Dynamic Adverse Selection and Belief Update in Credit Markets

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Abstract

We develop a dynamic model of debt contracts with adverse selection. Entrepreneurs borrow investment goods from lenders to run businesses whose returns depend on entrepreneurial productivity and common productivity. Entrepreneurial productivity is the entrepreneur's private information, and lenders construct beliefs about entrepreneurial productivity based on the entrepreneur's business operation history, common productivity history, and the terms of the contract. The model provides insights into the dynamic and cross-sectional relations between firm age and credit risk, persistency of the effects of a productivity shock, cyclical asymmetry of the business cycle, slow recovery after a crisis, and constructive and destructive economic downturns.

I.   Introduction

Lenders typically possess less information than borrowers regarding the attributes that affect borrowers' ability to repay, which generates the adverse selection problem. While the adverse selection problem in credit markets has been extensively explored in the literature, the majority of studies have examined its economic implications within single-period models.[1] However, in reality, less informed lenders make efforts to assess borrowers' credit risk by analyzing their historical data, aiming to overcome informational disadvantages. Specifically, lenders scrutinize borrowers' histories in conjunction with past aggregate economic conditions because the borrower's financial state and economic decisions are influenced by the overall state of the economy.

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[1]See Jaffee and Russell (1976), Stiglitz and Weiss (1981), Bester (1985), Besanko and Thakor (1987), and Figueroa and Leukhina (2015), for instance.

In this article, we develop an infinite horizon model of debt contracts with adverse selection to investigate how lenders construct their beliefs about the credit risks of borrowers with different histories by using the information on past aggregate economic conditions.[2] In particular, we scrutinize the macroeconomic implications of lenders' dynamic belief constructions. We examine the dynamic evolution of the borrowing cost as a borrower ages, the cross-sectional relation between the borrower's age and the borrowing cost in a given period, and the effects of positive and negative productivity shocks on macroeconomic performances through their influence on lenders' ongoing belief constructions.

Model preview. In the model economy, an entrepreneur can run his/her business using the lender's investment good as input in each period. The return from business operations is a product of common productivity and entrepreneurial productivity. Common productivity is a random variable that is independently and identically distributed over time. Entrepreneurs are heterogeneous with respect to entrepreneurial productivity which is the entrepreneur's private information. To run a business, an entrepreneur must borrow the investment good from a lender, subject to limited commitment. If an entrepreneur defaults, then he/she faces permanent exclusion from future credit and hence leaves the economy. Bankrupt entrepreneurs are replaced with new entrepreneurs whose productivity is randomly drawn from the given distribution.

The key novel ingredient of our model is that lenders have access to the entrepreneur's operation history, specifically the duration for which the business has been running, as well as the realized common productivities in the past, both of which align with reality.[3] However, lenders cannot observe the terms of debt contracts that an entrepreneur has made in the past, which is also consistent with reality.[4] The lender employs the entrepreneur's operation history, information on past realized common productivities, and the terms of the proposed contract to construct his/her beliefs regarding the entrepreneur's productivity. Based on the constructed beliefs, the lender decides whether to lend the investment good to the entrepreneur.

Results preview. In equilibrium, unsecured debts are feasible due to the threat of punishment for default, and entrepreneurs of the same age offer the same contract. Unlike standard debt contract models with limited commitment (e.g.,

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[2]A number of papers study the macroeconomic implications of adverse selection problems in asset markets in dynamic models. For instance, Guerrieri and Shimer (2014) and Chang (2018) analyze the condition for the existence of fire sales in asset markets. Rocheteau (2011) shows that payment arrangements exhibit a pecking-order property when asset qualities are asymmetrically informed, and Kurlat (2013) derives the amplification effects of an aggregate shock. In contrast to these studies, we study the macroeconomic implications of dynamic adverse selection problems in credit markets, not in asset markets. On a related point, Mankiw (1986) studies the role of asymmetric information in credit markets in a macroeconomic setting, but there is no dynamic belief update in Mankiw (1986).

[3]For example, many countries worldwide provide historical data on gross domestic production and total factor productivity, and a Certificate of Corporate Registration provides essential information about when a company was established.

[4]Credit reports issued by credit bureaus like Equifax and Experian in the U.S., upon which financial institutions rely to examine individuals' credit histories, show the default history, such as whether a borrower has made repayments on time in the past. However, these reports do not provide any information about the specific terms of contracts that the borrower has entered into previously.

Kehoe and Levine (1993), Sanches and Williamson (2010), Azariadis and Kass (2013), and Gu, Mattesini, Monnet, and Wright (2013)), defaults transpire in equilibrium because of aggregate productivity shocks, aligning with real-world observations. Nonetheless, it is optimal for entrepreneurs to make repayments whenever feasible, so entrepreneurs default only if they have no choice but to default in equilibrium. This implies that, given a level of realized common productivity and among a group of entrepreneurs of the same age, a threshold value of entrepreneurial productivity exists such that entrepreneurs whose productivity falls below this threshold default and leave the economy. On the other hand, those with higher productivity fulfill their debt obligations, ensuring continued access to the future credit market. Thus, in the next period, lenders can update their beliefs such that the productivity of the surviving entrepreneurs is distributed above the threshold.

Because more productive entrepreneurs tend to stay in the economy for a longer period and less productive entrepreneurs are more likely to exit early, the lender's belief about the entrepreneur's productivity weakly improves over time in terms of first-order stochastic dominance. As a result, the borrowing cost weakly decreases as the borrower ages. However, this result does not necessarily imply that older entrepreneurs always exhibit lower credit risks and borrowing costs than younger entrepreneurs in a given period. Depending on the realized common productivities in the past, younger entrepreneurs can experience lower borrowing costs than older entrepreneurs. On average, though, the model generates a negative cross-sectional correlation between the entrepreneur's age and borrowing cost.

In the model economy, a temporary shock on common productivity can have persistent impacts on the average entrepreneurial productivity and the aggregate output, depending on the size of the shock. Specifically, a transitory positive shock on common productivity can persistently increase the average entrepreneurial productivity by supporting productive entrepreneurs to survive and thereby be separated from less productive entrepreneurs. A negative shock, on the other hand, can have enduring consequences on the average entrepreneurial productivity by driving out a certain type of existing entrepreneurs from the economy. In particular, our model provides the following macroeconomic implications on economic downturns stemming from negative productivity shocks.

First, in the model, the arrival of a recession is prompt, and the recovery from a recession tends to be protracted due to the gradual replacement of less productive entrepreneurs with new entrepreneurs over time. In particular, a significant negative shock to common productivity results in the default of most (or all) existing entrepreneurs. Consequently, it can take a long time for aggregate output to return to the pre-shock level, which provides a narrative for the sluggish recovery of production after a crisis (e.g., Ikeda and Kurozumi (2019)).

Second, the model demonstrates that a recession caused by a mild negative shock to common productivity can yield cleansing effects that improve average entrepreneurial productivity in the long term through productive winnowing and, as a result, it can be constructive for the economy. However, a severe negative shock is always destructive, and the model suggests that the government should implement economic stimulus when a shock is sufficiently severe to prevent the collapse of good entrepreneurs.

Empirical content. Our predictions about the dynamic and cross-sectional relations between borrower's age and credit risk find support in a number of empirical papers. Agarwal and Gort (2002) show that firms' default risk decreases as firms age, and Berger and Udell (1995) and Bharath, Dahiya, Saunders, and Srinivasan (2011) find that firms' borrowing costs decline over time. Further, Eklund, Larsen, and Berhardsen (2001), Benito, Delgado, and Martínez Pagés (2004), Bhimani, Gulamhussen, and Lopes (2010), and Belaid (2014) document the negative relation between firms' ages and their credit risk using cross-sectional-data.

Our model captures the following empirical findings at the macroeconomic level: i) a temporary positive productivity shock can have persistent impacts on the economy (e.g., Blanchard and Quah (1989), Hvide and Meling (2023)), and ii) the economy behaves differently over the expansion and recession phases of the business cycle: the pace of increases in the output is slower than the pace of declines (e.g., Neftçi (1984), Hamilton (1989), and Morley and Piger (2012)).

Literature review. Ordoñez, Perez-Reyna, and Yogo (2019) study secured loan markets with asymmetric information and show that the usefulness of credit history depends on the degree of uncertainty in collateral value. Although no one would doubt that lenders will investigate the borrower's history before approving secured loans in reality, the importance of the borrower's history for secured loans would differ from that for unsecured loans because collateral limits lenders' loss when default occurs and collateral can also work as a signaling device (e.g., Bester (1985)). Further, Azariadis, Kaas, and Yi (2016) show that unsecured debts that we focus on in our article are still a major source of raising funds for firms in the U.S. market.

More relatedly, Diamond (1989) studies reputation formation in unsecured credit markets with adverse selection. However, he assumes that defaults destroy all output from projects so a borrower will never default strategically, while we allow a borrower to default strategically. Furthermore, in contrast to Diamond (1989) and Ordoñez et al. (2019), we introduce aggregate shocks on common productivity into the model to understand the interaction between aggregate shocks and lenders' belief construction.[5] As a result, we could provide greater macroeconomic implications, such as new explanations on business cycle properties, the short-run and long-run effects of productivity shocks, and the constructiveness and destructiveness of recessions, in a single framework.

Boot and Thakor (1994) study the dynamics of loan interest rates over the course of a borrower's life in a repeated game between a lender and a borrower with a moral hazard. While the distinction between adverse selection and moral hazard in credit markets is often subtle, the ways of incorporating the 2 frictions into the model differ profoundly: an asymmetric information problem occurs before the transaction in adverse selection and a moral hazard arises after the transaction. We take the view that informational asymmetries in credit markets often result from the prospects of projects that borrowers operate and contribute to the literature by investigating adverse selection problems in the credit market in a dynamic model.

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[5]On a related point, Ai and Bhandari (2021) investigate the optimal contracting problem with aggregate shocks in the model with limited commitment and moral hazard in labor markets.

The finding that a mild economic downturn can be constructive in the long term through productive winnowing is echoed in the related literature on the cleansing effect of recessions. For example, Caballero and Hammour (1994) and Osotimehin and Pappadà (2017) show that recessions improve resource allocation by driving out less productive firms and production units. Barlevy (2002) and Ouyang (2009) consider both the cleansing and scarring effects of recessions and show that the scarring effect dominates the cleansing effect. Our approach goes beyond these earlier studies by investigating the relation between the size of a negative shock and the constructiveness of the shock. Thus, our model can provide more precise prescriptions about government policies in response to the economic downturn.

Layout. The rest of this article is organized as follows: Section II presents the economic environment of the model. Section III characterizes the equilibrium, and Section IV presents a number of implications of our model. Section V concludes. The proofs of lemmas and propositions are in Appendix A of the Supplementary Material.

II.   Model

In this section, we present the environment of the model economy.

A.   Physical Environment

Time is discrete and continues forever. Each period t is divided into 2 sub-periods: morning and afternoon. Morning is the planning period, and consumption occurs in the afternoon. There are 2 risk-neutral agents: a unit measure of entrepreneurs and lenders. The instantaneous utility of both agents in each period equals the quantity of consumption in the afternoon (i.e., agents have a constant marginal utility of one).

An entrepreneur stays in the economy for multiple periods with a discount factor $\beta \in (0, 1)$ across periods until he/she leaves the economy. If an entrepreneur leaves the economy, then he/she is replaced by a new entrepreneur. On the other hand, we assume that lenders stay for one period, so lenders are more like an anonymous credit market rather than a financial institution in our model, similar to Diamond (1989).[6] Consequently, entrepreneurs face a new set of lenders each period.

Each lender receives an indivisible endowment of one unit of an investment good in the morning. The investment good can be either lent to an entrepreneur or invested in a saving technology that yields a certain return of $r > 0$ units of the consumption good in the afternoon. Entrepreneurs receive indivisible seed capital when they are born. Seed capital cannot be converted to a consumption good, but an entrepreneur can establish a new firm with seed capital to start his/her own business.

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[6]We impose this assumption to make the model tractable, avoiding a complicated dynamic contracting problem on the lender side. Additionally, it serves to ensure that the histories of an entrepreneur's economic decisions and aggregate economic conditions in the past are the only intertemporal linkage, which we focus on in this article.

Since the establishment of a company, the entrepreneur can run the business in the morning with one unit of the investment good as the only input to produce the consumption good in the afternoon. The return on the business operation in period $t \geq 0$, denoted by w_t, depends on common productivity, A_t, and entrepreneurial productivity θ, as $w_t = A_t \theta$. Common productivity, A_t, is independently and identically distributed across periods according to the uniform distribution with the support of $[0, 1]$, and it represents a productivity shock that affects the overall economy, such as the level of aggregate demands and natural disasters, for instance. Entrepreneurs are heterogeneous with respect to their productivity θ which is randomly drawn from the uniform distribution with the support of $\Theta = [\underline{\theta}, \overline{\theta}]$ when an entrepreneur is born, and θ is fixed until the entrepreneur leaves the economy. We assume that the distributions for A_t and new entrepreneurs' θ, respectively, are public information.

