

# Impact of neutrals on the plasma screening length

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In the classical treatment the screening phenomenon of electric fields in a plasma is solely caused by charged particles, i.e. electrons and ions. In contrast, the present consideration focuses on the role of neutrals in a situation when the correlations between the charged and neutral components of the plasma medium turn rather significant. The consideration is entirely based on the renormalization procedure for interparticle interactions, which takes into account collective events in the generalized Poisson–Boltzmann equation relating the true microscopic potentials with their effective macroscopic counterparts. A meaningful approach is proposed to analytically derive the screening length from an appropriate assumption on the asymptotic behaviour of the macroscopic potential at large interparticle separations. It is clearly demonstrated that the neutral component really affects the screening length when the plasma reaches states corresponding to warm dense matter conditions. It is also shown that, at certain critical values of the plasma parameters, the character of the screening changes from exponential to oscillatory decay.

**Key words:** plasma properties

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## 1. Introduction

In the spotlight of the following is a comprehensive examination of the screening process, occurring when an external test charge is embedded into a plasma medium. Because of the electrostatic forces, electrons and ions become rearranged in a way to weaken the external field of the electric charge, which eventually leads to a phenomenon called screening. Usually, the screening is studied by introducing the collective potential, which only takes into account the redistribution of charged plasma particles according to the Boltzmann law (Murillo 2004). Subsequent linearization of the thus obtained Poisson–Boltzmann equation allows one to rigorously derive the characteristic screening length known as the Debye screening radius. Further analysis of the solution proves that the expansion in the collective potential is valid at rather large distances from the electric field source, as compared with the Debye screening length (Li & Zhang 2003).

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Meanwhile, in various situations of general physical interest, there is an urgent need for careful handling of the emerging nonlinearities, which are known to manifest themselves in the capture of charged particles of the medium by the electric field of the test charge. In this case, an accurate analysis is typically implemented within the set of the Poisson and Vlasov equations (Krasovsky 2017), assuming that the screening is only due to those plasma particles which are capable of attaining the surface of the test charge (Tsytovich & Gusein-zade 2014). It is highly expected that such a nonlinear screening has a significant impact on the formation of ordered structures (Tsytovich & Gusein-zade 2013) and the physical characteristics of such strongly coupled systems as dusty plasmas (Semenov, Khrapak & Thomas 2015; Pandey & Vladimirov 2016; Martynova, Iosilevskiy & Shagayda 2018).

At present, the screening phenomenon is of extreme importance for explaining various situations that arise during the experimental production of warm dense matter (WDM), when specifically designed targets are irradiated with powerful laser and heavy ion beams (Falk *et al.* 2014; Zastra *et al.* 2014), as is the case, for instance, in inertial confinement fusion research (Nora *et al.* 2015; Betti & Hurricane 2016). At such compressed plasma states, both quantum and correlation effects play an important role (Moldabekov *et al.* 2015), so that the description of interionic interactions is conventionally carried out within the framework of the density-response theory with an appropriately defined dielectric function, in which the screening is exclusively due to surrounding electrons. Nevertheless, for the time being, the density functional theory (Stanton & Murillo 2015) seems to be more instructive for it provides a more universal description of the field screening process and enables detection of inconsistencies in such methods as quantum hydrodynamics (Vladimirov 2011). On the other hand, WDM is usually found in a state of partial ionization when neutrals, *viz.* atoms and molecules, must be somehow incorporated into the scrutiny. This is especially true for astrophysical applications such as the interiors of giant planets, white and brown dwarfs (Chabrier *et al.* 2000; Knudson *et al.* 2012; Soubiran *et al.* 2017) as well as for shock-compression experiments (Hamel *et al.* 2012; Guarguaglini *et al.* 2021). Despite recent progress in experimentation and computer simulation, WDM still poses significant problems from the viewpoint of theoretical understanding since it is a rather ambitious goal to simultaneously cope with a whole range of phenomena as dissimilar as quantum effects, well pronounced interparticle correlations, partial ionization, electron degeneracy, etc.

It should be clearly stressed that in the majority of current investigations dealing with the screening phenomena, the presence of the neutral plasma component is completely neglected, whereby treating it as a kind of inert uniform background. The present consideration is in a position to give an answer to the question of how and under what circumstances neutrals do really participate in the screening. Indeed, it is intuitively perceived that, if the charged and neutral components of the plasma are sufficiently correlated, mutual separation of electrons and ions in an external electric field inflicts collateral regrouping of neutrals, which then return their effect on the distribution of the charged component itself. In other words, charged and neutral plasma particles can no longer be separately treated and, as a result, the screening length should eventually become a more complicated function of the plasma parameters to embrace the physical characteristics of the neutral component.

It is also worthwhile mentioning that only a thermal plasma is systematically considered below, which can therefore be described in terms of the local thermodynamic equilibrium, so that the temperatures of the medium components, *i.e.* electrons, ions and neutrals, are assumed to all be equal. Typically, thermal plasmas are generated at a rather high temperature, that being the main source of ionization, which is typical for such physical

objects as astrophysical and thermonuclear plasmas, plasmas in ion thrusters and WDM experiments as well as working bodies in combustion and induction chambers. On the contrary, in various types of gas discharges, an external electromagnetic field is conventionally applied to ionize the medium, thereby causing the electron temperature to significantly exceed the temperature of heavier plasma components, and it then becomes obvious that, in this case, the proposed approach will require further development.

The sketch of this paper is concisely outlined as follows. Section 2 takes care of the general formalism for evaluating the screening length, which essentially stems from the previously developed generalized Poisson–Boltzmann equation (Arkhipov *et al.* 1999; Arkhipov, Baimbetov & Davletov 2003). Section 3 is devoted to the determination of the screening lengths in various types of plasmas to lucidly reveal the influence of quantum effects and partial ionization. Main conclusions are briefly compiled in § 4.

