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Some Adjunction Properties of Ample Vector Bundles

Hironobu Ishihara

Abstract. Let $\mathcal{E}$ be an ample vector bundle of rank r on a projective variety X with only log-terminal singularities. We consider the nefness of adjoint divisors $K_X + (t - r)\det\mathcal{E}$ when $t \geq \dim X$ and $t > r$. As an application, we classify pairs $(X, \mathcal{E})$ with c_r-sectional genus zero.

1 Introduction

Let X be a smooth projective variety and K_X the canonical bundle of X. For the study of X, it is useful to consider adjoint bundles $K_X + tL$, where t is a positive integer and L is an ample line bundle on X. We refer to the books [BS] and [F0] for the properties of $K_X + tL$; it is powerful when t is close to $\dim X$.

Recently, as a natural generalization of adjoint bundles, many authors have considered $K_X + \det\mathcal{E}$, where $\mathcal{E}$ is an ample vector bundle on X. (We say that a vector bundle $\mathcal{E}$ is ample if $\mathcal{O}_{\mathbb{P}(\mathcal{E})}(1)$ is ample on $\mathbb{P}(\mathcal{E})$.) In particular, Ye and Zhang [YZ] have given a classification for pairs $(X, \mathcal{E})$ when $\operatorname{rank}\mathcal{E} \geq n - 1$ and $K_X + \det\mathcal{E}$ is not nef. Many other results on $K_X + \det\mathcal{E}$ are obtained when $\operatorname{rank}\mathcal{E}$ is close to $\dim X$. It seems to be difficult to study the nefness of $K_X + \det\mathcal{E}$ when $\operatorname{rank}\mathcal{E}$ is small as compared with $\dim X$.

To overcome this difficulty, in the present paper, we consider the nefness of $K_X + (t - r)\det\mathcal{E}$ when $t \geq n = \dim X$ and $t > r = \operatorname{rank}\mathcal{E}$. We mainly use vanishing theorems and an estimate of the length of extremal rays, hence our argument works on projective varieties X with at worst log-terminal singularities. Our main result is Theorem 2.5 in which we show that $K_X + (n - r)\det\mathcal{E}$ is nef unless $(X, \mathcal{E}) \cong \bigl(\mathbb{P}^4, \mathcal{O}(1)^{\oplus 2}\bigr)$ when $1 < r < n - 1$.

As an application, we see that the c_r-sectional genus of the pairs $(X, \mathcal{E})$ is non-negative and we obtain the classification of $(X, \mathcal{E})$ with c_r-sectional genus zero in the case that X is log-terminal. We note that c_r-sectional genus is introduced in [I] and studied in the case that X is smooth (see also [FuI]).

2 Preliminaries

We work over the complex number field $\mathbb{C}$. Varieties are always irreducible and reduced. The tensor products of line bundles are denoted additively, while we use multiplicative notation for intersection products. The numerical equivalence is denoted

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by $\equiv$. We denote by $L^{\oplus n}$ the direct sum of n-copies of a line bundle L. The restriction $L|_Y$ of L to a variety Y is often written as L_Y. We denote by $\mathbb{Q}^n$ a (possibly singular) hyperquadric in $\mathbb{P}^{n+1}$. A polarized variety (X, L) is said to be a scroll over a variety W if $(X, L) \cong \big(\mathbb{P}_W(\mathcal{E}), \mathcal{O}_{\mathbb{P}(\mathcal{E})}(1)\big)$ for some vector bundle $\mathcal{E}$ on W. The number $\Delta(X, L) := \dim X + L^{\dim X} - h^0(X, L)$ is called the Δ-genus of a polarized variety (X, L).

The following facts are main tools of our argument.

Proposition 1.1 **([K, Theorem 1])** *Let Y be a projective variety with only log-terminal singularities and $f\colon Y \to Z$ a contraction morphism of an extremal ray of Y. Let E be an irreducible component of* $\operatorname{Exc}(f) := \{y \in Y \mid f$ *is not isomorphic at* $y\}$. *Then E is covered by a family of rational curves $\{C_i\}$ such that $f(C_i)$ are points and $-K_Y \cdot C_i \le 2\big(\dim E - \dim f(E)\big)$. Moreover, if f is birational, we have $-K_Y \cdot C_i < 2\big(\dim E - \dim f(E)\big)$.*

Proposition 1.2 **([Z1, Lemma 1]; see also [Z2, Lemma 1])** *Let Y be as in Proposition 1.1 and $f\colon Y \to Z$ a birational contraction morphism of an extremal ray R. Let F be an irreducible component of some positive-dimensional fiber of f. By taking a desingularization $\varphi\colon V \to F$ of F, we get $H^q\big(V, \varphi^*(-H_F)\big) = 0$ for any $H \in \operatorname{Pic} Y$ with $(K_Y + H)R \le 0$ and $q = \dim F$.*

3 Adjunction Properties

Throughout this section, let X be a projective variety with at worst log-terminal singularities, $n = \dim X \ge 2$, and let $\mathcal{E}$ be an ample vector bundle of rank r on X.