To run their business, entrepreneurs must borrow the investment good from lenders because they do not receive any investment goods. In the model, there is a decentralized credit market in which entrepreneurs and lenders are randomly matched in the morning.[7] We assume that in each bilateral meeting, an entrepreneur offers a contract and a lender either accepts or rejects the offer.

After the establishment of a company, an entrepreneur can temporarily close his/her company to stop running the business for some periods whenever he/she wants to. However, the entrepreneur must incur $\kappa > 0$ units of disutility in the morning to restart his/her business. For example, to restart the business after a close, a company must pay a search cost to hire new employees and might incur a cost to renovate an office in reality. We assume that $\kappa > \frac{\overline{\theta} - 2r}{2 - \beta - \beta\sqrt{1 - \frac{4r}{\overline{\theta}}}}$ to make the analysis straightforward. However, the main results do not hinge on this assumption on κ.[8]

Throughout, $U_{[a,b]}$ refers to the cumulative distribution function (cdf) of the uniform distribution on $[a, b]$. Additionally, for any cdf $\mathcal{F} : \mathbb{R} \to [0, 1]$, we define a probability measure $\mathbf{m}_{\mathcal{F}}$ on the Borel σ-field on $\mathbb{R}$ such that $\mathbf{m}_{\mathcal{F}}((a, b]) = \mathcal{F}(b) - \mathcal{F}(a)$ for all $-\infty < a \leq b < \infty$. For notational simplicity, we denote the probability measure for the cdf $U_{[a,b]}$ as $\mathbf{m}_{[a,b]}$ instead of $\mathbf{m}_{U_{[a,b]}}$. Finally, abusing notation, we use supp $\mathcal{F}$ to denote the support of the probability distribution function of cdf $\mathcal{F}$.

Parameter Assumption

We impose the following assumption on the parameters.

Assumption 1. $\beta > \dfrac{b(\underline{\theta}) - \sqrt{b(\underline{\theta})^2 - 4b(\underline{\theta})r}}{\underline{\theta}} > 0$ where $b(\theta') = \dfrac{\overline{\theta} - \theta'}{\int_{\theta'}^{\overline{\theta}} \frac{1}{\theta} d\theta}$ for all $\theta' \in [\underline{\theta}, \overline{\theta})$ and $b(\overline{\theta}) = \lim_{\theta' \to \overline{\theta}} b(\theta') = \overline{\theta}$.

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[7]Trades in decentralized markets with random matching have been often adopted in over-the-counter literature (e.g., Duffie, Gârleanu, and Pedersen (2005), Weill (2008), Lagos and Rocheteau (2009), and Lagos, Rocheteau, and Weill (2011)) and the money search literature (e.g., Kiyotaki and Wright (1993), Shi (1995), and Lagos and Wright (2005)).

[8]Specifically, the model can generate the same equilibrium outcomes even without the fixed cost κ of restarting the business as long as we construct lenders' belief off the equilibrium path appropriately.

Assumption 1 requires that agents are sufficiently patient and is a technical condition necessary for the existence of an equilibrium in which all entrepreneurs operate their business in every period. This assumption serves to streamline the analysis by restricting attention to relevant cases. Because $\beta < 1$, it must be verified that the set $\{\underline{\theta}, \overline{\theta}, r, \beta\}$ that satisfies Assumption 1 is not empty in advance before further analysis. The next lemma provides a sufficient condition for the set $\{\underline{\theta}, \overline{\theta}, r, \beta\}$ that satisfies Assumption 1 to be non-empty.

Lemma 1. If $\underline{\theta} \geq 4r$, then there exists $\beta \in \left(\dfrac{b(\underline{\theta}) - \sqrt{b(\underline{\theta})^2 - 4b(\underline{\theta})r}}{\underline{\theta}}, 1 \right)$.

B.   Information Structure

In this subsection, we describe the information structure (set of public and private information) in the model economy.

1.   Aggregate Production and Common Productivity History

Many countries in the world have an online portal system that provides time-series data on gross domestic production (GDP). We incorporate this reality into the model, allowing agents to access and observe the historical data on the aggregate production of consumption goods. This implies that one can correctly infer the history of common productivity in the past by forming a rational expectation about the cdf of θ of entrepreneurs who have run their business and the mass of lenders who have invested endowments in the saving technology along the equilibrium path.

To make the analysis straightforward and to simplify the notation, we assume that the history of past common productivity is public information. Specifically, in the morning in period t, all agents can observe $A^{t-1} \equiv \{A_{-1}, A_0, A_1, \ldots, A_{t-1}\}$, where $A_{-1} = \varnothing$. We let $\mathbb{A}^{t-1}$ be the set of all feasible sequences of A^{t-1} and $\mathbb{A} \equiv \underset{t \in \mathbb{N}_0}{\cup} \mathbb{A}^{t-1}$, where $\mathbb{N}_0 = \{0, 1, 2, \ldots\}$ is the set of non-negative integers. However, we assume that agents cannot directly observe A_t in the afternoon in period t, which is also consistent with the reality of GDP data being published with a lag.

2.   Business Operation History

Entrepreneurs could have different business operation histories – how long they have run their business – in the model economy because entrepreneurs could be born in different periods and entrepreneurs of the same age could establish their companies in different periods. We assume that the establishment period, denoted by $s \geq 0$, of any company is public information, so lenders can observe how long an entrepreneur has run his/her business in the economy. For instance, in practice, lenders can check the establishment period of a company by verifying the information stated in the company's Certificate of Corporate Registration.

In principle, an entrepreneur may not run his/her business occasionally after the establishment of the company. In such cases, the information about the establishment period may not provide a complete picture of the operation history. We can assume that lenders can observe whether the entrepreneur has operated a business during a given period since the establishment of his/her company, rather than solely

relying on the information about the establishment period.[9] However, under either assumption regarding the type of operational history available to lenders, we can show that entrepreneurs run their business every period until they leave the economy. Consequently, we obtain the same results. To simplify the notation and exposition of the analysis, we assume that lenders can observe the establishment period of the entrepreneur's firm.

3. Set of Public History

To simplify the notation, we combine individual-specific history s and the common productivity history A^{t-1} as $h_{t-1} = (s, A^{t-1})$ with the restriction $t \geq s$. Then, h_{t-1} captures all the public history information that lenders can use to evaluate each entrepreneur's credit risk in the morning in period t. We let $\mathbb{H}_t = \{0, 1, \ldots, t\} \times \mathbb{A}^t$ denote the set of all feasible h_t and $\mathbb{H} \equiv \bigcup_{t \in \mathbb{N}_0} \mathbb{H}_{t-1}$.

4. Information Friction and Entrepreneur's Type

We now discuss 2 informational frictions in the model economy. First, we assume that entrepreneurial productivity θ of individual entrepreneur is the entrepreneur's private information. Second, we assume that lenders are unable to observe the financial transaction history of entrepreneurs, although they can observe the operational history of entrepreneurs. Specifically, the terms of the contracts entered into by each entrepreneur in the past for borrowing the investment goods and the corresponding repayment amounts are not publicly observable, similar to Diamond (1989).[10]

Note that entrepreneurs differ in terms of their productivity and the establishment period of their companies. Thus, entrepreneur's types are 2-dimensional, characterized by $(\theta, s) \in \Theta \times \mathbb{N}_0$. Here, s is the observable type and θ is the unobservable type. In this article, we study how the observable type combined with common productivity history is used to infer the unobservable type in equilibrium.

C. Form of Contracts in a Bilateral Meeting

In a bilateral meeting, an entrepreneur offers a contract to borrow one unit of the investment good from a lender, and the contract must specify the repayment schedule. However, entrepreneurial productivity θ is the entrepreneur's private information and agents cannot directly observe common productivity realized in the current period. Thus, the lender cannot observe the return on the business of any entrepreneurs. Only the entrepreneur can observe the exact realized return of his/her business, so the repayment must depend on the information provided by the entrepreneur.

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[9]For example, for an entrepreneur who established a firm in period s, we can define o_τ for all $\tau \in \{-1, 0, 1, 2, \ldots\}$ as follows: i) $o_\tau = \varnothing$ if $\tau < s$, ii) $o_\tau = 1$ if the entrepreneur runs his/her business in period $\tau \geq s$, and iii) $o_{i,\tau} = 0$ otherwise. Then, a sequence $o^{t-1} \equiv \{o_{-1}, o_0, o_1, \ldots, o_{t-1}\}$ summarizes the business operation history of the entrepreneur in the morning in period $t \geq s$.

[10]This assumption is consistent with the practice in reality, wherein lenders rely on credit reports issued by credit bureaus, such as Equifax and Experian in the U.S., to investigate the credit history of an individual. These reports reveal any histories of default and late payments, but they do not disclose the specific terms of debt contracts that the individual has made in the past. Relatedly, Jang and Kang (2024) investigate the economic consequences of disclosing the borrower's financial transaction history to find the optimal information disclosure in credit markets.

Specifically, after observing the return on business operation $w_t = A_t\theta$ in the afternoon, the entrepreneur emits a signal $w' \in [0,\bar{\theta}]$ about the output from his/her business to the lender and pays $R(w')$ units of the consumption good, where $R(\cdot)$ is a repayment function from $[0,\bar{\theta}]$ to $[0,\bar{\theta}]$ pre-specified by the contract. However, lenders cannot observe the entrepreneur's financial transaction history as explained earlier. This implies that if an entrepreneur decides to honor the contract, he/she will always choose w' so as to minimize the payment to the lender. Thus, the payment is constant, denoted by $x = \min_{w' \in [0,\bar{\theta}]} R(w')$, so the contract has the form of the debt contract similar to results in Williamson (1986).

Next, we assume that there is no external source of enforcement in the credit market, creating a limited commitment problem. Thus, an entrepreneur can always choose not to make any payments and in this case, we say that the entrepreneur defaults on a loan. However, there is a device in the economy that records the default history of entrepreneurs. If an entrepreneur defaults on a loan, he/she will be permanently excluded from future credit, similar to Eaton and Gersovitz (1981) and Azariadis and Kass (2013).[11] For example, an entrepreneur can receive a discharge by filing bankruptcy, but the bankruptcy document is stored in the publicly available court archive, and no lender will provide a loan to this entrepreneur in the future. Because an entrepreneur cannot run a business without borrowing the investment good from a lender, bankrupt entrepreneurs are forced to exit and are replaced with new entrepreneurs. This assumption ensures that the measure of entrepreneurs is constant over time even though defaults occur in equilibrium.[12]

The terms of the contract may include specifications regarding the conditions under which the entrepreneur defaults, in addition to the repayment amount. However, the entrepreneur cannot commit to the default schedule. Furthermore, lenders in future periods cannot observe the terms of contracts that the entrepreneur made in the past. Consequently, adherence to the default schedule stipulated in the contract holds no significant consequence for the entrepreneur. Therefore, the entrepreneur will make an optimal decision regarding default, regardless of the default schedule specified in the contract, after observing the business's returns.[13] Consequently, including a default schedule in the contract would not effectively discipline the entrepreneur's behavior.

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[11]As explained in Eaton and Gersovitz (1981), imposing a strong punishment on default represents the fact in a handy way that default makes reentering the credit market arduous. Specifically, Eaton and Gersovitz (1981) and Azariadis and Kass (2013) assume that defaulters face permanent exclusion from all future loans. Furthermore, in our model, lenders exist for one period, and hence, the renegotiation scheme in Bulow and Rogoff (1989) is not applicable. Thus, adopting the assumption of permanent exclusion from credit markets as in Eaton and Gersovitz (1981) and Azariadis and Kass (2013) is appropriate.

[12]One of the objectives of this article is to study under which conditions defaults occur in equilibrium and if defaulted entrepreneurs can join the credit market with a positive probability instead of permanent exclusion, the mass of entrepreneurs would change over time as defaults occur. To avoid this problem while keeping the measure of entrepreneurs at a constant level over time, we assume that defaulters are permanently excluded and replaced with new entrepreneurs.

[13]Specifically, the entrepreneur's value decreases with the repayment amount x, which will be demonstrated later. Thus, if the terms of the contract include the default schedule, the entrepreneur will propose the default schedule that minimizes x, even if it requires cheating lenders about entrepreneurial productivity. However, after the realization of common productivity, the entrepreneur will optimally default, regardless of the default schedule specified in the contract.

Finally, the contract may potentially include specifications regarding the probability of loan provision. However, we assume that neither the entrepreneur nor the lender can commit to the contract. Specifically, suppose that a lender accepts a contract that specifies repayment x and probability of loan provision α. The lender accepts this contract because he/she can achieve a trade surplus by receiving the repayment. Then, in the case where the lender and entrepreneur should not enter the contract, which occurs with probability $1 - \alpha$, both parties have incentives to clinch the contract because it is optimal for both parties. Thus, loan provision probability is non-binding and cannot be an instrument of contracts.