## 2. General formalism

In order to analytically approach the screening phenomena in various kinds of plasmas, the generalized Poisson–Boltzmann equation is exploited, which couples together the true microscopic interaction potential  $\varphi_{ab}(\mathbf{r}_i^a, \mathbf{r}_j^b)$  and its macroscopic analogue  $\Phi_{ab}(\mathbf{r}_i^a, \mathbf{r}_j^b)$ , accounting for the collective events in the medium, as follows (Arkhipov *et al.* 1999, 2003):

$$\Delta_i \Phi_{ab}(\mathbf{r}_i^a, \mathbf{r}_j^b) = \Delta_i \varphi_{ab}(\mathbf{r}_i^a, \mathbf{r}_j^b) - \beta \sum_c n_c \int \Delta_i \varphi_{ac}(\mathbf{r}_i^a, \mathbf{r}_k^c) \Phi_{cb}(\mathbf{r}_j^b, \mathbf{r}_k^c) d\mathbf{r}_k^c. \quad (2.1)$$

Within this context the radius vector of the  $i$ th particle of species  $a$  is denoted as  $\mathbf{r}_i^a$  with  $\Delta_i$  being the corresponding Laplace operator,  $n_c$  stands for the number density of particle sort  $c$  and  $\beta = (k_B T)^{-1}$  designates the inverse temperature in energy units, with  $k_B$  being the Boltzmann constant.

The generalized Poisson–Boltzmann equation in the form of (2.1) is obtained on the basis of the renormalization procedure for the interaction of two selected plasma particles in the presence of a third one (Arkhipov, Baimbetov & Davletov 2011). In particular, the total macroscopic interaction force between the two particles is represented as a sum of their direct microscopic interaction and the positionally averaged interaction with the third particle, whose spatial distribution of the probability density is given by the Boltzmann law. It should be noted that the central equation (2.1) can still be rigorously derived from the Bogolyubov chain of equations for equilibrium distribution functions in the pair correlation approximation (Arkhipov *et al.* 2011) and turns into the ordinary Poisson–Boltzmann differential equation for the purely Coulomb interaction potential between plasma particles (Ecker 1972).

It is appropriate at this juncture to make a few remarks concerning the range of validity of the generalized Poisson–Boltzmann equation (2.1) and its subsequent solution. First of all, the linearization in  $\exp(-\Phi_{ab}(\mathbf{r}_i^a, \mathbf{r}_j^b)/k_B T)$  was actually used at the derivation, which formally restricts the present model to  $|\Phi_{ab}(\mathbf{r}_i^a, \mathbf{r}_j^b)/k_B T| \ll 1$ , i.e. to rather large distances between particles when  $\Phi_{ab}(\mathbf{r}_i^a, \mathbf{r}_j^b) \rightarrow 0$ . In reality, the solution of the generalized Poisson–Boltzmann equation retains its robustness in a much broader range of interparticle distances beyond the smallness of  $|\Phi_{ab}(\mathbf{r}_i^a, \mathbf{r}_j^b)/k_B T|$ , which is physically reasoned as follows. The first term on the right-hand side of (2.1) ensures that at rather short distances the macroscopic potential  $\Phi_{ab}(\mathbf{r}_i^a, \mathbf{r}_j^b)$  virtually coincides with the microscopic potential  $\varphi_{ab}(\mathbf{r}_i^a, \mathbf{r}_j^b)$ , whereas the second term firmly guarantees the screening at long interparticle separations  $|\mathbf{r}_i^a - \mathbf{r}_j^b| \rightarrow \infty$ . Therefore, the exact solution of (2.1) is practically the interpolation between the two correct asymptotic behaviours of the macroscopic potential

$\Phi_{ab}(\mathbf{r}_i^a, \mathbf{r}_j^b)$ , which effectively extends its range of validity far beyond satisfaction of the inequality  $|\Phi_{ab}(\mathbf{r}_i^a, \mathbf{r}_j^b)/k_B T| \ll 1$  and across the coupling regimes. The situation here can be forthright linked to the Yukawa (Debye–Hückel) potential, which also invokes the linearization procedure and whose viability was definitely demonstrated to span over the entire distance range and system coupling (Murillo 2004), thereby fully justifying the tremendous success of the Yukawa potential in describing various properties of strongly coupled dusty plasmas. Note, however, that the screening phenomena are only targeted in the subsequent consideration when the macroscopic potential vanishes exponentially with distance, thereby strictly obeying  $|\Phi_{ab}(\mathbf{r}_i^a, \mathbf{r}_j^b)/k_B T| \ll 1$ .

It is straightforward to prove that in the Fourier space the set of governing equations (2.1) is conveniently rewritten in a linear algebraic form, such that the corresponding solution for the Fourier transform of the macroscopic potential  $\tilde{\Phi}_{ab}(k)$  is explicitly expressed via the Fourier transform of the microscopic potential  $\tilde{\varphi}_{ab}(k)$  in the following tensor form:

$$\tilde{\Phi}_{ab}(k) = \sum_c \tilde{\varphi}_{ac}(k) \varepsilon_{cb}^{-1}(k), \tag{2.2}$$

where

$$\varepsilon_{ab}(k) = \delta_{ab} + \beta n_a \tilde{\varphi}_{ab}(k), \tag{2.3}$$

with  $\delta_{ab}$  signifying the Kronecker delta.

Note that the screening tensor  $\varepsilon_{ab}(k)$  is strictly specified in the space of particle species and can be cast in the matrix form as

$$\varepsilon_{ab}(k) = \begin{pmatrix} 1 + \beta n_a \tilde{\varphi}_{aa}(k) & \beta n_a \tilde{\varphi}_{ab}(k) & \dots & \beta n_a \tilde{\varphi}_{ac}(k) \\ \beta n_b \tilde{\varphi}_{ba}(k) & 1 + \beta n_b \tilde{\varphi}_{bb}(k) & \dots & \beta n_b \tilde{\varphi}_{bc}(k) \\ \vdots & \vdots & \ddots & \vdots \\ \beta n_c \tilde{\varphi}_{ca}(k) & \beta n_c \tilde{\varphi}_{cb}(k) & \dots & 1 + \beta n_c \tilde{\varphi}_{cc}(k) \end{pmatrix}. \tag{2.4}$$