Theorem 2.1 *When $r \le n+1$, $K_X + (n+2-r)\det\mathcal{E}$ is always nef. Moreover, $K_X + (t-r)\det\mathcal{E}$ is always nef when $t \ge n+2$ and $r \le t-1$.*

Theorem 2.2 *When $r \le n$, $K_X + (n+1-r)\det\mathcal{E}$ is nef unless $(X, \mathcal{E}) \cong \big(\mathbb{P}^n, \mathcal{O}(1)\big)$ or $\big(\mathbb{P}^n, \mathcal{O}(1)^{\oplus n}\big)$.*

These theorems are proved later; now we consider the nefness of $K_X + (n-r)\det\mathcal{E}$ when $r \le n-1$.

Theorem 2.3 **(*cf.* [F2, Theorem 3.4])** *When $r = 1$, $K_X + (n-1)\mathcal{E}$ is nef unless $\Delta(X, \mathcal{E}) = 0$ or $(X, \mathcal{E})$ is a scroll over a smooth curve.*

Proof The following argument is almost due to Fujita [F2], Andreatta and Wiśniewski [AW]. By the proof of [F2, Theorem 3.4], we find that Theorem 2.3 is true except the following case (we set $L := \mathcal{E}$ since $r = 1$):

$(*)$ there exists a birational contraction morphism $f\colon X \to Z$ of an extremal ray R such that $\big(K_X + (n-1)L\big)R < 0$ and $(F', L_{F'}) \cong \big(\mathbb{P}^{n-1}, \mathcal{O}(1)\big)$ for the normalization F' of an irreducible component F of some fiber of f.

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We show that the case $(*)$ does not occur. We consider the structure of f locally in a neighborhood of F. Since $\dim F = n-1$ and $K_X + (n-1)L$ is not nef, the evaluation morphism $f^* f_* L \to L$ is surjective at every point of F by relative spannedness [AW, Theorem 5.1]. Hence we have $(F, L_F) \cong \big(\mathbb{P}^{n-1}, \mathcal{O}(1)\big)$. Applying horizontal slicing [AW, Lemma 2.6] repeatedly, we get a birational morphism $\varphi\colon S \to W$ such that S is a surface with only log-terminal singularities and $(K_S + L_S)C < 0$ for an irreducible component $C \cong \mathbb{P}^1$ of some fiber of φ. Let $\pi\colon S' \to S$ be a minimal resolution of S and let C' be the strict transform of C. Then $K_{S'} \cdot C' < -1$ and C' deforms in an at least 1-dimensional family, which derives a contradiction. ■

Remark 2.3.1 Polarized varieties (X, L) with $\Delta(X, L) = 0$ have been classified in [F1].

***Theorem 2.4* (*cf.* [Me, Theorem 2])** *When $r = n-1$, $K_X + \det \mathcal{E}$ is nef unless $(X, \mathcal{E})$ is one of the following:*

(i) $\big(\mathbb{P}^n, \mathcal{O}(1)^{\oplus(n-1)}\big)$;
(ii) $\big(\mathbb{P}^n, \mathcal{O}(1)^{\oplus(n-2)} \oplus \mathcal{O}(2)\big)$;
(iii) $\big(\mathbb{Q}^n, \mathcal{O}(1)^{\oplus(n-1)}\big)$;
(iv) *$X \cong \mathbb{P}_C(\mathcal{F})$ for a vector bundle $\mathcal{F}$ of rank n on a smooth curve C and $\mathcal{E}|_F = \mathcal{O}_{\mathbb{P}^{n-1}}(1)^{\oplus(n-1)}$ for every fiber $F \cong \mathbb{P}^{n-1}$ of the bundle projection $X \to C$;*
(v) *There exists a very ample line bundle L on X such that (X, L) is a generalized cone on $\big(\mathbb{P}^2, \mathcal{O}(2)\big)$ or $\big(\mathbb{P}^1, \mathcal{O}(e)\big)$ $(e \geq 3)$, and $\mathcal{E} = L^{\oplus(n-1)}$.*

Remark 2.4.1 The case (v) is overlooked in [Me, Theorem 2], but we can recover it. We refer to [BS, (1.1.8)] for generalized cones.