Although ruling out the loan provision probability from the terms of the contract makes the analysis straightforward without unnecessary distraction, it is not critical for obtaining the main results. Even if we explicitly consider the loan provision probability as a contracting instrument, we can still obtain the same results by appropriately constructing lenders' out-of-equilibrium beliefs, which is standard in the signaling literature.

Note that repayment x fully describes the terms of the contract because the loan size is fixed, similar to Stiglitz and Weiss (1981), and we say that contract x_1 is lower than contract x_2 whenever $x_1 < x_2$. Next, because lenders will never accept contract $x = 0$, offering $x = 0$ is equivalent to not making an offer to a lender. Without loss of generality, we assume that an entrepreneur offers $x = 0$ if he/she chooses not to offer a contract to the lender.

III.   Equilibrium

In this section, we characterize the equilibrium of the model economy as follows: First, we describe the game between an entrepreneur and a lender in each period. Second, we investigate agents' strategies, value functions, and belief system. Then, we characterize equilibrium by analyzing the agents' optimal strategies.

Game structure in a single period. Once an entrepreneur establishes his/her company, the entrepreneur is randomly matched with a lender in the morning in every period, generating a dynamic game between the long-lived entrepreneur and short-lived lenders, until the entrepreneur leaves the economy after default. A sequence of moves in each period is as follows: In a pairwise meeting, an entrepreneur offers a contract to the matched lender. Then, the lender decides whether to accept the offered contract or not. If the lender rejects the offer, the match is terminated. On the other hand, if the lender accepts the offer, the lender transfers the investment good to the entrepreneur and the entrepreneur runs his/her business in the morning with the investment good. Then, in the afternoon, the entrepreneur observes the return from the business and decides whether to make the repayment to the lender or to default.

Agents' strategies. Consider an entrepreneur who has established a company in the morning in period $s \geq 0$.[14] A strategy of the entrepreneur is a mapping

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[14]An entrepreneur sets about being matched with a lender, thereby starting the dynamic game only after he/she has established a company. Therefore, from this point onward, when referring to the entrepreneur's strategy, we specifically mean the strategy of an entrepreneur who has already established a company. Once we have identified the entrepreneur's value through the optimal strategy, we can then examine the entrepreneur's optimal decision regarding the timing of establishing a company.

$(x, D) : \Theta \times \mathbb{H} \to \mathbb{R}_+ \times \mathbb{P}^{[0,1]}$, where $\mathbb{P}^{[0,1]}$ is the power set of $[0, 1]$, such that given the entrepreneur's productivity θ and public history h_{t-1} in period t, the entrepreneur offers $x(\theta, h_{t-1})$ to a lender in the morning and defaults on the loan in the afternoon if $A_t \in D(\theta, h_{t-1})$. In what follows, we use $x_t = x(\cdot, h_{t-1})$ and $D_t = D(\cdot, h_{t-1})$ as the entrepreneur's behavioral strategy given h_{t-1} unless it causes any confusion. Furthermore, abusing notations, we also use x_t to represent a contract that the entrepreneur offers to a lender in the morning in period t.

Next, as a short-lived player of the game, the lender's strategy is relatively simple. A lender who is alive and matched with an entrepreneur in the morning in period t chooses whether to accept the offered contract or reject it. If the lender rejects the offer, then he/she invests the investment good in the saving technology which yields $r > 0$ units of consumption goods in the afternoon with certainty.

Entrepreneur's payoffs. If the entrepreneur does not borrow the investment good from a lender in the morning in period t, the entrepreneur cannot run the business and moves to the next period with the updated history of common productivity as $A^t = \{A^{t-1}, A_t\}$. The entrepreneur can restart running his/her business whenever he/she wants but must incur κ units of disutility to restart the business.

On the other hand, if the entrepreneur offers x_t and the lender accepts it, then the entrepreneur produces $A_t \theta$ units of consumption goods in the afternoon. Then, the entrepreneur repays x_t units of goods to the lender and proceeds to the next period if $A_t \in [0,1] \setminus D_t$ and defaults otherwise. Consequently, in the morning in period t, the entrepreneur's expected payoff from offering the acceptable contract x_t with default strategy D_t is given as

$$(1) \qquad \mathbb{E}_{A_t}[A_t \theta] + (1 - |D_t|)\mathbb{E}_{A_t}[-x_t + \beta V_{t+1}(\theta, h_t)|A_t \notin D_t],$$

where $|D_t|$ is the measure of D_t, $h_t = (s, A^t)$, and $V_{t+1}(\theta, h_t)$ is the value of an entrepreneur with θ and history h_t in the morning in period $t + 1$.

Lender's payoffs and belief system. Given the entrepreneur's period-t strategy (x_t, D_t), if a lender rejects the offer, the lender invests the investment good in the saving technology that yields r units of consumption goods in the afternoon. On the other hand, if the lender accepts the offer, then, the lender receives x_t units of consumption goods in the afternoon provided $A_t \notin D_t$ and receives nothing otherwise. Thus, the lender's expected payoff from accepting the contract x_t is given as $(1 - |D_t|)x_t$.

However, the lender cannot directly observe the entrepreneur's default strategy D_t that depends on θ. Thus, the lender must form beliefs about θ before making an acceptance decision because θ is the entrepreneur's private information. To construct the belief in the morning in period t, the lender uses all available information which includes the terms of the offered contract x_t and history h_{t-1}. Specifically, we write $\mu : \mathbb{R}_+ \times \mathbb{H} \to \mathcal{M}$, where $\mathcal{M}$ is the set of all feasible cdfs on Θ, for the lenders' belief function: $\mu(x_t, h_{t-1})$ is the lender's conditional belief about the distribution of θ of an entrepreneur upon observing (x_t, h_{t-1}).

In reality, aggregate economic condition matters to the entrepreneurs' economic decisions only if the company exists at that time. Thus, what had happened in the economy before the establishment of a company should not affect the lender's evaluation of the company's credit risk. Based on this rationale, we impose the following assumption on the properties of the lender's belief system: For any

$s_1, s_2 \in \mathbb{N}_0$ and $A^{s_1-1}, A^{s_2-1} \in \mathbb{A}$, we have $\mu\big(\cdot, (s_1, A^{s_1-1})\big) = \mu\big(\cdot, (s_2, A^{s_2-1})\big)$. This implies that entrepreneurs face the same belief of lenders in the credit market in the establishment period of their companies.

Given the lender's belief system μ, history h_{t-1}, and the entrepreneur's default decision rule $D(\cdot, h_{t-1})$, the lender's expected payoff from accepting a contract $\hat{x}$ is given as

(2) $$\omega_\mu(\hat{x}, D(\cdot, h_{t-1}), h_{t-1}) = \int_\Theta (1 - |D(\theta, h_{t-1})|)\hat{x} d\mu(\hat{x}, h_{t-1}).$$

Optimal strategies of agents. In the morning, lenders can always invest his/her investment good in the saving technology to earn a certain return of $r > 0$ in the afternoon. Thus, for any contract x_t and $h_{t-1} \in \mathbb{H}$, lenders will accept the contract if $\omega_\mu(x_t, D_t, h_{t-1}) \geq r$ and rejects x_t otherwise.[15] Next, regarding the entrepreneur's default decision, note that an entrepreneur with productivity θ has no choice but to default on contract x_t in period $t \geq 0$ if $A_t < \frac{x_t}{\theta}$ because the entrepreneur does not have sufficient goods to make a repayment (exogenous default). Furthermore, the entrepreneur can also default if the repayment is higher than the value of staying in the economy even though he/she has enough goods to make a repayment (strategic default). Specifically, the entrepreneur with θ and h_{t-1} will opportunistically default on x_t even though $A_t \theta \geq x_t$ if

$$x_t > \beta V_{t+1}(\theta, h_t)$$

where $h_t = \big(s, \{A^{t-1}, A_t\}\big)$.

In the model, entrepreneurs can offer $x_t = 0$, which will be rejected by a lender, to take a break from business whenever they want. However, later we will show that it is always optimal for entrepreneurs to never stop running a business since the establishment of their companies. Thus, entrepreneurs will always offer an incentive-compatible contract to lenders in any equilibrium.

Based on these observations, the entrepreneur's optimal period-t strategy $(x(\cdot, h_{t-1}), D(\cdot, h_{t-1}))$ and value $V_t(\theta, h_{t-1})$ given θ and history h_{t-1} are obtained by solving the following problem for all $\theta \in \Theta$:

(3) $V_t(\theta, h_{t-1}) =$

$$\max_{\hat{x} \in \mathbb{R}_+, \hat{D} \in \mathbb{P}^{[0,1]}} \left\{ \mathbb{E}_{A_t}[A_t \theta] + \big(1 - |\hat{D}|\big) \mathbb{E}_{A_t}\big[-\hat{x} + \beta V_{t+1}(\theta, h_t) | A_t \notin \hat{D}\big] \right\}$$

subject to

(4) $$r \leq \omega_\mu(x(\theta, h_{t-1}), D(\cdot, h_{t-1}), h_{t-1})$$

(5) $$D(\theta, h_{t-1}) = \left\{ A_t \in [0,1] : A_t < \frac{x(\theta, h_{t-1})}{\theta} \text{ or } x(\theta, h_{t-1}) > \beta V_{t+1}(\theta, h_t) \right\},$$

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[15]We assume that lenders accept loan contracts when they are indifferent.

where $h_t = \left(s, \left\{A^{t-1}, A_t\right\}\right)$. Here, (4) is the lender's incentive compatibility constraint and (5) is the entrepreneur's incentive compatibility constraint for defaults.

Equilibrium characterization. We now characterize equilibrium. The game between an entrepreneur and a lender in the morning has the structure of a signaling game and we adopt Perfect Bayesian Equilibrium (PBE), which is formally stated in the following definition.

Definition 1. An equilibrium is the entrepreneur's strategy (x, D) and a belief system μ such that (x, D) solves (3) for all $\theta \in \Theta$ and $h_{t-1} \in \mathbb{H}$, and μ is consistent with Bayes' rule whenever it is applicable for all $(x_t, h_{t-1}) \in \mathbb{R}_+ \times \mathbb{H}$.

As is standard in signaling models, equilibrium outcomes depend on how the lender's belief system μ off the equilibrium path is constructed. In particular, equilibrium may exist in which any entrepreneur cannot make an acceptable offer for all periods $t \geq 0$. In this case, no entrepreneurs run their businesses in the model economy. To rule out this extreme case, we impose a restriction on the belief system such that for any $h_{t-1} \in \mathbb{H}_{t-1}$, there exists $x_t \in \mathbb{R}_+$ such that $\omega_\mu(x_t, D_t, h_{t-1}) \geq r$. That is, for any common productivity history at any period, there exists a contract $x > 0$ that an entrepreneur can offer without violating the lender's incentive compatibility.

However, this restriction does not necessarily imply that entrepreneurs always run their businesses. An entrepreneur may not establish a company for certain periods after he/she was born or the entrepreneur may stop running the business even after the establishment of the company whenever he/she wants. However, the next lemma shows that entrepreneurs will never do this in equilibrium.

Lemma 2. Entrepreneurs establish their companies when they are born and run their businesses every period until they leave the economy.

In the model, the history $h_{t-1} = \left(s, A^{t-1}\right)$ of each entrepreneur is public information. Thus, we can group entrepreneurs based on the history h_{t-1}. Here, note that all entrepreneurs have the same common productivity history A^{t-1}. Furthermore, all entrepreneurs who were born in the same period established their companies on the same date by the result of Lemma 2. Thus, all entrepreneurs who were born in the same period have the same h_{t-1}. We let $\hat{\Omega}_{h_{t-1}}$ denote the cdf of θ of entrepreneurs with the history h_{t-1} in the morning in period t. Abusing notation, we use $\hat{\theta}_t$ to represent minsupp$\hat{\Omega}_{h_{t-1}}$. In what follows, we call a group of entrepreneurs who were born in period $s \geq 0$ the s-cohort.

Proposition 1. For any $h_{t-1} \in \mathbb{H}$ and $t \geq 0$, all entrepreneurs with the same history h_{t-1} offer the same contract in equilibrium.

Proposition 1 implies that all entrepreneurs in the same cohort offer the same contract in any PBE. The logic of this finding goes as follows: In the model, lenders cannot observe the terms of the contract that an entrepreneur made in the past.

Furthermore, as the repayment amount x_t decreases, the entrepreneur gains more discretion over the default decision, and the expected payoff from honoring the contract increases. Thus, for all $\theta \in \operatorname{supp} \hat{\Omega}_{h_{t-1}}$, the entrepreneur's expected payoff in the morning in period t decreases with x_t whenever the lender accepts the offer x_t in period t as shown in (1). Consequently, the terms of contract x_t cannot work as a signaling device. Thus, all entrepreneurs in the same cohort offer the minimum contract x_t among the incentive-compatible contracts in each period.