Without any further loss of generality, we consider a partially ionized plasma consisting of electrons, singly charged ions and neutrals, denoted in what follows by the subscripts  $e, i$  and  $n$ , respectively. In addition, to study the screening phenomenon in its pure form, a widely accepted setting is applied when two external point-like charges  $z_1 e$  and  $z_2 e$  are placed into the plasma medium with  $e$  being the elementary charge and  $z_i, i = \overline{1, 2}$  standing for their charge numbers. Therefore, the main purpose is to figure out the influence of the plasma environment on the interaction between these external charges, whose microscopic potentials obey the Coulomb law to have the following Fourier transforms:

$$\tilde{\varphi}_{z_i z_j}(k) = z_i z_j \tilde{\varphi}(k) = z_i z_j \frac{4\pi e^2}{k^2}, \quad i, j = \overline{1, 2}. \tag{2.5}$$

To avoid unnecessary complications the same regulation is imposed on the interaction of the external charges with electrons and ions of the plasma medium such that the corresponding Fourier transforms of the microscopic potentials read as

$$\tilde{\varphi}_{z_i z_i}(k) = -\tilde{\varphi}_{e z_i}(k) = z_i \tilde{\varphi}(k), \quad i = \overline{1, 2}. \tag{2.6}$$

As for neutrals in the system, their interaction with the external test charges is completely omitted for the sake of simplicity, while the interaction of neutral plasma particles with electrons and ions is extremely essential for the whole consideration and its

effect on the screened interaction of the external charges is one of the key issues addressed hereinafter.

Although the external charges are indeed solitary, they are nevertheless dealt with as individual medium constituents with the number densities being equal to zero, such that the solution (2.2) and (2.3) for the Fourier transform of the interaction potentials of the external particles is ultimately represented as

$$\tilde{\Phi}_{z_i z_j}(k) = z_i z_j \tilde{\Phi}(k), \quad (2.7)$$

with

$$\begin{aligned} \tilde{\Phi}(k) = & \tilde{\varphi}(k) - \frac{\beta \tilde{\varphi}^2(k)}{\Delta(k)} \sum_a n_a (1 - \delta_{an}) \left( 1 - \beta \sum_{b, b \neq a} n_b [\tilde{\varphi}_{bb}(k) + \tilde{\varphi}_{ab}(k)(1 - \delta_{bn})] \right. \\ & - \beta^2 \sum_{c, c \neq b} n_b n_c (1 - \delta_{ab})(1 - \delta_{ac})(1 - \delta_{bn}) \delta_{cn} \\ & \left. \times [\tilde{\varphi}_{ac}(k) \tilde{\varphi}_{bc}(k) + \tilde{\varphi}_{ab}(k) \tilde{\varphi}_{cc}(k)] \right), \end{aligned} \quad (2.8)$$

and

$$\begin{aligned} \Delta(k) = & 1 + \frac{\beta}{1!} \sum_a n_a \tilde{\varphi}_{aa}(k) + \frac{\beta^2}{2!} \sum_{a, b} n_a n_b [\tilde{\varphi}_{aa}(k) \tilde{\varphi}_{bb}(k) - \tilde{\varphi}_{ab}(k)^2] \\ & + \frac{\beta^3}{3!} \sum_{a, b, c} n_a n_b n_c [2\tilde{\varphi}_{ab}(k) \tilde{\varphi}_{bc}(k) \tilde{\varphi}_{ac}(k) + \tilde{\varphi}_{aa}(k) \tilde{\varphi}_{bb}(k) \tilde{\varphi}_{cc}(k) \\ & - 3\tilde{\varphi}_{aa}(k) \tilde{\varphi}_{bc}(k)^2]. \end{aligned} \quad (2.9)$$

Note that the summations in the above formulas are implied over electrons, ions and neutral particles of the plasma medium. Hence, it is readily concluded that the interaction between the external charges is, to some extent, influenced by the neutral plasma component, since expressions (2.7)–(2.9) explicitly contain the number density of neutrals together with the corresponding microscopic interaction potentials. Moreover, it is the present investigation that is utterly aimed at elucidating the physical conditions under which the neutral component of the plasma medium exerts a noticeable effect on the interaction between the two external charges.

In particular, a close attention is paid to how the screening length  $r_D$  is modified both qualitatively and quantitatively due to the presence of neutrals, which has been hitherto abandoned in the plasma theory. Thus, it is natural to assume that, in the ordinary configuration space, the interaction potential  $\Phi(r)$  between the external charges obeys the following asymptotic behaviour at large distances:

$$\Phi(r) \propto \frac{1}{r} \exp\left(-\frac{r}{r_D}\right), \quad r \rightarrow \infty, \quad (2.10)$$

which, in turn, requires that its Fourier transform  $\tilde{\Phi}(k)$  in (2.8) complies with the corresponding asymptotics at small wavenumbers

$$\tilde{\Phi}(k) \propto \frac{4\pi}{k^2 + r_D^{-2}}, \quad k \rightarrow 0. \quad (2.11)$$

Consequently, the primary idea behind the whole proposed technique is to analyse the asymptotics in formula (2.8), which, in principle, should ultimately yield the screening length as a function of all plasma parameters. However, as is to be demonstrated below, for a certain domain of plasma parameters the squared screening length can acquire negative values and, formally rewriting equation (2.11) as  $\tilde{\Phi}(k) \propto 4\pi/(k^2 - r_D^{-2})$  at  $k \rightarrow 0$ , we unambiguously conclude that at large interparticle distances the following asymptotics holds:

$$\Phi(r) \propto \frac{1}{r} \cos\left(\frac{r}{r_D}\right), \quad r \rightarrow \infty, \quad (2.12)$$

i.e. the oscillatory decay of the interaction potential should be observed. Note that a non-monotonic behaviour of the interaction potential was heretofore reported in the literature for quantum plasmas (Shukla & Eliasson 2012), which is to be coped with in § 3.2. Strictly speaking, oscillatory screening behaviour (2.12) is also known for gravitational plasmas (stars interacting collectively in a galaxy), where the screening radius goes under the name of the Jeans length (Bertin 1999; Griv, Gedalin & Yuan 2006). This analogy can be extended even further because, in order to physically justify the non-monotonic behaviour of the interaction potential, additional phenomena must be incorporated into the analysis that compete the screened electrostatic forces.