Theorem 2.5 *When $1 < r < n-1$, $K_X + (n-r)\det \mathcal{E}$ is nef unless $(X, \mathcal{E}) \cong \big(\mathbb{P}^4, \mathcal{O}(1)^{\oplus 2}\big)$.*

Remark 2.5.1 This theorem is proved by [I] in the case that X is smooth.

Proof of Theorems 2.1, 2.2 and 2.5 Suppose that $t \geq n$ and $r \leq t-1$ and $K_X + (t-r)\det \mathcal{E}$ is not nef. When $r = 1$, we have $t \geq n$ and $K_X + (t-1)\det \mathcal{E}$ is not nef. Then we are done by [M1, Proposition 2.1] and Theorem 2.3. When $r = t-1$, we have $r \geq n-1$ and $K_X + \det \mathcal{E}$ is not nef. Then we are done by [Z2, Theorem 1] and Theroem 2.4. Thus we may suppose that $1 < r < t-1$ in the following.

Let $p\colon \mathbb{P}_X(\mathcal{E}) \to X$ be the bundle projection. We set $Y := \mathbb{P}_X(\mathcal{E})$ and denote by L the tautological line bundle of Y. We can take an extremal ray R of Y such that $p^*\big(K_X + (t-r)\det \mathcal{E}\big) \cdot R < 0$ by an argument similar to that in [Z1, Claim IV]. Let $f\colon Y \to Z$ be a contraction morphism of R and let E be an irreducible component of $\mathrm{Exc}(f)$. By Proposition 1.1, there exists a rational curve $C \subset E$ belonging to R such that

$$-K_Y \cdot C \leq 2\big(\dim E - \dim f(E)\big) \leq 2n$$

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since $p|_F\colon F \to X$ is a finite morphism for every fiber F of $E \to f(E)$. On the other hand, we have

$$\begin{aligned}-K_Y \cdot C &= \bigl(rL - p^*(K_X + \det \mathcal{E})\bigr) C\\ &= r \cdot LC - p^*\bigl(K_X + (t-r)\det \mathcal{E}\bigr) \cdot C + (t-r-1)(p^* \det \mathcal{E}) \cdot C\\ &> r + (t-r-1)r\\ &= (t-r)r\\ &\geq 2(t-2),\end{aligned}$$

hence $t = n$ or $n+1$, and $LC = 1$ or 2. If $LC = 2$, we see that $t = n$ and $\dim E - \dim f(E) = n$.

Case 2.6 $LC = 1$**.** We have $(K_Y + sL)C < 0$ for $s \leq t$. We use Zhang's idea in [Z1] and [Z2]. If f is birational, by Proposition 1.2, $H^q\bigl(V, \varphi^*(-sL_F)\bigr) = 0$ for $s \leq t$, where $\varphi\colon V \to F$ is a desingularization of an irreducible component F of some positive-dimensional fiber of f and $q = \dim F$. We get $\chi\bigl(V, \varphi^*(-sL_F)\bigr) = 0$ for $1 \leq s \leq t$ by Kawamata-Viehweg vanishing theorem. Then it follows that $q = n = t$. Let $\mu\colon W \to F$ be the normalization that factors φ. We get $\bigl(W, \mu^*(L_F)\bigr) \cong \bigl(\mathbb{P}^n, \mathcal{O}(1)\bigr)$ by using [F2, Theorem 2.2]. Set $\lambda := (p|_F) \circ \mu$. Then $\lambda\colon W \to X$ is a finite surjective morphism. We can write $\lambda^*\bigl(K_X + (n-r)\det \mathcal{E}\bigr) = \mathcal{O}_{\mathbb{P}^n}(m)$. Let l be a line in $W \cong \mathbb{P}^n$ such that $\lambda(l) \subset X \setminus \operatorname{Sing} X$. Then we have $m = \lambda^*\bigl(K_X + (n-r)\det \mathcal{E}\bigr) \cdot l \in \mathbb{Z}$. Set $C' := \mu_* l$ as a 1-cycle. We find that

$$\begin{aligned}(K_Y + sL)C' &= \mu^*[(s-r)L + p^*(K_X + \det \mathcal{E})]_F \cdot l\\ &\leq (s-r) + m - (n-r-1)r\\ &\leq 0\end{aligned}$$

for $s \leq n+1$. Since $C' \equiv \alpha C$ for some $\alpha > 0$, we get $(K_Y + sL)C \leq 0$ for $s \leq n+1$. Then we infer that $\chi\bigl(V, \varphi^*(-sL_F)\bigr) = 0$ for $1 \leq s \leq n+1$ as before. This is a contradiction, thus f is of fiber type.