The pooling result in Proposition 1 simplifies the equilibrium analysis. To be specific, consider a group of entrepreneurs who are born in period $s \geq 0$. By the results of Lemma 2 and Proposition 1, all entrepreneurs in the s-cohort offer the same contract in the morning in period s. Next, given that $x(\theta, h_{s-1}) = x_s$ for some $x_s > 0$ for all $\theta \in \operatorname{supp} \hat{\Omega}_{h_{s-1}}$, an entrepreneur with higher θ is less likely to default exogenously because $\frac{x_s}{\theta}$ decreases with θ. Furthermore, note that $V_{s+1}(\theta, h_s)$, where $h_s = (s, A^t)$, weakly increases with θ since a more productive entrepreneur can always mimic a less productive entrepreneur. Thus, a more productive entrepreneur has less incentive to default strategically as shown in (5). Consequently, if a type (θ', s) entrepreneur makes the repayment x_s in the afternoon, then for all $\theta \geq \theta'$, a type (θ, s) entrepreneur honors the debt. This implies that θ for surviving entrepreneurs in the morning in period $s + 1$ will be uniformly distributed over $\left[\hat{\theta}_{s+1}, \overline{\theta}\right]$ for some $\hat{\theta}_{s+1} \in \Theta$ (i.e., $\hat{\Omega}_{h_s} = U_{\left[\hat{\theta}_{s+1}, \overline{\theta}\right]}$). By using the above argument inductively, we obtain the next lemma.

Lemma 3. For any $h_{t-1} \in \mathbb{H}$, whenever $\operatorname{supp} \hat{\Omega}_{h_{t-1}} \neq \varnothing$, there exists $\hat{\theta}_t < \overline{\theta}$ such that $\hat{\Omega}_{h_{t-1}} = U_{\left[\hat{\theta}_t, \overline{\theta}\right]}$ in equilibrium.

The result of Lemma 3 implies that for any equilibrium contract x_t offered by an entrepreneur with the history h_{t-1}, it must be that $\mu(x_t, h_{t-1}) = U_{\left[\hat{\theta}_t, \overline{\theta}\right]}$ for the lender's belief system to be consistent on the equilibrium path. Thus, the lender's expected payoff from accepting x_t in equilibrium is given as

$$(6) \qquad \omega_\mu(x_t, D(\cdot, h_{t-1}), h_{t-1}) = \int_\Theta \int_{[0,1]\setminus D(\theta, \mathrm{h}_{t-1})} x_t \mathbf{m}_{[0,1]}(dA_t) \mathbf{m}_{\left[\hat{\theta}_t, \overline{\theta}\right]}(d\theta),$$

because all entrepreneurs with h_{t-1} offer the same contract x_t. Then, it must be that $\omega_\mu(x_t, D_t, h_{t-1}) \geq r$ for equilibrium contract x_t to be acceptable. Note that $\omega_\mu(x_t, D_t, h_{t-1})$ decreases with the measure of D_t in (6). By imposing the smallest default set, $D_t = \left[0, \frac{x_t}{\theta}\right)$, into (6), we obtain $\omega_\mu(x_t, D_t, h_{t-1}) = x_t - \frac{x_t^2}{b(\hat{\theta}_t)}$, where $b(\cdot)$ is defined in Assumption 1, and this is the highest expected payoff that lenders can achieve from accepting x_t given that $\hat{\Omega}_{h_{t-1}} = U_{\left[\hat{\theta}_t, \overline{\theta}\right]}$.

Now define a function $x^* : \Theta \to \mathbb{R}$ such that

$$(7) \qquad x^*(\theta) \equiv \frac{b(\theta) - \sqrt{b(\theta)^2 - 4b(\theta)r}}{2},$$

which is well-defined given Assumption 1. Then, $x^*\left(\hat{\theta}_t\right)$ is the lowest x that satisfies $x - \frac{x^2}{b\left(\hat{\theta}_t\right)} = r$. Thus, for any $x_t < x^*\left(\hat{\theta}_t\right)$, it must be that $\omega_\mu(x_t, D_t, h_{t-1}) < r$ and, hence, contract x_t will be rejected as long as the lender forms the correct belief about θ of the s-cohort, which must hold on the equilibrium path. As a result, an equilibrium offer cannot be lower than $x^*\left(\hat{\theta}_t\right)$. This argument leads to the following lemma.

Lemma 4. Take any $h_{t-1} \in \mathbb{H}$ such that $\text{supp}\hat{\Omega}_{h_{t-1}} \neq \varnothing$, and let $\hat{\theta}_t \equiv \text{minsupp}\hat{\Omega}_{h_{t-1}}$. Then, any equilibrium contract x_t offered by entrepreneurs with history h_{t-1} in the morning in period $t \geq 0$ is bounded below by $x^*\left(\hat{\theta}_t\right)$ (i.e., $x_t \geq x^*\left(\hat{\theta}_t\right)$), where $x^*(\cdot)$, defined in (7), is a convexly decreasing function.

A signaling model, in general, often generates multiple Perfect Bayesian Equilibria because the concept of PBE imposes little discipline on the lender's belief system. This also holds true for our model. Specifically, we show in Appendix B of the Supplementary Material that for any h_{t-1} there exist $\underline{x} < \overline{x}$ such that for any $x' \in [\underline{x}, \overline{x})$, equilibrium exists with $\{x_t, D_t\} = \{x', [0, \frac{x'}{\theta}]\}$ for all $\theta \in \text{supp}\hat{\Omega}_{h_{t-1}}$. Thus, the model generates a continuum of equilibria.

In this article, we restrict our attention to equilibrium with the lowest x for each $h_{t-1} \in \mathbb{H}$, which we denote as the e^* equilibrium. Specifically, if x is offered in the e^* equilibrium by a cohort whose θ is distributed according to a cdf $\hat{\Omega}'$, then there is no equilibrium in which some cohort with the same cdf $\hat{\Omega}'$ of θ offers $x' < x$. We adopt the e^* equilibrium for the following 2 reasons.

First, the e^* equilibrium provides a specific form of contract x as a function of the set of available information to lenders and the value of fundamental parameters. Except for the e^* equilibrium, the terms of the equilibrium contract are indeterminate and depend on arbitrary choices. For example, suppose that x' is a contract offered by a type (θ, s) entrepreneur in period t in the e^* equilibrium. Then, there exists an equilibrium in which the type (θ, s) entrepreneur offers $x' + \varepsilon$ for sufficiently low $\varepsilon > 0$ in period t. Here, the value of ε is not determined by the model's environment and can be chosen arbitrarily.[16] Second, the e^* equilibrium is the default-minimizing equilibrium because entrepreneurs have a higher incentive to default when x is high. Thus, if there is a cost for default, such as legal costs of bankruptcies, then the e^* equilibrium is the equilibrium that minimizes social costs caused by defaults.[17]

The following proposition characterizes the e^* equilibrium, showing the main result of the article.

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[16]Furthermore, the main implications of equilibrium outcome do not change even though we choose equilibrium with $x' + \varepsilon$ as an equilibrium offer after fixing ε.

[17]For example, we can construct a model such that an entrepreneur incurs $\delta > 0$ units of disutility to capture the cost of processing legal documents for filing bankruptcy when he/she defaults. Introducing the default cost in this way does not change the main implications except that the optimal default condition is now given as $x - \delta > \beta V_{t+1}(\theta, h_t)$.

Proposition 2. The e^* equilibrium exists. In the e^* equilibrium, for any $h_{t-1} \in \mathbb{H}$, if $\mathrm{supp}\,\hat{\Omega}_{h_{t-1}} \neq \varnothing$, all entrepreneurs with the history h_{t-1} offer contract $x^*\left(\hat{\theta}_t\right)$, where $\hat{\theta}_t \equiv \mathrm{minsupp}\,\hat{\Omega}_{h_{t-1}}$, and do not default opportunistically (i.e., $D(\theta, h_{t-1}) = \left[0, \frac{x^*(\hat{\theta}_t)}{\theta}\right)$), in period t.

In the e^* equilibrium, the repayment x is sufficiently low to deter entrepreneurs from opportunistic default: Entrepreneurs default only if they cannot honor the debt contract. This result is similar to that the repayment size is restricted by the incentive constraint that prevents defaults in standard models of debt contracts with limited commitment, such as Kehoe and Levine (1993) and Azariadis and Kass (2013), although defaults do occur in our model in contrast to those models. Diamond (1989) also shows a similar result: a borrower repays the loan whenever feasible. However, in Diamond (1989), defaults imply the destruction of all output from the project, and a borrower takes nothing, which rules out the possibility of strategic default.

The outcome that entrepreneurs have no incentives to opportunistically default on contract $x^*\left(\hat{\theta}_t\right)$ is essential for contract $x^*\left(\hat{\theta}_t\right)$ not to violate the lender's incentive compatibility constraint. Specifically, $x^*\left(\hat{\theta}_t\right)$ is obtained from the binding incentive constraint (4) after exogenously imposing the minimum default set into (2). Thus, given $\hat{\Omega}_{h_{t-1}} = U_{[\hat{\theta}, \overline{\theta}]}$, if some of the entrepreneurs with the history h_{t-1} default opportunistically, the lender's expected payoff from accepting contract $x^*\left(\hat{\theta}_t\right)$ is strictly lower than r, which makes the entrepreneur's period-t strategy not incentive-compatible. Thus, it is required in equilibrium that all entrepreneurs with h_{t-1} have an incentive to honor the debt whenever it is feasible after offering contract $x^*\left(\hat{\theta}_t\right)$.

The result that entrepreneurs do not default opportunistically makes it easy to trace the dynamic changes in the distribution of θ of entrepreneurs in the same cohort in the e^* equilibrium. To obtain intuition, consider any s-cohort in period $\tau \geq s$. By the result of Lemma 3, there exists $\hat{\theta}_\tau \in \Theta$ such that $\hat{\Omega}_{h_{\tau-1}} = U_{[\hat{\theta}_\tau, \overline{\theta}]}$, where $h_{\tau-1} = (s, A^{\tau-1})$. In the morning in period τ, all entrepreneurs in the s-cohort offer $x^*\left(\hat{\theta}_\tau\right)$, and in the afternoon, type (θ, s) entrepreneurs default only if $A_\tau \theta < x^*\left(\hat{\theta}_\tau\right)$. Thus, if $\frac{x^*(\hat{\theta}_\tau)}{A_\tau} \leq \hat{\theta}_\tau$, all entrepreneurs in the s-cohort make the repayment. On the other hand, if $\frac{x^*(\hat{\theta}_\tau)}{A_\tau} > \hat{\theta}_\tau$, only entrepreneurs with $\theta \in \left[\frac{x^*(\hat{\theta}_\tau)}{A_\tau}, \overline{\theta}\right]$ can honor the debt. Combined together, θ of surviving entrepreneurs in the morning in period $\tau + 1$ is uniformly distributed over $\left[\max\left\{\frac{x^*(\hat{\theta}_\tau)}{A_\tau}, \hat{\theta}_\tau\right\}, \overline{\theta}\right]$. By applying the same logic inductively, we obtain the next proposition that describes the dynamics of the distribution $\hat{\Omega}_{h_{t-1}}$, and hence, the lender's belief in the path of the e^* equilibrium.

Proposition 3. Take any $h_{t-1} = \left(s, A^{t-1}\right) \in \mathbb{H}$ such that $\mathrm{supp}\,\hat{\Omega}_{h_{t-1}} \neq \varnothing$, and for any $\tau = s, \ldots, t$, let $A^{\tau-1}$ be the truncated subsequence of A^{t-1} such that $A^{\tau-1} = \{\varnothing, \ldots, A_{\tau-1}\}$ and $h_{\tau-1} = \left(s, A^{\tau-1}\right)$. Then, in the $\boldsymbol{e^*}$ equilibrium, $\hat{\Omega}_{h_{\tau-1}} = U_{\left[\hat{\theta}_\tau, \overline{\theta}\right]}$, where $\hat{\theta}_\tau$ is given as

$$(8) \qquad \hat{\theta}_s = \underline{\theta} \text{ and } \hat{\theta}_\tau = \max\left\{\frac{x^*(\hat{\theta}_{\tau-1})}{A_{\tau-1}}, \hat{\theta}_{\tau-1}\right\} \text{ for } \tau = s+1, \ldots, t.$$

IV.   Applications

In this section, we consider 2 applications of our model. In Section IV.A, we assess the relation between the entrepreneur's age and credit risk. In Section IV.B, we study the effects of common productivity shocks on the dynamics of aggregate production in the model economy. In the following analysis, when we say equilibrium, we mean the $\boldsymbol{e^*}$ equilibrium.

A.   Entrepreneur Age and Credit Risk

Extensive studies have been conducted on the factors that affect firms' credit risks, and firm age has been identified as a determinant of default probability. In this subsection, we use our model to study the relation between the entrepreneur's age and credit risks, both dynamically and cross-sectionally.