### 3. Characteristic screening lengths

This section is purely designed for studying the screening phenomena in the interaction between the external charges immersed in various types of plasmas by taking the limit  $\lim_{k \rightarrow 0} \tilde{\Phi}(k)$ , which, as is convincingly seen from asymptotics (2.11), provides an extraordinary opportunity to instantly obtain an analytical expression for the characteristic screening length. In so doing, the first and foremost proposition is literally adopted that the interaction between the external charges is well described by the universal Fourier transform in expressions (2.7)–(2.9), whereas all possible physical effects that may play a role for a given specific medium are thoroughly mimicked by the corresponding microscopic interaction potentials of plasma particles. Specifically, an interest of the following lays in unveiling the impact the neutral particles have on the screening phenomena in general and the screening length in particular.

#### 3.1. Fully ionized classical plasmas

First of all, we consider as a test case an obvious example of a classical fully ionized plasma, in which the microscopic interactions between electrons and ions are delineated by the Coulomb law, so that the corresponding Fourier transforms are found as

$$\tilde{\varphi}_{ee}(k) = -\tilde{\varphi}_{ei}(k) = -\tilde{\varphi}_{ii}(k) = \frac{4\pi e^2}{k^2}. \quad (3.1)$$

Taking the above mentioned limit in (2.7)–(2.9), the classical expression for the squared Debye radius is ultimately recovered, which is totally determined by the electrons and ions of the plasma medium as

$$r_{D0}^2 = \frac{k_B T}{4\pi(n_e + n_i)e^2}. \quad (3.2)$$

Note that, in this special case only, the well-known Debye–Hückel potential is exactly retrieved for the interaction between the external charges after taking the proper backward Fourier transform.

It is learnt from formula (3.2) that the square of the screening radius in a purely Coulomb plasma is always positive and due to the clouds of positively charged ions and negatively charged electrons formed as a response to the potential of an external charge. Indeed, if a positive external charge is embedded into a plasma, it starts to repel ions, forming a depleted ion cloud and, at the same time, to attract electrons forming a thickened electron cloud in the surrounding plasma environment. For the following, it is very important to figure out how various phenomena affect those screening clouds, that is why all the screening lengths  $r_D$  are expressed below in terms of the classical Debye radius  $r_{D0}$  in (3.2). The primary logic to be used in the following is that the sharper the separation of these ion and electron clouds, the smaller the screening length since, for example, if the plasma charge separation is completely absent, there is no screening at all with the screening length being equal to infinity.

### 3.2. Semiclassical plasmas

The second meaningful example, important from the theoretical point of view, corresponds to the so-called semiclassical plasma of electrons and ions, in which quantum effects are classically treated. It is rather typical that the microscopic interaction potentials, involving both the electrons and ions, remain finite at the origin and their Fourier transforms are merely derived as (Arkhipov *et al.* 1999; Minoo, Gombert & Deutsch 1981)

$$\tilde{\varphi}_{ee}(k) = \frac{4\pi e^2}{k^2(1 + k^2\lambda_{ee}^2)}, \quad \tilde{\varphi}_{ei}(k) = -\frac{4\pi e^2}{k^2(1 + k^2\lambda_{ei}^2)}, \quad \tilde{\varphi}_{ii}(k) = \frac{4\pi e^2}{k^2(1 + k^2\lambda_{ii}^2)}, \quad (3.3a-c)$$

where  $\lambda_{ab} = \hbar/(2\pi\mu_{ab}k_B T)^{1/2}$  denotes the thermal de Broglie wavelength,  $\hbar$  symbolizes the Planck constant and  $\mu_{ab} = m_a m_b / (m_a + m_b)$  stands for the reduced mass of interacting particles with masses  $m_a$  and  $m_b$ , respectively.

Using the above described procedure, the screening length sought is finally deduced in the following form:

$$\frac{r_D^2}{r_{D0}^2} = 1 - \frac{4\pi e^2}{k_B T} (n_e \lambda_{ee}^2 + n_i \lambda_{ii}^2) - \frac{16\pi^2 e^4 n_e n_i}{k_B^2 T^2} (\lambda_{ei}^4 - \lambda_{ee}^2 \lambda_{ii}^2). \quad (3.4)$$

To strictly prove the validity of the asymptotic behaviour (2.10) figure 1 is deliberately drawn to show the linearized part  $\ln(R\Phi(R)/\Gamma k_B T)$  of the interaction potential between the external charges in a plasma as a function of the dimensionless distance  $R = r/a$  with the following notation being used: the number densities of electrons and ions  $n_e = n_i = n$ ; the Wigner–Seitz radius  $a = (4\pi n/3)^{-1/3}$ ; the coupling parameter  $\Gamma = e^2/ak_B T$ ; the density parameter  $r_s = a/a_B$ , with  $a_B = \hbar^2/m_e e^2$  being the first Bohr radius. Straight lines are evidently observed to confirm the exponential screening of the interaction potential at large interparticle separations and, by finding the relevant slopes, formula (3.4) has been numerically checked to quite accurately describe the screening length, which is evidently seen to fall off with an increase in the coupling parameter. Moreover, as the plasma density rises, the screening length steadily diminishes such that it can even turn negative for a certain range of plasma parameters. According to (3.4) such a situation turns possible when the electron–electron thermal de Broglie wavelength becomes of the order of the classical Debye radius. In virtue of (2.12) this eventually results in a non-monotonic character of the potential dependence against distance, which is clearly depicted in figure 2 for the hydrogen plasma. Note that, in contrast to Arkhipov *et al.* (1999), the interaction potential  $\Phi(r)$  preserves its Coulomb-like nature at short distances and, at the same time, remains screened at rather large interparticle separations.