Let F be a general fiber of f. Since $(K_Y + tL)C < 0$, we see that $K_F + tL_F$ is not nef. Then $t = n$ and $(F, L_F) \cong \bigl(\mathbb{P}^n, \mathcal{O}(1)\bigr)$ by [M1, Proposition 2.1]. Let U be a smooth open subset of Z such that $f^{-1}(z) \cong \mathbb{P}^n$ for every $z \in U$. Set $V := f^{-1}(U)$. We see that $f|_V\colon V \to U$ is a smooth morphism. It follows that V is smooth and so is X. Then we obtain that $(X, \mathcal{E}) \cong \bigl(\mathbb{P}^4, \mathcal{O}(1)^{\oplus 2}\bigr)$ by Remark 2.5.1.

Case 2.7 $LC = 2$**.** We have $(K_Y + sL)C < 0$ for $s \leq n-1$. If f is of fiber type, then $-\bigl(K_F + (n-1)L_F\bigr)$ is ample for a general fiber F of f. Note that $\dim F = \dim E - \dim f(E) = n$. Using Vanishing theorem, we get $\chi(s) := \chi(F, sL_F) = 0$ for $-(n-1) \leq s \leq -1$, $\chi(0) = h^0(F, \mathcal{O}_F) = 1$ and $\chi(1) = h^0(F, L_F)$. Then we find that $\Delta(F, L_F) = 0$ by Riemann-Roch theorem. Hence (F, L_F) is one of the following [F1]:

(a) $\bigl(\mathbb{P}^n, \mathcal{O}(1)\bigr)$;

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(b) $\left(\mathbb{Q}^n, \mathcal{O}(1)\right)$;
(c) a scroll over $\mathbb{P}^1$;
(d) a generalized cone over a smooth subvariety $V \subset F$ with $\Delta(V, L_V) = 0$.

Then there exists a rational curve $l \subset F$ such that $L_F \cdot l = 1$. We see that $C \equiv 2l$ and we get

$$2n \geq -K_Y \cdot C > 2r(n-r) \geq 4(n-2),$$

a contradiction. Thus f is birational. Since

$$2n > -K_Y \cdot C > (n-r+1)r \geq 2(n-1),$$

we find that $r = 2$ or $(r, n) = (3, 5)$. If $(r, n) = (3, 5)$, then we have $(p^* \det \mathcal{E}) \cdot C = 3$. Set $A := 2L - p^* \det \mathcal{E}$. Since $AC = 1$, A is an f-ample line bundle on Y and we have $(K_Y + sA)C < 0$ for $s \leq 2n - 2 = 8$. Then we get a contradiction by using Proposition 1.2 as in Case 2.6. Thus we see that $r = 2$. Since $\dim E - \dim f(E) = n$, there exists an n-dimensional irreducible component F of some fiber of f. Since $\dim Y = n + 1$ and $K_Y + (n-1)L$ is not nef, we infer that $\Delta(F, L_F) = 0$ from the argument in the proof of [A, Theorem 2.1]. Then we get a contradiction by the same argument that is used when f is of fiber type. ■

4 An Application on c_r-Sectional Genus

Definition 3.1 Let X be an n-dimensional normal projective variety and $\mathcal{E}$ an ample vector bundle of rank $r < n$ on X. The *c_r-sectional genus* $g(X, \mathcal{E})$ of a pair $(X, \mathcal{E})$ is defined by the formula

$$2g(X, \mathcal{E}) - 2 := \left(K_X + (n-r)c_1(\mathcal{E})\right) c_1(\mathcal{E})^{n-r-1} c_r(\mathcal{E}),$$

where K_X is the canonical divisor of X.

Remark 3.1.1 Let $(X, \mathcal{E})$ be as above. When $r = 1$, $g(X, \mathcal{E})$ is called the *sectional genus* of a polarized variety $(X, \mathcal{E})$. We refer to [F0] for the general properties of sectional genus. When $r = n - 1$, $g(X, \mathcal{E})$ is called the *curve genus* of a generalized polarized variety $(X, \mathcal{E})$. We refer to [Ba], [LMS], [LM] and [M2] for the properties of curve genus in the case that X is smooth. We have good properties of $g(X, \mathcal{E})$ for general $r < n$ in the case that X is smooth (see [I] and [FuI]).