1.   Measuring Credit Risk

What is the entrepreneur's credit risk that a lender faces when he/she lends the investment good to an entrepreneur? In a bilateral meeting, the lender cannot directly observe the entrepreneur's productivity, and the lender estimates the entrepreneur's credit risk based on the updated belief.

Productivity θ of the s-cohort is uniformly distributed over $\left[\hat{\theta}_t, \overline{\theta}\right]$, where $\hat{\theta}_t$ is given by (8), in the $\boldsymbol{e^*}$ equilibrium, and all entrepreneurs in the s-cohort offer $x^*\left(\hat{\theta}_t\right)$, as described in Proposition 2. On the equilibrium path, the lender's belief follows Bayes' rule; hence, it must be that $\mu\left(x^*\left(\hat{\theta}_t\right), h_{t-1}\right) = U_{\left[\hat{\theta}_t, \overline{\theta}\right]}$. Then, given that an entrepreneur with θ defaults only if $A_t < \frac{x^*(\hat{\theta}_t)}{\theta}$, the lender perceives that the ex ante default probability, which is denoted by λ, of an entrepreneur in the s-cohort with public history h_{t-1} in period t is

$$(9) \qquad \lambda(h_{t-1}) = \int_\Theta \frac{x^*\left(\hat{\theta}_t\right)}{\theta} dU_{\left[\hat{\theta}_t, \overline{\theta}\right]}.$$

Lemma 5. The default probability $\lambda(h_{t-1})$, defined in (9), decreases with $\hat{\theta}_t$.

The result of Lemma 5 implies that $\hat{\theta}_t$ inversely captures the credit risk of an entrepreneur with h_{t-1} in period $t \geq 0$. The intuitive explanation for this finding is as

follows: As $\hat{\theta}_t$ rises, the average productivity of entrepreneurs with h_{t-1} increases. Furthermore, the repayment function $x^*\left(\hat{\theta}\right)$ decreases with $\hat{\theta}$ as described in Lemma 4. Combined together, the default probability λ decreases with $\hat{\theta}_t$.

2.   Evolution of Credit Risk Over Time

We first analyze the dynamic evolution of the entrepreneur's credit risk perceived by lenders over the entrepreneur's life. Consider an entrepreneur in the s-cohort in period $t > s$. The lender's belief about the entrepreneur's productivity θ in a past period $\tau \in \{s, \ldots, t-1\}$ is given as $\mu\left(x^*\left(\hat{\theta}_\tau\right), h_{\tau-1}\right) = U_{[\hat{\theta}_\tau, \overline{\theta}]}$, where $h_{\tau-1} = \left(s, A^{\tau-1}\right)$ and $\hat{\theta}_\tau$ is given by (8). As one can see from (8), $\hat{\theta}_\tau$ weakly increases over time τ until the entrepreneur leaves the economy, which means that the lender's belief about the entrepreneur's productivity weakly improves in terms of first-order stochastic dominance as the entrepreneur ages. The improvement in the belief, in turn, reduces the entrepreneur's perceived credit risk, $\lambda(h_{\tau-1})$, and the repayment, $x^*\left(\hat{\theta}_\tau\right)$, which is consistent with the finding in Diamond (1989).[18] In summary, we have the following proposition, whose proof is omitted.

Proposition 4. In the e^* equilibrium, the entrepreneur's credit risk and demanded repayment weakly decrease as the entrepreneur ages.

The results of Proposition 4 are consistent with the empirical findings in Berger and Udell (1995) and Agarwal and Gort (2002), which document a decline in firms' default risk and borrowing costs, respectively, over time. The intuition for the results of Proposition 4 is in line with our earlier observations. In equilibrium, each entrepreneur honors the debt contract as much as possible and defaults only if he/she does not have sufficient income which is a product of common productivity and entrepreneurial productivity. Thus, honoring the debt contract in each period indicates that the entrepreneur's productivity is above a certain level, which updates the lender's belief. This update, in turn, decreases the entrepreneur's perceived credit risk and the demanded repayment.

On a related point, Boot and Thakor (1994) construct a repeated game between a lender and a borrower with a moral hazard problem and demonstrate that loan interest rates decline over time. Although their theoretical prediction is similar to that of ours, the primary mechanism is different. In Boot and Thakor (1994), the borrowing cost decreases as a borrower ages, because a decreasing sequence of interest rates incentivizes a borrower to invest more effort in his/her project. In contrast, we show that borrowing costs decrease as borrowers age as a result of information learning in a credit market, in which adverse selection problems exist, thus complementing previous studies.

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[18]Specifically, Diamond (1989) shows that a borrower with a safe project improves his/her reputation over time as borrowers with risky projects leave the economy. Consequently, the interest rate decreases as a borrower ages. However, in Diamond (1989), there is no uncertainty on aggregate productivity, and hence, a borrower with a safe project always builds his/her reputation over time while reputation building in our model depends on the level of common productivity and a borrower with the highest productivity can leave the economy after default if common productivity is sufficiently low. Furthermore, we also study the relation between the borrower's age and the borrowing cost cross-sectionally, while Diamond (1989) focuses on the dynamic relation.

3.   Cross-Sectional Differences in Credit Risk

In the model economy, entrepreneurs leave the economy after defaulting on debt contracts and are replaced by new entrepreneurs. Thus, the economy consists of different age groups of entrepreneurs in a given period, and each age group could have a different credit risk. We show, in Proposition 4, that the credit risk of an individual entrepreneur weakly decreases throughout his/her life. This result implies that, when new entrepreneurs are born, they have a weakly higher credit risk than the existing entrepreneurs who were born in previous periods. Then, do these features imply that old entrepreneurs always have a lower credit risk than young entrepreneurs in a given period? The answer is explored in the next proposition that describes the dynamics of the relative credit risk among cohorts with different operation histories.

Proposition 5. Take any $h_{t-1}^o, h_{t-1}^y \in \mathbb{H}$ for any $t \geq 0$, and suppose that $\lambda(h_{t-1}^o) < \lambda(h_{t-1}^y)$. Then, in the e^* equilibrium, there exist $0 < A_L < A_H < 1$ such that i) $\lambda(h_t^o) \leq \lambda(h_t^y)$ for all $A_t \in [A_H, 1]$, ii) $\lambda(h_t^o) > \lambda(h_t^y)$ for all $A_t \in [A_L, A_H)$, and iii) all entrepreneurs with history h_{t-1}^y leave the economy after defaulting in period t for all $A_t \in [0, A_L)$.

The main implication of Proposition 5 is that a reversal of the credit risk rank among cohorts can occur over time. This is illustrated graphically in Figure 1, where $\hat{\theta}_t^o = \text{minsupp}\,\hat{\Omega}_{h_{t-1}^o}$ and $\hat{\theta}_t^y = \text{minsupp}\,\hat{\Omega}_{h_{t-1}^y}$.[19] This result implies that young entrepreneurs can have a lower credit risk than old entrepreneurs in a given period,

FIGURE 1

Credit Risk of Entrepreneurs with h_t^o and h_t^y in Period $t + 1$

Figure 1 illustrates how the perceived credit risk $\lambda(s^i, A^t)$ for each group $i \in \{o, y\}$ in the morning in period $t + 1$ depends on the realized common productivity in period t. In particular, the figure shows that when $A_t \in [A_L, A_H]$, a reversal of credit risk occurs.

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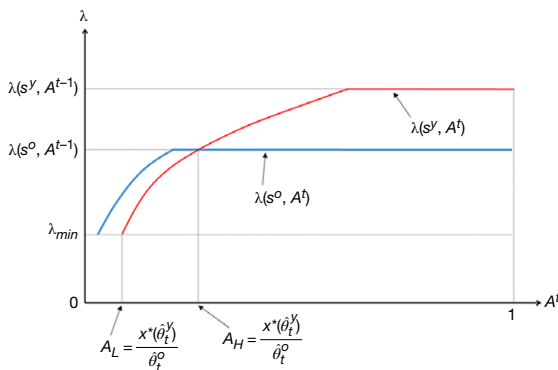
[19]In Figure 1, $h_{t-1}^o = (s^o, A^{t-1})$ and $h_{t-1}^y = (s^y, A^{t-1})$ for some $s^o, s^y \in \mathbb{N}_0$, so h_{t-1}^o and h_{t-1}^y are different only in terms of the establishment period.

depending on the realization of common productivity. In particular, the reversal of credit risk occurs when common productivity is low enough $(A_t < A_H)$ to remove a sufficient fraction of low-productive entrepreneurs with h_{t-1}^y, but not too low $(A_t \geq A_L)$ to drive out all of them. However, if the threshold value A_H is low enough as illustrated in Figure 1, credit risk reversal would not occur on average. This inference holds in the model under some conditions as stated below.

Proposition 6. If $\theta \geq 4r$, in the e^* equilibrium, an entrepreneur with a lower credit risk than another entrepreneur in the current period maintains a lower credit risk, on average, in the next period.

The result of Proposition 6 implies that old entrepreneurs tend to have a lower credit risk than young entrepreneurs because when young entrepreneurs were born, old entrepreneurs had a weakly lower credit risk than new entrepreneurs. As a result, the model generates a cross-sectional negative correlation between entrepreneur age and credit risk in a given period.

The negative relation between firm age and the firm's credit risk has been well documented in empirical studies that use cross-sectional data (e.g., Altman (1968), Eklund et al. (2001), Benito et al. (2004), Bhimani et al. (2010), and Belaid (2014)). The supporting argument of these studies is that young firms are more sensitive to external shocks and are therefore expected to have higher bankruptcy probabilities than old firms. Through the lens of our model, old firms' adaptiveness to shocks results from the fact that only good firms can handle negative external shocks, survive for a long time, and can thus get older.

B.   Common Productivity and Aggregate Production

In this subsection, we study the effects of common productivity on the dynamics of aggregate production. In equilibrium, the aggregate production in period t is given as $Y_t = A_t \int_\Theta \theta d\Omega_t$, where Ω_t is the cdf of θ of entrepreneurs who are alive in the morning in period t. Common productivity affects aggregate production through 2 channels.

First, A_t has a direct effect on Y_t in period t because entrepreneurs' return on their projects is a product of entrepreneurial productivity and common productivity. Second, the history of realized common productivity in the past, A^{t-1}, influences the current aggregate output, Y_t, in period t by affecting the cdf Ω_t because the types of defaulted entrepreneurs in period $\tau < t$ depend on A_τ. For instance, all entrepreneurs offer $x^*(\underline{\theta})$ in period 0, and only entrepreneurs with $\theta \geq \frac{x^*(\underline{\theta})}{A_0}$ survive in period 0. If $\frac{x^*(\underline{\theta})}{A_0} > \underline{\theta}$, then entrepreneurs with $\theta \in [\underline{\theta}, \frac{x^*(\underline{\theta})}{A_0})$ default in period 0 and are replaced with new entrepreneurs in period 1. Thus, the cdf, Ω_1, in the morning in period 1 is the average of 2 distributions, namely, $U_{[\frac{x^*(\underline{\theta})}{A_0}, \bar{\theta}]}$ and $U_{[\underline{\theta}, \bar{\theta}]}$ weighted by the mass of entrepreneurs in each distribution. Thus, Ω_1 depends on the realization of A_0. Then, by induction, we can express the cdf Ω_t as a function of A^{t-1} such that $\Omega_t = \hat{\Omega}_{A^{t-1}}$ in the e^* equilibrium. Specifically, $\hat{\Omega}_{A^{t-1}}$ is the average of $\hat{\Omega}_{h_{t-1}}$ weighted by the mass of entrepreneurs with public history h_{t-1}.

Accordingly, given a common productivity history A^t, we can express the aggregate production in period t as a function of A^t, such that

$$(10) \qquad Y_t = A_t \int_\Theta \theta d\hat{\Omega}_{A^{t-1}} \equiv \hat{Y}(A^t).$$

Note that the cdf $\hat{\Omega}_{A^{t-1}}$ in the morning in period t is a function of A^{t-1}, while $\hat{Y}(A^t)$ is a function of $A^t = \{A^{t-1}, A_t\}$ because the aggregate production in the afternoon in period t depends on the realization of A_t.

In general, it is difficult to trace $\hat{\Omega}_{A^{t-1}}$ and $\hat{Y}(A^t)$ over time because the realization of common productivity in each period is randomly drawn from $U_{[0,1]}$. To gain insight into the dynamics of $\hat{\Omega}_{A^{t-1}}$ and $\hat{Y}(A^t)$ over time, we study a special case in which the realized common productivity is constant such that $A_\tau = \tilde{A}$ for all $\tau \geq 0$. For notational convenience, when $A_\tau = \tilde{A}$ for all $\tau \geq 0$, we let $\tilde{A}^t = \{A_\tau\}_{\tau=-1}^t$ denote such sequence of common productivity for each t. In what follows, we focus our attention on the case where $\tilde{A} \in (0,1]$ because $\tilde{A} = 0$ implies $\hat{Y}(\tilde{A}^t) = 0$ for all $t \geq 0$.