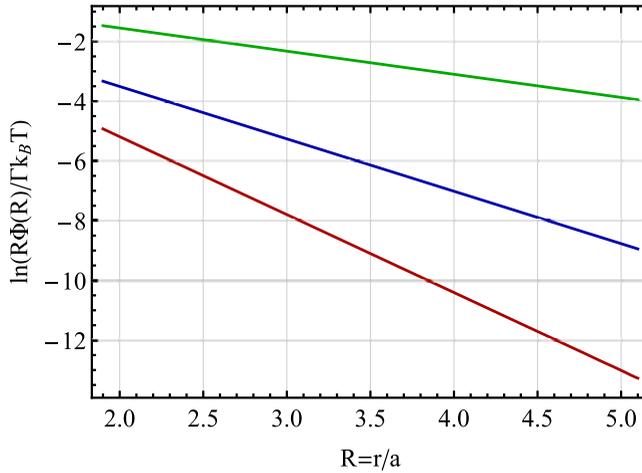


FIGURE 1. The asymptotic behaviour of the linearized part  $\ln(R\Phi(R)/\Gamma k_B T)$  of the macroscopic potential of external charges as a function of the dimensionless distance  $R = r/a$  in a semiclassical hydrogen plasma at  $r_s = 10$ . Green line:  $\Gamma = 0.1$ ; blue line:  $\Gamma = 0.5$ ; red line:  $\Gamma = 1.0$ .

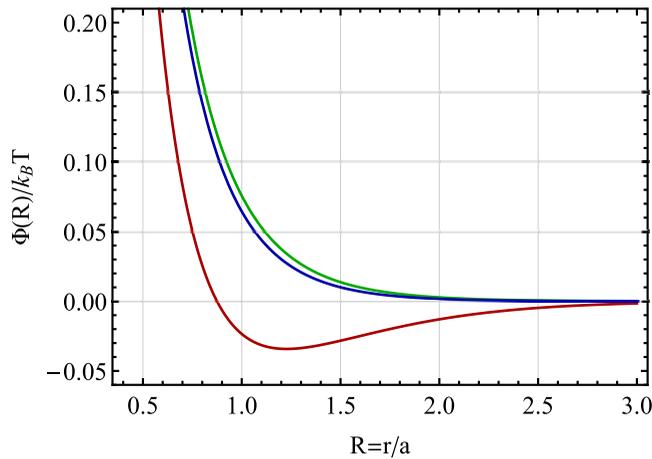


FIGURE 2. The macroscopic potential  $\Phi(R)/k_B T$  of external charges as a function of the dimensionless distance  $R = r/a$  in a semiclassical hydrogen plasma at  $\Gamma = 1.0$ . Green line:  $r_s = 10$ ; blue line:  $r_s = 5$ ; red line:  $r_s = 1$ .

The screening length (3.4) in a semiclassical plasma, denoted as  $r_D$ , is always less in magnitude than the classical Debye radius  $r_{D0}$  in (3.2) and a quick glance at its structure unambiguously reveals that the difference from the classical plasma in the second term is separately caused by changes in the states of electron and ion screening clouds, while the third term appears due to the interaction of these clouds with each other. Indeed, in order to produce a clear-cut physical reason behind the decrease in the screening radius, let us imagine a positive external charge being placed into the plasma medium, which results in the formation of electron and ion screening clouds. As in the case of classical plasmas, the cloud electrons, being attracted by the external positive charge, are mutually repelled, but the quantum effects in their interaction are responsible for an effective attraction, which

leads to the thickening of the electron screening cloud. The opposite undoubtedly happens to the ion screening cloud, which is further depleted in the vicinity of the external charge because the quantum effects in the interionic forces are also reciprocally attractive and, therefore, the cloud ions are much more easily pushed away from the positive external charge. Thus, a sharper plasma charge separation develops in a semiclassical plasma, which is finally disclosed by the second term in formula (3.4) as a decrease in the screening length, additively proportional to the number densities of electrons and ions. In contrast, the last correction in formula (3.4) is proportional to the product of the number densities of electrons and ions and is, therefore, due to the joint attraction of the screening clouds, which is also weakened by the quantum effects, thereby reinforcing cloud separation and producing a negative contribution to the screening length. Note that the whole situation changes dramatically when the scales of the screening phenomena and of the quantum effects become comparable such that the effective quantum interactions can virtually overcome the screened electrostatic forces at certain interparticle distances, which ultimately manifests itself in the oscillatory behaviour of the collective interaction potential, already demonstrated above in figure 2.

It has to be admitted that, when it comes to the WDM, the semiclassical approach, based on the interaction potentials (3.3a–c) only, is subject to known physical limitations, which are due to many-body effects in the Slater sum (Jones & Murillo 2007). Note that, herein, we only study the linear screening process in the pairwise macroscopic potential at large interparticle distances, at which such density effects simply disappear.

### 3.3. Partially ionized plasmas

It is now rather timely to proceed with one of the main topics of the present consideration, which is to establish a possible influence of the neutral component on the screening phenomena in partially ionized plasmas. For definiteness, a hydrogen plasma is systematically examined below, which consists of free electrons and protons together with an admixture of hydrogen atoms.

To begin with, we assume that electrons and protons are both classical and the Fourier transforms of their microscopic potentials are again written as represented by expression (3.1). The microscopic potentials, involving neutral hydrogen atoms, are chosen in the plain form of Mott & Massey (1987) that embodies short-range (hard-core) interaction to admit the following Fourier transforms:

$$\tilde{\varphi}_{in}(k) = -\tilde{\varphi}_{en}(k) = \frac{4\pi e^2(k^2 + 8/a_B^2)}{(k^2 + 4/a_B^2)^2}, \quad \tilde{\varphi}_{nn}(k) = \frac{4\pi e^2}{(k^2 + 2/a_B^2)}, \quad (3.5a,b)$$

such that the screening length is legitimately extracted, according to the developed general formalism, as

$$\frac{r_D^2}{r_{D0}^2} = 1 - \frac{4\pi^2 n_n (n_e + n_i) e^4 a_B^4}{k_B T (k_B T + 2\pi n_n e^2 a_B^2)}. \quad (3.6)$$