Lemma 3.2 *Let $(X, \mathcal{E})$ be as in Definition 3.1. Then $g(X, \mathcal{E})$ is an integer.*

Proof Let $\pi \colon X' \to X$ be a desingularization of X. We get $g(X', \pi^* \mathcal{E}) \in \mathbb{Z}$ by an argument in [I]. We have

$$\begin{aligned} 2g(X', \pi^*\mathcal{E}) - 2 &= \left(K_{X'} + (n-r)\pi^* c_1(\mathcal{E})\right) \left(\pi^* c_1(\mathcal{E})\right)^{n-r-1} \pi^* c_r(\mathcal{E}) \\ &= \left(\pi_* K_{X'} + (n-r)c_1(\mathcal{E})\right) c_1(\mathcal{E})^{n-r-1} c_r(\mathcal{E}) \\ &= 2g(X, \mathcal{E}) - 2, \end{aligned}$$

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hence $g(X,\mathcal{E}) = g(X', \pi^*\mathcal{E}) \in \mathbb{Z}$. ■

As corollaries of Theorems 2.3, 2.4 and 2.5, we obtain the following theorems.

***Theorem 3.3* (*cf.* [F2, Corollary 3.8])** *Let L be an ample line bundle on a projective variety X with only log-terminal singularities. Then $g(X,L) \geq 0$, and $g(X,L) = 0$ if and only if $\Delta(X,L) = 0$.*

Proof First we note that $\Delta(X,L) = 0$ implies $g(X,L) = 0$ (see [F1]). Assume that $g(X,L) \leq 0$. Then $K_X + (n-1)L$ is not nef and it follows that $g(X,L) = \Delta(X,L) = 0$ by Theorem 2.3. ■

Theorem 3.4 *Let $(X,\mathcal{E})$ be as in Definition 3.1. Suppose that $2 \leq r = n-1$ and X has at worst log-terminal singularities. Then $g(X,\mathcal{E}) \geq 0$, and $g(X,\mathcal{E}) = 0$ if and only if $(X,\mathcal{E})$ is one of the following:*

(i) $\left(\mathbb{P}^n, \mathcal{O}(1)^{\oplus(n-1)}\right)$;
(ii) $\left(\mathbb{P}^n, \mathcal{O}(1)^{\oplus(n-2)} \oplus \mathcal{O}(2)\right)$;
(iii) $\left(\mathbb{Q}^n, \mathcal{O}(1)^{\oplus(n-1)}\right)$;
(iv) *$X \cong \mathbb{P}_{\mathbb{P}^1}(\mathcal{F})$ for a vector bundle $\mathcal{F}$ of rank n on $\mathbb{P}^1$ and $\mathcal{E}|_F = \mathcal{O}_{\mathbb{P}^{n-1}}(1)^{\oplus(n-1)}$ for every fiber $F \cong \mathbb{P}^{n-1}$ of the bundle projection $X \to \mathbb{P}^1$;*
(v) *There exists a very ample line bundle L on X such that (X,L) is a generalized cone on $\left(\mathbb{P}^2, \mathcal{O}(2)\right)$ or $\left(\mathbb{P}^1, \mathcal{O}(e)\right)$ $(e \geq 3)$, and $\mathcal{E} = L^{\oplus(n-1)}$.*

Proof Assume that $g(X,\mathcal{E}) \leq 0$. Then $K_X + \det\mathcal{E}$ is not nef and $(X,\mathcal{E})$ is one of the cases in Theroem 2.4. In the cases (i), (ii), (iii) and (v) of Theorem 2.4, we have $g(X,\mathcal{E}) = 0$. In the case (iv) of Theorem 2.4, we have $g(X,\mathcal{E}) = g(C)$, hence $g(X,\mathcal{E}) = 0$ and $C \cong \mathbb{P}^1$ by assumption. ■

Theorem 3.5 *Let $(X,\mathcal{E})$ be as in Definition 3.1. Suppose that $1 < r < n-1$ and X has at worst log-terminal singularities. Then $g(X,\mathcal{E}) \geq 0$, and $g(X,\mathcal{E}) = 0$ if and only if $(X,\mathcal{E}) \cong \left(\mathbb{P}^4, \mathcal{O}(1)^{\oplus 2}\right)$.*

This is shown as in the proof of Theorem 3.4 by using Theorem 2.5.

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