CONJUGACY OF FREE FINITE GROUP ACTIONS ON INFRANILMANIFOLDS

by MICHAŁ SADOWSKI

(Received 14 December, 1988; revised 23 March, 1989)

In this note we give the proof of the following result (previously known for homotopically trivial and free actions on infranilmanifolds [**3**, Theorem 5.6]).

THEOREM 1. *Let G be a finite group acting freely and smoothly on a closed infranilmanifold M. Assume that* $\dim M \neq 3, 4$. *Then the action of G is topologically conjugate to an affine action.*

The following notions are used here. A diffeomorphism f of a Lie group N onto a Lie group N' is said to be *affine* if $f = L_g \circ \Phi$, where $\Phi : N \to N'$ is an isomorphism, $g \in G$, and $L_g : N' \to N'$ is given by $L_g(x) = gx$. An *infranilmanifold* is an orbit space $M = N/\Gamma$, where N is a nilpotent simply connected Lie group, Γ is a discrete group acting affinely, freely, and properly discontinuously on N and such that $N \cap \Gamma$ has finite index in Γ. Note that Γ is the deck group of M. This group is virtually nilpotent (that is Γ is a finite extension of a nilpotent group). A diffeomorphism of one infranilmanifold onto another one is *affine* if it is covered by an affine diffeomorphism of nilpotent Lie groups.

Proof of Theorem 1. Since the group G acts freely, the orbit space M/G is a closed manifold. The group $\Pi_1(M/G)$ is virtually nilpotent. According to [**1**, Theorem 6.3], [**2**, Section 3.2, Corollary 1] there is an infranilmanifold V_0 and a homeomorphism $f_0 : M/G \to V_0$.

If $p : M \to M/G$ is the canonical projection, then the following diagram commutes.

$$\begin{array}{ccc} M & \xrightarrow{p} & M/G \\ \downarrow{\scriptstyle f} & & \downarrow{\scriptstyle f_0} \\ V & \xrightarrow{q} & V_0 \end{array}$$

Here $q : V \to V_0$ is the covering induced by f_0^{-1} and f covers f_0. The map q induces the structure of an infranilmanifold on V. For any $g \in G$ the homeomorphism $f \circ \sigma(g) \circ f^{-1}$ (where $\sigma(g)$ is the action of g on M) induces the identity map on V_0 so that $f \circ \sigma(g) \circ f^{-1}$ is an affine transformation of V.

By [**2**, Section 4.2], there is an affine diffeomorphism $h : V \to M$. The formula $\rho(g) = h \circ f \circ \sigma(g) \circ (h \circ f)^{-1}$ defines an affine action of G on M that is topologically conjugate to the original action of G. The proof of Theorem 1 is complete.

REMARK 1. The conjugating homeomorphism $h \circ f$ can be chosen in such a way that it is homotopic to the identity, because every homeomorphism of a closed infranilmanifold is homotopic to an affine diffeomorphism ([**2**, Section 4]).

Glasgow Math. J. **32** (1990) 239–240.

REFERENCES

1. F. T. Farrell and W. C. Hsiang, Topological characterisation of flat and almost flat Riemannian manifolds M^n ($n \neq 3, 4$), *Amer. J. Math.* **105** (1983), 641–673.

2. Y. Kamishima, K. B. Lee and F. Raymond, The Seifert construction and its applications to infranilmanifolds, *Quart. J. Math. Oxford* **34** (1983), 433–452.

3. K. B. Lee and F. Raymond, Geometric realisation of group extension by the Seifert construction, *Contemporary Math.*, **333** (1984), 353–411.

DEPARTMENT OF MATHEMATICS
THE UNIVERSITY OF GDAŃSK
80-952 GDAŃSK, WITA STWOSZA 57
POLAND