Let us closely analyse formula (3.6), which constitutes a cornerstone result of the current investigation. First of all, if the number density of neutrals  $n_n$  goes to zero, the screening length regains its classical value (3.2), as should be the case. In order to simplify the further examination, two dimensionless parameters  $x_p = a_B^2/2r_{D0}^2$  and  $x_n = a_B^2/2r_{Dn}^2$  are introduced with  $r_{Dn}^2 = k_B T/4\pi n_n e^2$ , which allow one to rewrite equation (3.6) as follows:

$$\frac{r_D^2}{r_{D0}^2} = 1 - \frac{x_n}{1 + x_n} x_p. \quad (3.7)$$

Thus, if  $x_p \ll 1$ , then  $r_D \approx r_{D0}$  since  $x_n/(1 + x_n) < 1$  for any arbitrary magnitude of  $x_n$ . It is indispensable to acknowledge from the numerical point of view that the inequality  $x_p \ll 1$  is literally fulfilled for a very wide range of plasma parameters. For instance, it certainly holds for all known dusty plasma research, thereby corroborating a rather intuitive assumption that, for an ordinary dusty plasma, the presence of neutrals can be absolutely ignored as far as the electrostatic interactions between dust particles are concerned. However, quite an opposite inference can be made for WDM conditions when the plasma density turns so high that the parameter  $x_p$  can become comparable to unity. Moreover, for neutrals to take an active part in the screening phenomena it is also obligatory, according to formula (3.7), that  $x_n \sim 1$ , which is automatically guaranteed for a WDM plasma with substantially moderate but not full ionization.

To demonstrate the significance of neutrals, we pick out a numerical example for warm dense hydrogen with the coupling and density parameters  $\Gamma = 1$  and  $r_s = 0.55$ , which are both evaluated for the total number density of protons  $n = n_i + n_n = 9.68 \cdot 10^{24} \text{ cm}^{-3}$  and the temperature  $T = 5.74 \cdot 10^5 \text{ K}$ . In this particular situation, the ionization degree is provided in Davletov, Kurbanov & Mukhametkarimov (2020) as  $\alpha = n_i/n \approx 0.96$  and formula (3.6) finally gives rise to  $r_D \approx 0.81r_{D0}$ , i.e. the screening length is decreased by approximately 19%, which can have a drastic impact on theoretical predictions of various plasma properties.

To verify the asymptotic behaviour (2.11), the linearized part  $\ln(R\Phi(R)/\Gamma k_B T)$  of the interaction potential is displayed in figure 3 for a partially ionized hydrogen as a function of the dimensionless distance  $R = r/a$  at the fixed value of the density parameter  $r_s = 10$  and different values of the coupling parameter  $\Gamma$ . Note that the ionization degree  $\alpha$  has been evaluated as in Kumar *et al.* (2021) to guarantee that the formation of hydrogen molecules is effectively prevented. An important observation is that when the number density of the neutral component reaches significant values, the screening length can again turn negative as evidenced by formula (3.6), which virtually enacts the non-monotonic character of the potential dependence as a function of distance, in complete accord with asymptotics (2.12). The last statement is showcased in figure 4 for partially ionized hydrogen plasma at the fixed value of the coupling parameter  $\Gamma = 1$  and different values of the density parameter  $r_s$  with the ionization degree calculated as in Kumar *et al.* (2021).

All in all, it follows from (3.6) that the presence of neutrals is responsible for a decrease of the screening length in comparison with the classical result (3.2), which can be perceived as above by immersing an imaginary external positive charge into the plasma medium. Once again, the electron and ion screening clouds occur that are somehow affected by neutrals whose interaction with oppositely charged plasma particles is of opposite character in the sense of repulsion and attraction as exemplified by the Fourier transforms (3.5a,b). In particular, neutrals are attracted by electrons, thereby concentrating near the electron screening cloud, and, at the same time, neutrals repel ions of the ion screening cloud. As a consequence, neutrals create their own neutral cloud in the vicinity of the external charge that, in turn, attracts the electron screening cloud and repels the ion screening cloud, which results in a much stronger plasma charge separation and, therefore, in a decrease of the screening length. Thus, the physical explanation behind the effect of neutrals on the screening length is that, because of the rearrangement of electrons and ions in the external electric field, the distribution of neutrals also turns non-uniform, which then returns its effect on the electron and ion screening clouds. A very clear sign that such an interpretation is truly valid is that the correction to the screening length (3.6) is proportional to the product of the number densities of neutrals and charged plasma particles. Again, non-monotonic behaviour of the potential only becomes possible when the scale of the charge–neutral interaction, which can be roughly approximated as the size

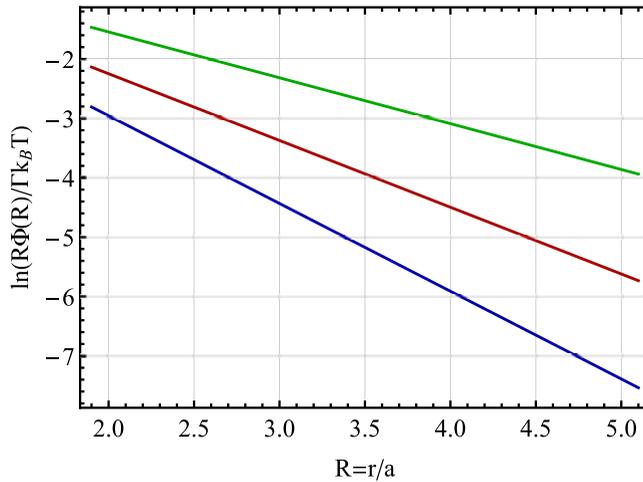


FIGURE 3. The asymptotic behaviour of the linearized part  $\ln(R\Phi(R)/\Gamma k_B T)$  of the macroscopic potential of external charges as a function of the dimensionless distance  $R = r/a$  in a partially ionized hydrogen plasma at  $r_s = 10$ . Green line:  $\Gamma = 0.1$  with  $\alpha = 0.9939$ ; blue line:  $\Gamma = 0.5$  with  $\alpha = 0.7279$ ; red line:  $\Gamma = 1.0$  with  $\alpha = 0.2106$ . The ionization degree  $\alpha$  is evaluated as in Kumar *et al.* (2021) to ensure the absence of hydrogen molecules.