Proposition 7. Suppose that the realized common productivity is constant at $\tilde{A} \in (0,1]$ (i.e., $A_t = \tilde{A}$ for all $t \geq 0$), in the e^* equilibrium.

1. If $\tilde{A} \in \left(0, \frac{x^*(\underline{\theta})}{\overline{\theta}}\right] \cup \left[\frac{x^*(\overline{\theta})}{\underline{\theta}}, 1\right]$, then $\hat{\Omega}_{\tilde{A}^{t-1}} = U_{[\underline{\theta},\overline{\theta}]}$ and $\hat{Y}(\tilde{A}^t) = \frac{\tilde{A}(\underline{\theta}+\overline{\theta})}{2}$ for all $t \geq 0$.

2. If $\tilde{A} \in \left(\frac{x^*(\overline{\theta})}{\overline{\theta}}, \frac{x^*(\underline{\theta})}{\underline{\theta}}\right)$, then, letting $\Delta \equiv \frac{\frac{x^*(\underline{\theta})}{\tilde{A}} - \underline{\theta}}{\overline{\theta}-\underline{\theta}}$, we have

$$(11) \qquad \hat{\Omega}_{\tilde{A}^{t-1}} = \begin{cases} \Delta^t \dfrac{\theta - \underline{\theta}}{\overline{\theta}-\underline{\theta}} & \text{for } \theta \leq \dfrac{x^*(\underline{\theta})}{\tilde{A}} \\[2em] \Delta^t \dfrac{\theta - \underline{\theta}}{\overline{\theta}-\underline{\theta}} + (1-\Delta^t)\dfrac{\theta - \dfrac{x^*(\underline{\theta})}{\tilde{A}}}{\overline{\theta} - \dfrac{x^*(\underline{\theta})}{\tilde{A}}} & \text{for } \theta > \dfrac{x^*(\underline{\theta})}{\tilde{A}} \end{cases},$$

$$(12) \qquad \hat{Y}(\tilde{A}^t) = \Delta^t \frac{\tilde{A}(\underline{\theta}+\overline{\theta})}{2} + (1-\Delta^t)\frac{x^*(\underline{\theta})+\tilde{A}\overline{\theta}}{2},$$

for all $t \geq 0$.

Proposition 7 describes the dynamics of $\hat{\Omega}_{\tilde{A}^{t-1}}$ and $\hat{Y}(\tilde{A}^t)$ over time when the realized common productivity is constant at $\tilde{A} \in (0,1]$ for all $t \geq 0$. The first part of Proposition 7 is straightforward: If $\tilde{A} \in \left(0, \frac{x^*(\underline{\theta})}{\overline{\theta}}\right]$, then all entrepreneurs with $\theta < \overline{\theta}$ default and are replaced with new entrepreneurs in every period, and if

FIGURE 2

Dynamics of $\widehat{\Omega}_{\tilde{A}^{t-1}}$ when $\tilde{A} \in \left(\frac{x^*(\underline{\theta})}{\overline{\theta}}, \frac{x^*(\underline{\theta})}{\underline{\theta}} \right)$

Figure 2 depicts the dynamics of the distribution $\widehat{\Omega}_{\tilde{A}^{t-1}}$ over time. Specifically, it clarifies that $\widehat{\Omega}_{\tilde{A}^{t-1}}$ is first-order stochastically dominated by $\Omega_{\tilde{A}^{t+s-1}}$ for all $s > 0$.
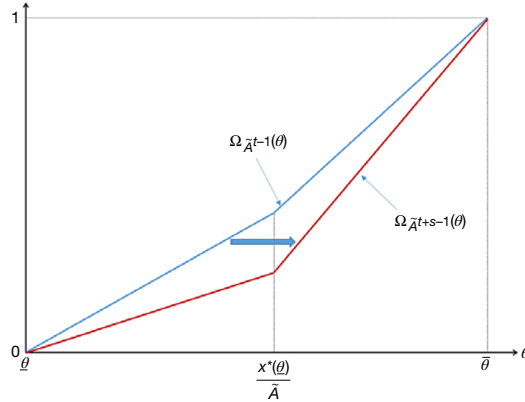
$\tilde{A} \in \left[\frac{x^*(\underline{\theta})}{\underline{\theta}}, 1 \right]$, then no entrepreneurs default in every period. In either case, $\widehat{\Omega}_{\tilde{A}^{t-1}} = U_{[\underline{\theta}, \overline{\theta}]}$ for all $t \geq 0$, and hence, $\widehat{Y}\left(\tilde{A}^t \right) = \frac{\tilde{A}(\underline{\theta}+\overline{\theta})}{2}$.[20] On the other hand, when $\tilde{A} \in \left(\frac{x^*(\underline{\theta})}{\overline{\theta}}, \frac{x^*(\underline{\theta})}{\underline{\theta}} \right)$, a certain fraction of entrepreneurs leave the economy after default and are replaced with new entrepreneurs, thereby changing the cdf $\widehat{\Omega}_{\tilde{A}^{t-1}}$ and $\widehat{Y}\left(\tilde{A}^t \right)$ over time, as stated in (11) and (12), respectively.

Note that when $\tilde{A} \in \left(\frac{x^*(\underline{\theta})}{\overline{\theta}}, \frac{x^*(\underline{\theta})}{\underline{\theta}} \right)$, $\widehat{\Omega}_{\tilde{A}^{t-1}}$ in (11) improves over time in the sense of first-order-stochastic dominance as described in Figure 2, because $\Delta < 1$ and $\frac{\theta - \underline{\theta}}{\overline{\theta} - \underline{\theta}} > \frac{\theta - \frac{x^*(\underline{\theta})}{A}}{\overline{\theta} - \frac{x^*(\underline{\theta})}{A}}$ for all $\theta \in \Theta$. Consequently, $\widehat{Y}\left(\tilde{A}^t \right)$ increases over time and converges to its limit $\frac{x^*(\underline{\theta}) + \tilde{A}\overline{\theta}}{2}$. The intuitive explanation for these findings is as follows: All new entrepreneurs offer $x^*(\underline{\theta})$ to lenders when they are born. Among them, $1 - \Delta$ fraction of entrepreneurs with $\theta \geq \frac{x^*(\underline{\theta})}{A}$ repay $x^*(\underline{\theta})$ in that period and offer $x^* \left(\frac{x^*(\underline{\theta})}{A} \right)$ to lenders for all succeeding periods, remaining in the economy.[21]

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[20]When $\tilde{A} = \frac{x^*(\underline{\theta})}{\overline{\theta}}$, entrepreneurs with $\overline{\theta}$ do not default and survive to the next period. However, the measure of the surviving entrepreneurs is 0 in every period, so they do not affect the cdf $\widehat{\Omega}_{\tilde{A}^{t-1}}$.

[21]Note that $x^* \left(\frac{x^*(\underline{\theta})}{A} \right) < x^*(\underline{\theta})$ given that $\tilde{A} < \frac{x^*(\underline{\theta})}{\underline{\theta}}$. Hence, entrepreneurs with $\theta \geq \frac{x^*(\underline{\theta})}{A}$ can honor contract $x^* \left(\frac{x^*(\underline{\theta})}{A} \right)$ for all succeeding periods.

On the other hand, Δ fraction of new entrepreneurs with $\theta < \frac{x^*(\theta)}{A}$ leave the economy after default, and they are replaced with new entrepreneurs who undergo the same process. In summary, only entrepreneurs with $\theta \geq \frac{x^*(\theta)}{A}$ survive in each period, and the process of survival of the fittest continues until θ of all entrepreneurs is distributed over $\left[\frac{x^*(\theta)}{A}, \overline{\theta}\right]$, and, hence, $\lim_{t\to\infty} \hat{Y}(\tilde{A}^t) = \frac{x^*(\theta) + \overline{A}\overline{\theta}}{2}$.

1. Effects of a Temporary Common Productivity Shock

We now study the dynamics of aggregate production after a temporary shock on common productivity when the economy stays in the stationary e^* equilibrium. By stationarity, we mean that the cdf Ω_t does not change over time. For example, if $\tilde{A} \in \left(0, \frac{x^*(\theta)}{\overline{\theta}}\right] \cup \left[\frac{x^*(\theta)}{\underline{\theta}}, 1\right]$, then the economy stays in a stationary equilibrium because $\hat{\Omega}_{\tilde{A}^{t-1}} = U_{[\underline{\theta}, \overline{\theta}]}$ for all $t \geq 0$. When $\tilde{A} \in \left(\frac{x^*(\theta)}{\overline{\theta}}, \frac{x^*(\theta)}{\underline{\theta}}\right)$, $\hat{\Omega}_{\tilde{A}^{t-1}}$ changes over time, but for a sufficiently high $T > 0$, we have $\hat{\Omega}_{\tilde{A}^{t-1}} \approx \hat{\Omega}_{\tilde{A}^t}$ for all $t \geq T$. In this case, we also say that the economy is in a stationary equilibrium, and let $\hat{\Omega}_{\tilde{A}^{t-1}} = \hat{\Omega}_{\tilde{A}^t}$ for all $t \geq T$ without loss of generality.

Define a sequence $\hat{A}^t = \{A_\tau\}_{\tau=-1}^t$ for each $t \geq 0$ such that

(13) $$A_\tau = \tilde{A} \text{ for all } \tau \neq \eta \text{ and } A_\eta = A'.$$

Suppose that the economy has reached the stationary equilibrium in period $\eta' < \eta$ (i.e., $\hat{\Omega}_{\tilde{A}^{t-1}} = \hat{\Omega}_{\tilde{A}^t}$ for $t \in \{\eta', \ldots, \eta - 1\}$). Clearly, the aggregate production in period η, when the shock arrives, is given as $\hat{Y}(\hat{A}^\eta) = \frac{A'}{\tilde{A}} \hat{Y}(\hat{A}^{\eta-1})$. However, a temporary productivity shock might have persistent impacts on aggregate productions, which requires more detailed analysis. In particular, we analyze the dynamics of $\hat{Y}(\hat{A}^t)$ for $t > \eta$ by dividing the shock into 2 groups depending on the nature of the shock: whether the shock is positive (i.e., $A' > \tilde{A}$) or negative (i.e., $A' < \tilde{A}$).

Proposition 8 describes the effects of a positive productivity shock $(A' > \tilde{A})$ in period $t = \eta$ on $\hat{Y}(\hat{A}^t)$ for $t \geq \eta + 1$.

Proposition 8. Take the sequence $\hat{A}^t$ given by (13) for some $\tilde{A} \in (0, 1)$ and $A' \in (\tilde{A}, 1]$, and assume that the economy has reached the stationary e^* equilibrium in period $\eta' < \eta$. If $\tilde{A} \in \left(0, \frac{x^*(\theta)}{\overline{\theta}}\right]$ and $A' \in \left(\frac{x^*(\theta)}{\overline{\theta}}, \frac{x^*(\theta)}{\underline{\theta}}\right)$, then

$\hat{Y}(\hat{A}^{\eta+1}) = \hat{Y}(\hat{A}^{\eta-1}) + \frac{\tilde{A}\left(\overline{\theta} - \frac{x^*(\theta)}{A'}\right)\left(\frac{x^*(\theta)}{A'} - \underline{\theta}\right)}{2(\overline{\theta} - \underline{\theta})}$, and for all $t \geq \eta + 2$, there exists $A^* \in \left[\frac{x^*(\theta)}{\overline{\theta}}, \frac{x^*(\theta)}{\underline{\theta}}\right)$ such that

$$
\hat{Y}\left(\hat{A}^t\right) =
\begin{cases}
\hat{Y}\left(\hat{A}^{\eta+1}\right) & \text{when } A' \leq A^* \\[2ex]
\hat{Y}\left(\hat{A}^{\eta-1}\right) + \max\left\{0, \dfrac{\left(\tilde{A}\overline{\theta} - x^*\left(\frac{x^*(\theta)}{A'}\right)\right)\left(x^*\left(\frac{x^*(\theta)}{A'}\right) - \tilde{A}\underline{\theta}\right)}{2\tilde{A}\left(\overline{\theta} - \underline{\theta}\right)}\right\} & \text{when } A' > A^*.
\end{cases}
$$

Otherwise, $\hat{Y}\left(\hat{A}^t\right) = \hat{Y}\left(\hat{A}^{\eta-1}\right)$ for all $t \geq \eta + 1$.

