$$\begin{aligned} \frac{d^2\theta}{ds^2} &= \frac{-a^2 \sin \theta \cos \theta}{\Sigma \Delta} \dot{r}^2 - \frac{2r}{\Sigma} \dot{r} \dot{t} + \frac{a^2 \sin \theta \cos \theta}{\Sigma} \dot{\theta}^2 \\ &+ \frac{2Mra^2 \sin \theta \cos \theta}{\Sigma^3} \dot{t}^2 - \frac{4Mra(r^2 + a^2)}{\Sigma^3} \sin \theta \cos \theta \dot{\phi} \dot{t} + \frac{\sin \theta \cos \theta}{\Sigma^3} \\ &\times [(r^2 + a^2)^3 - (r^2 + a^2 + \Sigma)\Delta a^2 \sin^2 \theta] \dot{\phi}^2 + \frac{q}{\Sigma} (A_{\phi,r} \dot{\phi} + A_{t,r} \dot{t}). \end{aligned}$$

$$\frac{d^2r}{ds^2} = \frac{M(r^2 - a^2 \cos^2 \theta) - ra^2 \sin^2 \theta}{\Sigma \Delta} \dot{r}^2 + \frac{r\Delta}{\Sigma} \dot{\theta}^2 + \frac{2a^2 \sin \theta \cos \theta}{\Sigma} \dot{r} \dot{\theta}$$

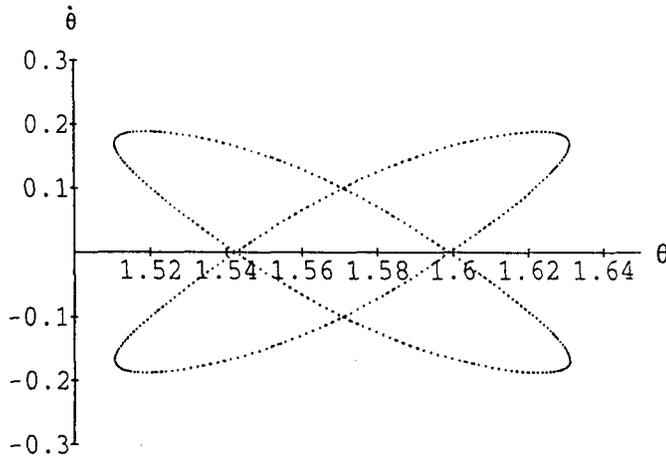


Fig. 1. A regular pattern in motion with initial values of $\theta_0 = 0.10$.

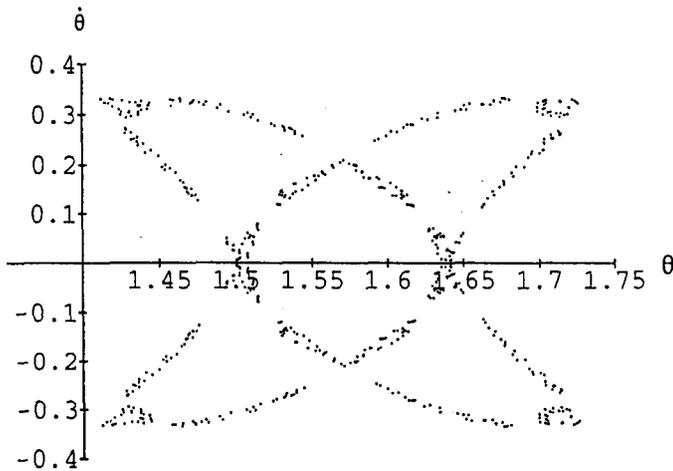


Fig. 2. An intermediate case of $\theta_0 = 0.21$.

$$\begin{aligned}
 &-\frac{M\Delta}{\Sigma^3}(r^2 - a^2 \cos^2 \theta)\dot{\theta} + \frac{\Delta \sin^2 \theta}{\Sigma^3}[r^5 + 2r^3 a^2 \cos^2 \theta - Mr^2 a^2 \sin^2 \theta \\
 &+ ra^4 \cos^2 \theta + (M - r)a^4 \sin^2 \theta \cos^2 \theta]\dot{\phi}^2 + \frac{2\Delta Ma \sin^2 \theta}{\Sigma^3} \\
 &\times (r^2 - a^2 \cos^2 \theta)\dot{\phi}\dot{t} + \frac{q\Delta}{\Sigma}(A_{\phi,r}\dot{\phi} + A_{t,r}\dot{t}),
 \end{aligned}$$

where q is the charge of the particle, and Σ , Δ , ϕ , t , r , and θ are usual variables in the Kerr metric.

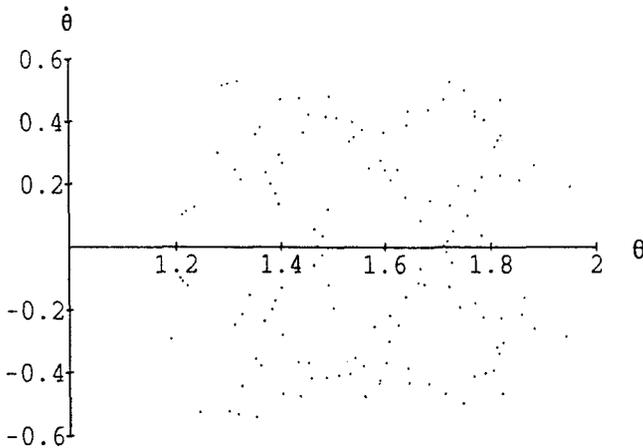


Fig. 3. An irregular case of $\dot{\theta}_0 = 0.25$.

3. Results

As parameters used in these equations we select 0.999, 5, 100, and 300 for a , E_0 , L , and B in geometrical units. Furthermore, we consider the case where the particle starts from $r = 4.559$ to any direction with various initial values of θ , but with $\dot{r} = 0$ and $\dot{\phi} = 0$.

Figure 1 shows a case of regular pattern in which initial value of θ , $\dot{\theta}_0$, is equal to 0.1. The case of $\dot{\theta}_0 = 0.21$ near irregular one is shown in figure 2. In the figure 3 irregular case of $\dot{\theta}_0 = 0.25$ is given.

From these figures we find characteristic patterns which are similar to the Poincaré's surface appearing in the Hamiltonian system with two degrees of freedom. So we can conclude that motions of a particle around the black hole immersed in the magnetic field show chaotic characters.

References

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