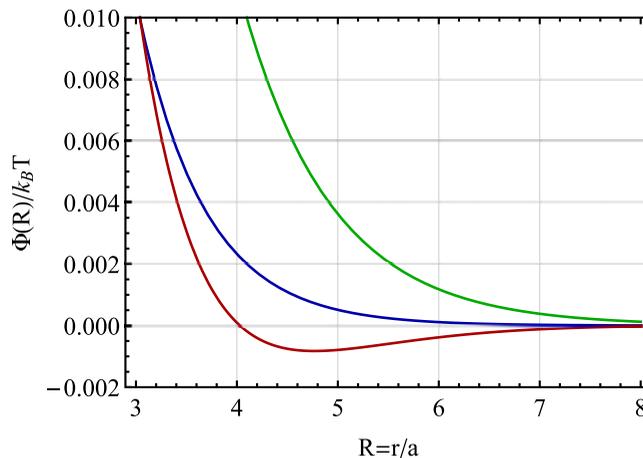


FIGURE 4. The macroscopic potential  $\Phi(R)/k_B T$  of external charges as a function of the dimensionless distance  $R = r/a$  in a partially ionized hydrogen plasma at  $\Gamma = 1.0$ . Green line:  $r_s = 10$  with  $\alpha = 0.2106$ ; blue line:  $r_s = 5$  with  $\alpha = 0.3824$ ; red line:  $r_s = 0.5$  with  $\alpha = 0.2087$ . The ionization degree  $\alpha$  is evaluated as in Kumar *et al.* (2021) to ensure the absence of hydrogen molecules.

of the neutrals, becomes comparable to the classical screening radius under the additional requirement of a rather moderate ionization degree.

### 3.4. Semiclassical partially ionized plasmas

It has been demonstrated in the previous § 3.3 that the neutral component can considerably affect the screening phenomena when the plasma somehow reaches the WDM states. Under such extreme conditions the electron thermal de Broglie wavelength turns out

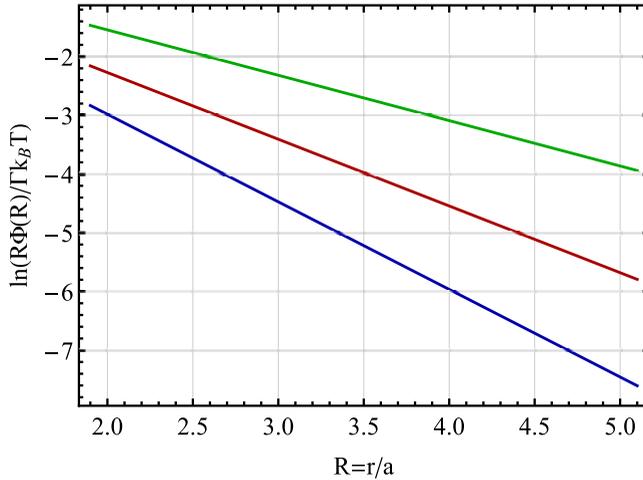


FIGURE 5. The asymptotic behaviour of the linearized part  $\ln(R\Phi(R)/\Gamma k_B T)$  of the macroscopic potential of external charges as a function of the dimensionless distance  $R = r/a$  in a semiclassical partially ionized hydrogen plasma at  $r_s = 10$ . Green line:  $\Gamma = 0.1$  with  $\alpha = 0.9939$ ; blue line:  $\Gamma = 0.5$  with  $\alpha = 0.7279$ ; red line:  $\Gamma = 1.0$  with  $\alpha = 0.2106$ . The ionization degree  $\alpha$  is evaluated as in Kumar *et al.* (2021) to ensure the absence of hydrogen molecules.

to be of the order of the mean interparticle spacing, which necessitates simultaneous accounting for the presence of neutrals and quantum effects. This can be naturally realized in the framework of the present approach by jointly applying the microscopic interaction potentials (3.3a–c) and (3.5a,b), which finally yields the following analytical expression for the screening length in a semiclassical partially ionized plasma:

$$\frac{r_D^2}{r_{D0}^2} = 1 - \frac{4\pi^2 n_n (n_e + n_i) e^4 a_B^4}{k_B T (k_B T + 2\pi n_n e^2 a_B^2)} - \frac{4\pi e^2}{k_B T} (n_e \lambda_{ee}^2 + n_i \lambda_{ii}^2) - \frac{16\pi^2 e^4 n_e n_i}{k_B^2 T^2} (\lambda_{ei}^4 - \lambda_{ee}^2 \lambda_{ii}^2). \tag{3.8}$$

It is rather curious to discover that, as evidenced by comparing formula (3.8) with expressions (3.4) and (3.6), neutrals and quantum effects contribute independently to the modification of the squared screening length, which seems to be a consequence of the linearity of the governing generalized Poisson–Boltzmann equation (2.1).

To properly confirm the robustness of relation (3.8), figure 5 is plotted to estimate the slopes in the curve of the linearized part  $\ln(R\Phi(R)/\Gamma k_B T)$  of the interaction potential of the external charges versus the dimensionless interparticle separation  $R = r/a$  at various sets of plasma parameters. At the same time, it is anticipated from formula (3.8) that the squared screening length can turn negative at some values of the plasma parameters, whose straightforward consequence is a non-monotonic behaviour of the interaction potential as a function of distance, as duly revealed by figure 6.