   Proposition 8 shows that whether a positive productivity shock has temporary or persistent effects on aggregate production depends on the nature of a stationary equilibrium before the shock. Specifically, the economy can land in a stationary equilibrium for 2 different reasons. When $\tilde{A} > \frac{x^*(\theta)}{\underline{\theta}}$, the cdf Ω_t does not change over time in a stationary equilibrium because all existing entrepreneurs honor debts staying in the economy.[22] In this case, a positive productivity shock in period η does not change the composition of entrepreneurs in the economy. Therefore, $\Omega_t = \Omega_{\eta-1}$ for all $t > \eta$, and the aggregate production immediately returns to the pre-shock level (i.e., $\hat{Y}\left(\hat{A}^t\right) = \hat{Y}\left(\hat{A}^{\eta-1}\right)$ for all $t > \eta$). On the other hand, when $\tilde{A} \in \left(0, \frac{x^*(\theta)}{\overline{\theta}}\right]$, all entrepreneurs with $\theta < \overline{\theta}$ default and are replaced with new entrepreneurs in every period. As a result, $\hat{\Omega}_{\hat{A}^{t-1}} = U_{[\underline{\theta},\overline{\theta}]}$ for all $t < \eta$ and the economy is in a stationary equilibrium.[23] In this case, a positive shock in period η might allow some entrepreneurs to honor debts, which causes changes in Ω_t for all succeeding periods after the shock, depending on the level of shock A'.

   Specifically, if $A' \in \left(\tilde{A}, \frac{x^*(\theta)}{\overline{\theta}}\right]$, all entrepreneurs with $\theta < \overline{\theta}$ default in period η same as to the pre-shock state, and, hence, $\Omega_{\eta+1} = U_{[\underline{\theta},\overline{\theta}]}$. On the other hand, if $A' \in \left[\frac{x^*(\theta)}{\underline{\theta}}, 1\right]$, then a positive shock is sufficiently high, such that all entrepreneurs survive in period η, and, hence, $\Omega_{\eta+1} = U_{[\underline{\theta},\overline{\theta}]}$. In both cases, the economy returns to the equilibrium in which all entrepreneurs default in every period once common productivity returns to the pre-shock level $\tilde{A}$. As a result, $\hat{Y}\left(\hat{A}^t\right) = \frac{\tilde{A}(\underline{\theta}+\overline{\theta})}{2} = \hat{Y}\left(\hat{A}^{\eta-1}\right)$ for all $t \geq \eta + 1$. Finally, if $A' \in \left(\frac{x^*(\theta)}{\overline{\theta}}, \frac{x^*(\theta)}{\underline{\theta}}\right)$, entrepreneurs with $\theta \in \left[\frac{x^*(\theta)}{A'}, \overline{\theta}\right]$ honor contract $x^*(\theta)$ when a positive shock arrives in period η

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[22]Specifically, if $\tilde{A} \in \left[\frac{x^*(\theta)}{\underline{\theta}}, 1\right]$, then $\hat{\Omega}_{\hat{A}^{t-1}} = U_{[\underline{\theta},\overline{\theta}]}$, and if $\tilde{A} \in \left(\frac{x^*(\theta)}{\overline{\theta}}, \frac{x^*(\theta)}{\underline{\theta}}\right)$, then $\hat{\Omega}_{\hat{A}^{t-1}} = U_{\left[\frac{x^*(\theta)}{\tilde{A}}, \overline{\theta}\right]}$ in a stationary equilibrium without any defaults.

[23]When $\tilde{A} = \frac{x^*(\theta)}{\overline{\theta}}$, entrepreneurs with $\theta = \overline{\theta}$ do not default and survive. However, the measure of the surviving entrepreneurs is 0 and, hence, we obtain $\Omega_{\hat{A}^{t-1}} = U_{[\underline{\theta},\overline{\theta}]}$ for all $t < \eta$.

and entrepreneurs with $\theta < \frac{x^*(\theta)}{A'}$ default. The group of surviving entrepreneurs has a higher average entrepreneurial productivity than the group of new entrepreneurs. Consequently, the aggregate production in period $\eta + 1$ is higher than the pre-shock level even though common productivity returns to the pre-shock level, $\tilde{A}$. In the afternoon in period $\eta + 1$, a fraction of the survived entrepreneurs might default given $\tilde{A}$. However, unless $\tilde{A}$ is sufficiently low to drive out all survived entrepreneurs, the aggregate production stays at a higher level than the pre-shock level (i.e., $\hat{Y}\left(\hat{A}^t\right) > \hat{Y}\left(\hat{A}^{\eta-1}\right)$ for all $t > \eta + 1$). Thus, a positive productivity shock can have long-term effects on aggregate production in our model consistent with the empirical finding that a positive demand shock could have persistent impacts on the economy (see Hvide and Meling (2023)).

We now study the effects of a negative common productivity shock $\left(A' < \tilde{A}\right)$ in a stationary equilibrium on the dynamics of aggregate production, which is described in the following proposition.

Proposition 9. Take the sequence $\hat{A}^t$ given by (13) for some $\tilde{A} \in (0,1]$ and $A' \in \left(0, \tilde{A}\right)$, and assume that the economy has reached the stationary e^* equilibrium in period $\eta' < \eta$. Let $\tilde{\theta} = \frac{x^*(\theta)}{A}$, $\Delta = \frac{\frac{x^*(\theta)}{\tilde{A}} - \theta}{\bar{\theta} - \underline{\theta}}$, $\Delta' = \frac{\frac{x^*(\theta)}{A'} - \theta}{\bar{\theta} - \underline{\theta}}$, and $\tilde{\Delta}' = \min\left\{1, \frac{\frac{x^*(\bar{\theta})}{A'} - \bar{\theta}}{\bar{\theta} - \tilde{\theta}}\right\}$.

Then, for $t \geq \eta + 1$, $\hat{Y}\left(\hat{A}^t\right)$ is given as follows:

1. If $\tilde{A} \in \left[\frac{x^*(\theta)}{\theta}, 1\right]$ and $A' \in \left(\frac{x^*(\theta)}{\bar{\theta}}, \frac{x^*(\theta)}{\underline{\theta}}\right)$, then

$$\hat{Y}\left(\hat{A}^t\right) = \Delta' \frac{\tilde{A}\left(\underline{\theta} + \bar{\theta}\right)}{2} + [1 - \Delta'] \frac{\tilde{A}}{2}\left(\frac{x^*(\theta)}{A'} + \bar{\theta}\right).$$

2. If $\tilde{A} \in \left(\frac{x^*(\theta)}{\bar{\theta}}, \frac{x^*(\theta)}{\underline{\theta}}\right)$ and $A' \in \left(0, \frac{x^*(\bar{\theta})}{\bar{\theta}}\right)$, then

$$\hat{Y}(\hat{A}^t) = \tilde{\Delta}'\left\{\Delta^{t-(\eta+1)} \frac{\tilde{A}\left(\underline{\theta} + \bar{\theta}\right)}{2} + [1 - \Delta^{t-(\eta+1)}] \frac{x^*(\theta) + \tilde{A}\bar{\theta}}{2}\right\} + [1 - \tilde{\Delta}'] \frac{\tilde{A}}{2}\left(\frac{x^*(\tilde{\theta})}{A'} + \bar{\theta}\right).$$

3. Otherwise, $\hat{Y}\left(\hat{A}^t\right) = \hat{Y}\left(\hat{A}^{\eta-1}\right)$.

In the model, if $\tilde{A} \leq \frac{x^*(\theta)}{\bar{\theta}}$, the economy has experienced continuous defaults and replacements of all entrepreneurs before the negative productivity shock, and the economy experiences the same process when the shock arrives in period η. Thus, for all $t \geq \eta + 1$, $\Omega_t = \Omega_{\eta-1} = U_{[\underline{\theta},\bar{\theta}]}$ and $\hat{Y}\left(\hat{A}^t\right) = \hat{Y}\left(\hat{A}^{\eta-1}\right)$. On the other hand, if $\tilde{A} > \frac{x^*(\theta)}{\bar{\theta}}$, a negative shock could have long-term effects on aggregate production by driving existing entrepreneurs to default. Specifically, Proposition 9 shows that the dynamics of $\hat{Y}\left(\hat{A}^t\right)$ depend on the measure of defaulted entrepreneurs when the negative shock arrives in period s.

First, if $A' \in \left(0, \tilde{A}\right)$ is sufficiently high, all existing entrepreneurs survive without defaulting in period s. This implies that for all $t \geq \eta + 1$, $\Omega_t = \Omega_{\eta-1}$ and

$\hat{Y}\left(\hat{A}^t\right) = \hat{Y}\left(\hat{A}^{\eta-1}\right)$. Second, if A' is low enough, a certain fraction (Δ' and $\tilde{\Delta}'$ for cases 1 and 2, respectively) of existing entrepreneurs default in period η and are replaced with new entrepreneurs. Thus, $\hat{Y}\left(\hat{A}^t\right)$ for $t \geq \eta+1$ consists of the following 2 parts: i) goods produced by entrepreneurs who were born after the negative shock and ii) goods produced by existing entrepreneurs who did not default in period η when the shock arrived. In particular, if A' is sufficiently low, then all existing entrepreneurs leave the economy, and the economy starts with all new entrepreneurs in period $\eta+1$.

Thus far, we have focused on the effects of a common productivity shock in a stationary equilibrium. However, the results that a positive shock supports more entrepreneurs to honor debts while a negative shock causes more entrepreneurs to default hold in a non-stationary equilibrium. Thus, given a sequence of $\{A_t\}_{t=0}^{\infty} \in \mathbb{A}$, where A_t is independently distributed over time, the pattern of the dynamics of the aggregate output is similar to the results in Propositions 8 and 9, although the aggregate output fluctuates in response to changes in A_t over time.

Note, from Proposition 9, that when $\tilde{A} \in \left(\frac{x^*(\underline{\theta})}{\bar{\theta}}, \frac{x^*(\underline{\theta})}{\underline{\theta}}\right)$, the time it takes for the aggregate production to recover back to the pre-shock level after a negative shock depends on the size of the shock, which is measured by $\frac{\tilde{A}-A'}{\tilde{A}}$. Specifically, when A' is not excessively low, such as $A' \in \left[\frac{x^*(\underline{\theta})}{\underline{\theta}}, \tilde{A}\right)$, no entrepreneurs default in period η and the aggregate production $\hat{Y}\left(\hat{A}^t\right)$ moves back to the pre-shock level in the next period. On the other hand, if A' is sufficiently low, such as $A' \leq \frac{x^*(\bar{\theta})}{\bar{\theta}}$, then all entrepreneurs with $\theta < \bar{\theta}$ default when the shock arrives in period η, and $\hat{Y}\left(\hat{A}^t\right)$ increases for all $t \geq \eta+1$, converging to $\frac{x^*(\underline{\theta})+\tilde{A}\bar{\theta}}{2}$.[24] Finally, suppose that $A' \in \left(\frac{x^*(\bar{\theta})}{\bar{\theta}}, \frac{x^*(\underline{\theta})}{\bar{\theta}}\right)$. Then, from case 2 of Proposition 9 and the fact that $\hat{Y}\left(\hat{A}^{\eta-1}\right) = \frac{x^*(\underline{\theta})+\tilde{A}\underline{\theta}}{2}$ when $\tilde{A} \in \left(\frac{x^*(\underline{\theta})}{\bar{\theta}}, \frac{x^*(\underline{\theta})}{\underline{\theta}}\right)$, we obtain
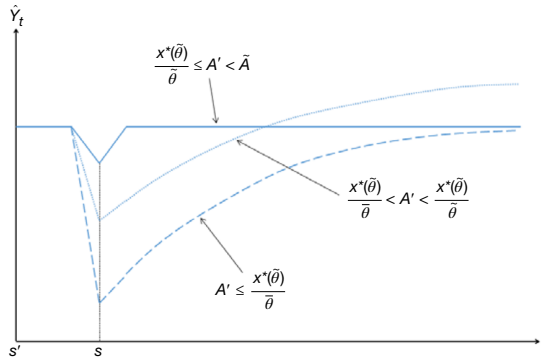
$$(14) \quad \hat{Y}\left(\hat{A}^t\right) - \hat{Y}\left(\hat{A}^{\eta-1}\right) = \frac{1-\tilde{\Delta}'}{2}\left(x^*(\bar{\theta})\frac{\tilde{A}}{A'} - x^*(\underline{\theta})\right) - \frac{\tilde{\Delta}'\Delta^{t-\eta-1}}{2}\left(x^*(\underline{\theta}) - \tilde{A}\underline{\theta}\right)$$

for $t \geq \eta+1$. Substituting $\Delta = \frac{\frac{x^*(\underline{\theta})}{A'}-\underline{\theta}}{\bar{\theta}-\underline{\theta}}$ and $\tilde{\Delta}' = \frac{\frac{x^*(\bar{\theta})}{A'}-\tilde{\theta}}{\bar{\theta}-\underline{\theta}}$ into (14) and using the definition $\tilde{\theta} = \frac{x^*(\underline{\theta})}{\tilde{A}}$ and the assumptions that $\tilde{A} > \frac{x^*(\underline{\theta})}{\bar{\theta}}$ and $A' < \frac{x^*(\bar{\theta})}{\underline{\theta}}$, we obtain that $\hat{Y}\left(\hat{A}^t\right) \geq \hat{Y}\left(\hat{A}^{\eta-1}\right)$ for all $t \geq \hat{\imath}(\tilde{A}, A') + \eta + 1$, where

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[24]When $A' = \frac{x^*(\bar{\theta})}{\bar{\theta}}$, entrepreneurs with $\theta = \bar{\theta}$ survive, but their measure is 0 and, hence, $\Omega_{\eta+1} = U_{[\underline{\theta},\bar{\theta}]}$.