**4. Conclusion**

A comprehensive approach has been systematically developed to numerically estimate the influence of the neutral component on the screening length in a partially ionized plasma. To be exact, two external charges, immersed in a partially ionized plasma,

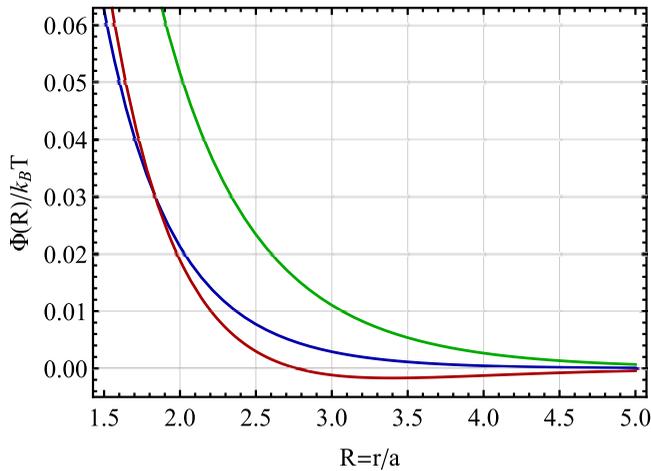


FIGURE 6. The macroscopic potential  $\Phi(R)/k_B T$  of external charges as a function of the dimensionless distance  $R = r/a$  in a semiclassical partially ionized hydrogen plasma at  $\Gamma = 1.0$ . Green line:  $r_s = 10$  with  $\alpha = 0.2106$ ; blue line:  $r_s = 5$  with  $\alpha = 0.3824$ ; red line:  $r_s = 0.5$  with  $\alpha = 0.2087$ . The ionization degree  $\alpha$  is evaluated as in Kumar *et al.* (2021) to ensure the absence of hydrogen molecules.

have been considered, and then, the renormalization procedure has been thoroughly adopted to take into account the plasma medium within the framework of the generalized Poisson–Boltzmann equation. Thus, the Fourier transform of the interaction potential of the external charges has been obtained, which explicitly depends on the characteristics of the neutral component, such as its interaction potentials and number density. Subsequent physically reasonable assumption on the asymptotic behaviour of the potential has made it possible to analytically derive the screening length for various types of plasmas.

In particular, in a classical fully ionized plasma, as expected, the screening length has been shown to strictly coincide with the well-known Debye radius, whereas the interaction potential between the external charges has been remarked to reproduce the notorious Debye–Hückel potential. It is well established within the classical picture that the screening occurs due to the formation of oppositely charged electron and ion plasma clouds developing in external electric fields. On the other hand, as the present study shows, the state of plasma polarization clouds of electrons and ions can be determined not only by mutual electrostatic interactions in the system, but also by other physical phenomena that may seriously affect the screening length. In particular, that is why all screening lengths for different types of plasmas have been expressed above in terms of the classical Debye radius to clearly identify changes in the structure of electron and ion screening clouds.

For a so-called semiclassical plasma, in which quantum effects are effectively embraced with appropriate microscopic interaction potentials, the screening length has been demonstrated to significantly depend on the squared ratio of the thermal de Broglie wavelength to the classical Debye radius. At the same time, the general analysis has undoubtedly confirmed that at a certain critical value of the plasma number density, the character of the interaction potential vanishing changes from monotonic to oscillatory decay at long interparticle distances. It has to be mentioned that such a remarkable behaviour is actually a result of the competition between the quantum effects and screening

phenomenon when the scales of their actions become comparable. Note that the decrease in the screening length is ultimately ascribed to quantum softening of mutual electrostatic interactions of plasma particles, which only reinforces electron and ion cloud detachment as compared with the classical plasma case. In particular, it has been stated that corrections to the classical Debye radius, proportional to the number densities of charged particles, appear due to a change in the state of the screening clouds themselves, whereas modifications, proportional to the product of number densities, are a consequence of a change in the state of interaction of the corresponding clouds with each other.

As for a partially ionized plasma, the effect of the neutral component on the real screening length has been shown to be essential under WDM conditions, when the hard-core size of neutrals becomes comparable to the classical Debye radius. Moreover, at very high densities of the plasma medium, the appearance of local maxima and minima in the curve of the interaction potential has been detected as a function of distance. In order to reach a reasonable explanation as to why neutral particles may have some effect on the screening length, consider the following. It is well understood that when an external test charge is placed into a plasma, a separation of negatively and positively charged particles arises, i.e. clouds of electrons and ions are formed and these are responsible for the appearance of the screening phenomena. The electric field of the test charge has no direct influence on the neutrals, which nevertheless may strongly interact with the electrons and ions of the polarization clouds, thereby straightforwardly causing a non-uniform distribution of neutrals, i.e. a cloud of neutrals develops as well. It is then this cloud of neutrals that turns back its impact on the electron and ion screening clouds to manifest itself in the screening length decrease, which means that the true explanation is attributed to fairly strong correlations between the charged and neutral plasma components. Note that such an interpretation is in full agreement with the above described picture of how various effects exert influence on the screening length, since the contribution of neutrals is proportional to the product of their number density and the total concentration of charged plasma particles.

For a range of WDM conditions simultaneous handling of the quantum effects and the presence of the neutral component has been proved to be a key issue, and it has been clearly justified that they contribute independently to the correction of the classical expression for the squared screening length. Moreover, the modification of the screening length due to neutrals has been numerically revealed to be as noticeable as that due to quantum effects at rather high plasma densities.

It should be especially emphasized that such a plasma parameter as the screening length is vastly important for the theoretical description of interionic interaction in a WDM state (Vorberger & Gericke 2013; Lv *et al.* 2021), since it directly determines the static structure factor measured experimentally by the amplitude of elastic scattering and absorption of X-ray radiation in the near-surface plasma layers. The static structure factor, in turn, can seriously alter such characteristics as the electrical conductivity (Arkhipov *et al.* 2002), and can even be used to restore its dynamic counterpart by the method of moments (Arkhipov *et al.* 2007).

It must also be borne in mind that while treating the neutral component we have completely omitted the individual polarization of neutral particles in external electric fields, only hard-core effects have been neatly treated for appropriate plasma particle interactions. Nevertheless, it is important from a practical point of view to study how and at what distances the screening of the external electric field by charged plasma particles is actually replaced by the neutral component response governed by a dielectric constant as in standard dielectric media, which is a provision for future improvements to the present research.

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## Declaration of interest

The authors report no conflict of interest.

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## Data availability statement

Relevant data which aid in reproducing the findings of this work are available upon request.

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