FIGURE 3

Dynamics of $\hat{Y}\left(\hat{A}^{t}\right)$ when $\tilde{A} \in \left(\frac{x^{*}(\underline{\theta})}{\bar{\theta}}, \frac{x^{*}(\underline{\theta})}{\underline{\theta}}\right)$

Figure 3 delineates the dynamics of $\hat{Y}(\hat{A}^{t})$ after a shock to common productivity A' in period s. It demonstrates that the pace of output recovery slows as the size of the negative shock increases. The figure also illustrates that when the shock is in an intermediate range, $A' \in (\frac{x^{*}(\bar{\theta})}{\bar{\theta}}, \frac{x^{*}(\underline{\theta})}{\bar{\theta}})$ $\hat{Y}(\hat{A}^{t})$ can surpass the initial level of output $\hat{Y}(\hat{A}^{\eta-1})$ before the shock, after a sufficiently long period of time.

$$(15) \qquad \hat{t}\left(\tilde{A},A'\right) \equiv \frac{\log\left(\tilde{\theta} - \underline{\theta}\right) - \log\left(\overline{\theta} - \frac{x^{*}\left(\tilde{\theta}\right)}{A'}\right)}{\log\left(\overline{\theta} - \underline{\theta}\right) - \log\left(\tilde{\theta} - \underline{\theta}\right)}.$$

Note that $\hat{t}\left(\tilde{A},A'\right)$ in (15) decreases with A'. Thus, it takes more time for the aggregate production to return to the pre-shock level as A' decreases. The analysis of the above 3 cases shows that the time for the recovery of aggregate production increases as the size of the negative shock increases. Figure 3 summarizes the above analysis.

The results described in Propositions 8 and 9 (in particular, in Figure 3) imply that, on average, the pace of increases in the output is slower than the pace of declines in the model economy, consistent with empirical findings (see Neftçi (1984), Hamilton (1989), and Morley and Piger (2012)). Thus, the model can generate the cyclical asymmetry in which the economy behaves differently over the expansion and recession phases of the business cycle.

A number of studies have attempted to provide explanations for the cyclical asymmetry of aggregate time-series data. For example, Acemoglu and Scott (1997) show that intertemporal increasing return can generate persistence in output over the expansion phases, and Chalkley and Lee (1998) derive similar results using risk-averse agents and noisy information on the aggregate state. Veldkamp (2005), Van Nieuwerburgh and Veldkamp (2006), and Ordoñez (2013) construct learning models to explain the asymmetry of the business cycle. More recently, Gorton and Ordoñez (2014) show the cyclical asymmetry of production and credit in a model of asset exchanges with costly information acquisition. Although these studies provide similar theoretic predictions to those of our model, the economic mechanisms that derive the cyclical asymmetry are different from the mechanism of our model.

Specifically, in the context of our model economy, the cyclical asymmetry of the business cycle and the slow recovery of output after a large shock are symptomatic of the improvement in entrepreneurial productivity over time through the continuous replacement of less productive entrepreneurs with new ones, complementing previous studies. Furthermore, once we interpret the total factor productivity as the product of common productivity and the average entrepreneurial productivity, our model provides better insights than the previous studies cited above into the recent empirical finding that a protracted drop in productivity is an essential factor of the slow recovery after a crisis (see Reifschneider, Wascher, and Wilcox (2015), Ikeda and Kurozumi (2019)).

2.   Constructive and Destructive Economic Downturns

One interesting result of Proposition 9 is that although aggregate production drops when a negative shock arrives, aggregate production can exceed the pre-shock level after the shock unless all existing entrepreneurs leave the economy or survive. Specifically, when $\tilde{A} \in \left[\frac{x^*(\underline{\theta})}{\underline{\theta}}, 1 \right]$, if $A' \in \left(\frac{x^*(\underline{\theta})}{\overline{\theta}}, \frac{x^*(\underline{\theta})}{\underline{\theta}} \right)$, then we obtain, from part 1 of Proposition 9, that $\hat{Y}\left(\hat{A}^t\right) > \hat{Y}\left(\hat{A}^{\eta-1}\right)$ for all $t \geq \eta + 1$. Similarly, when $\tilde{A} \in \left(\frac{x^*(\underline{\theta})}{\overline{\theta}}, \frac{x^*(\underline{\theta})}{\underline{\theta}} \right)$, if $A' \in \left(\frac{x^*(\overline{\theta})}{\overline{\theta}}, \frac{x^*(\overline{\theta})}{\underline{\theta}} \right)$, then $\hat{Y}\left(\hat{A}^t\right) \geq \hat{Y}\left(\hat{A}^{\eta-1}\right)$ for all $t \geq \hat{t}(\tilde{A}, A') + \eta + 1$, where $\hat{t}(\tilde{A}, A')$ is given in (15). This is because a negative productivity shock drives out less productive entrepreneurs and the average entrepreneurial productivity therefore increases. This result is consistent with the view that recessions cleanse out less efficient firms and hence raise the average firm-level productivity (see Caballero and Hammour (1994), Osotimehin and Pappadà (2017)).

Although a negative shock can increase long-term aggregate production, it still reduces aggregate production, $\hat{Y}\left(\hat{A}^\eta\right)$, when the shock arrives. The natural question is, then, whether a negative shock is beneficial. To conduct a cost–benefit analysis of a negative productivity shock, we use the sum of discounted aggregate production as our criterion for the constructiveness of a negative shock, which resonates with the utilitarian social welfare function.[25] Specifically, we compare $\sum_{t=0}^{\infty} \beta^t \hat{Y}\left(\tilde{A}^t\right)$ and $\sum_{t=0}^{\infty} \beta^t \hat{Y}\left(\hat{A}^t\right)$ for 2 sequences $\tilde{A}^t$ and $\hat{A}^t$, where $\hat{A}^t$ is given by (13) for some $\tilde{A} \in (0, 1]$ and $A' \in (0, \tilde{A})$. Note that $\hat{Y}\left(\tilde{A}^t\right) = \hat{Y}\left(\hat{A}^t\right)$ for all $t < \eta$. Given $\tilde{A}$ and β, define the set of A' as

$$I(\tilde{A}, \beta) = \left\{ A' \in (0, \tilde{A}) : \sum_{t=0}^{\infty} \beta^t \left[\hat{Y}\left(\hat{A}^t\right) - \hat{Y}\left(\tilde{A}^t\right) \right] > 0 \right\}.$$

[25]We adopt the classical utilitarian perspective of the Benthamite social welfare function, in which the goal of a social choice function is to maximize the sum of all individual's utility (see Arrow (1951), Harsanyi (1955)).

Then, for all $A' \in I(\tilde{A}, \beta)$, the negative shock is constructive; otherwise, the shock is destructive.

Proposition 10. Take the sequence $\hat{A}^t$ given by (13) for some $\tilde{A} \in (0,1]$ and $A' \in (0, \tilde{A})$. If β is sufficiently high, there exists $\tilde{A} \in \left(\frac{x^*(\theta)}{\bar{\theta}}, 1 \right]$ such that $I(\tilde{A}, \beta)$ is an open interval with the following properties:

1. If $\tilde{A} \in \left[\frac{x^*(\theta)}{\underline{\theta}}, 1 \right]$, then $I(\tilde{A}, \beta) \subset \left(\frac{x^*(\theta)}{\bar{\theta}}, \frac{x^*(\theta)}{\underline{\theta}} \right)$, and for any $\tilde{A}_1$, $\tilde{A}_2 \in \left[\frac{x^*(\theta)}{\underline{\theta}}, 1 \right]$ with $\tilde{A}_1 < \tilde{A}_2$, $I(\tilde{A}_2, \beta) \subset I(\tilde{A}_1, \beta)$.

2. If $\tilde{A} \in \left(\frac{x^*(\theta)}{\bar{\theta}}, \frac{x^*(\theta)}{\underline{\theta}} \right)$, then $I(\tilde{A}, \beta) \subset \left(\frac{x^*(\bar{\theta})}{\bar{\theta}}, \frac{x^*(\bar{\theta})}{\theta} \right)$.

Proposition 10 describes that the constructiveness of the negative shock depends on 3 factors. First, for a negative shock to be constructive, the shock should remove less productive entrepreneurs and improve long-term average entrepreneurial productivity. Thus, constructive economic downturns occur only if A' is in the subset of $\left(\frac{x^*(\theta)}{\bar{\theta}}, \frac{x^*(\theta)}{\underline{\theta}} \right)$ or of $\left(\frac{x^*(\bar{\theta})}{\bar{\theta}}, \frac{x^*(\bar{\theta})}{\theta} \right)$, depending on $\tilde{A}$.[26] Second, it takes time for a negative shock to raise aggregate production above the pre-shock level; thus, a shock is more likely to be constructive when the discount factor β is higher. Third, $\tilde{A}$ is important because the cdf Ω_t before the shock and the size of the shock, $\frac{\tilde{A}-A'}{\tilde{A}}$, depend on $\tilde{A}$. Specifically, when $\tilde{A} \in \left[\frac{x^*(\bar{\theta})}{\bar{\theta}}, 1 \right]$, a decrease in $\tilde{A}$ alleviates only the temporary negative effect of the shock given A' by reducing the size of the shock, $\frac{\tilde{A}-A'}{\tilde{A}}$, and does not affect Ω_t for all $t \geq 0$. Thus, the measure of $I(\tilde{A}, \beta)$ decreases with $\tilde{A}$. Next, when $\tilde{A} \in \left(\frac{x^*(\theta)}{\bar{\theta}}, \frac{x^*(\theta)}{\underline{\theta}} \right)$, a decrease in $\tilde{A}$ also alleviates the temporary negative effect of the shock, which expands the set $I(\tilde{A}, \beta)$. However, in this case, θ is uniformly distributed over $\left[\frac{x^*(\theta)}{\tilde{A}}, \bar{\theta} \right]$ in the steady state. Thus, as $\tilde{A}$ decreases, the average productivity of existing entrepreneurs before the shock increases and the positive effects of the negative shock on long-term aggregate output decreases, thereby contracting the set $I(\tilde{A}, \beta)$. Combined, the effect of $\tilde{A}$ on $I(\tilde{A}, \beta)$ is unclear.

The results of Proposition 10 imply that a severe negative shock tends to be destructive, while a mild negative shock is more likely to be constructive. This is

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[26]Proposition 9 shows that i) when $\tilde{A} \in \left[\frac{x^*(\theta)}{\underline{\theta}}, 1 \right]$, the measure of defaulting entrepreneurs Δ' is in $(0,1)$ for $A' \in \left(\frac{x^*(\theta)}{\bar{\theta}}, \frac{x^*(\theta)}{\underline{\theta}} \right)$, and ii) when $\tilde{A} \in \left(\frac{x^*(\theta)}{\bar{\theta}}, \frac{x^*(\theta)}{\underline{\theta}} \right)$, the measure of defaulting entrepreneurs $\tilde{\Delta}'$ is in $(0,1)$ for $A' \in \left(\frac{x^*(\bar{\theta})}{\bar{\theta}}, \frac{x^*(\bar{\theta})}{\theta} \right)$.

because a severe shock drives out not only bad entrepreneurs but also good entrepreneurs, while a mild shock improves the average entrepreneurial productivity through productive winnowing. Thus, our model suggests that if government stimulus measures alleviate the effects of a negative shock on the economy, the government should intervene in markets when a shock is sufficiently severe to prevent the collapse of good entrepreneurs. This implication is consistent with the view that government stimulus measures are more effective and necessary when the economy is in a severe economic downturn (Auerbach and Gorodnichenko (2012)).

V.   Conclusion

In this article, we have constructed a dynamic equilibrium model of debt contracts with adverse selection and studied how lenders use information about aggregate economic conditions in the past to construct beliefs about the credit risks of borrowers with different business operation histories. We have shown that borrowers' credit risk perceived by lenders decreases as borrowers age because more productive borrowers tend to stay in the economy for longer periods without defaults, and, hence, the borrowing cost weakly decreases with the borrower's age. Cross-sectionally, old borrowers pay lower borrowing costs than young borrowers on average, although the reverse is also possible under some conditions. We have shown that the model was tractable for analyzing impulse responses after an aggregate productivity shock. We used the model to provide theoretical explanations for the cyclical asymmetry of aggregate output over the business cycle and a narrative for the sluggish recovery of aggregate production after a crisis. The model also shows that a mild negative productivity shock can be constructive by increasing long-term aggregate output, while a severe negative shock is always destructive.

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Supplementary Material

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