RADIAL PULSATION ANALYSES OF DA DWARFS*

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I. INTRODUCTION

There are now twelve ZZ Ceti stars known and their properties are summarized by Nather (1978) and McGraw (1977, 1979). They have normal DA spectra and their colors range from a B-V of 0.18 mag to 0.22 mag. This places them on the HR diagram in a region which is an extension to the white dwarfs of the Cepheid instability strip. This would suggest that the cause of their pulsation could be analogous to the κ and γ mechanisms of the Cepheids except for the fact that the periods of the ZZ Ceti stars range from 200 to 1200 seconds, three orders of magnitude larger than the radial pulsation periods for white dwarfs.

Previous analyses of white dwarfs have shown no instabilities in the radial modes (c.f. Dziembowski 1977). While Dziembowski (1977) did find some evidence for an instability in nonradial modes, his periods were much too short for the ZZ Ceti variables and the particular modes that have been identified with some ZZ Ceti stars (the low order g-modes) were stable.

Therefore, we felt that it was necessary to study realistic white dwarf envelopes as carefully as possible with the new Los Alamos opacities and try to identify any destabilizing mechanisms present in the models with a future application to a non-radial study. Our initial results with the Los Alamos extension of Castor's (1971) linear nonadiabatic computer program (hereafter, LNA) showed that the high order radial overtones of DA dwarfs were unstable (Cox, Hodson, and Starrfield 1979). We have now extended that work to various chemical compositions and slightly more realistic radii and report on that work in this paper.

II. CONSTRUCTION OF THE MODELS

The observations of the ZZ Ceti variables imply that their effective temperatures range from 9×10^3 K to 1.5×10^4 K (McGraw 1979). This implies luminosities of $\sim 10^{-3}\ L_\odot$. We chose a mass of 0.6 $M_\odot$ and obtained radii from both the Hamada and Salpeter (1961) mass-radius relation for carbon white dwarfs with thin hydrogen envelopes and a white

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dwarf cooling curve computed with the Kutter and Sparks (1972, see also Starrfield, et. al. 1978) hydrodynamic stellar evolution code. The evolutionary results showed that the outer radius shrank by nearly ten percent as the dwarf cooled from 1.8×10^4 K to 8×10^3 K. The different radii that we used changed the periods and unstable modes only minimally.

One of our primary concerns was the effect of composition on the unstable modes and their periods. Therefore, we varied the helium content in the surface layers from Y = 0.00 to Y = 0.88. At temperatures of 10^6 K we switch to more helium rich mixtures and then to helium plus carbon mixtures. At densities greater than 10^4 gm cm^{-3} (T ~10^7 K) we assume a pure carbon mixture. We use the Los Alamos Opacity Library (Huebner, et. al. 1977) for many mixtures at temperatures greater than 1.2×10^4 K (1 ev); below that value we use the Stellingwerf (1975) fit to the King tables (Cox and Tabor 1976). All models had regions of convective instability and we used a standard mixing-length formulation (c.f. Cox and Giuli 1968) with $\ell/H_p = 1.0$. Comparison of our model envelopes with those of Fontaine and Van Horn (1976) showed good agreement. We treated the luminosity variation with depth by a relationship of the form: $L(r) = (1-q) L_s$ where $q = M_r/M$ and L_s is the surface luminosity of the star (Van Horn 1978, private communication). All of the envelopes were integrated to a q of 0.01.

Figure 1 shows the run of opacity, density, and temperature as a function of surface mass fraction, 1-q, for a typical model (10,000 K) in which we found an instability to radial pulsations. The small irregularities in the opacity curve are the regions of composition discontinuity where we changed tables. These discontinuities have no effect on the instabilities that we have found since they lie considerably deeper into the star than the region of any pulsation driving or damping.

An important feature of our envelopes is that there exists a significant convective region near the surface of all models with T_e less than 10^4 K. In Figure 2 we show the temperature at the bottom of the convective zone as a function of the effective temperature of the star. Y is the mass fraction of helium and the points for Y = 0.0 were obtained from Fontaine and Van Horn (1976). Our Y = 0.0 envelopes give essentially the same results. Regions at T ~ 40,000 K and 150,000 K labeled "driving" are obtained from the LNA analysis of these envelopes. Since a strongly convective region cannot be a driving region (there is no radiation to modulate the κ and α mechanisms), it will act to stabilize those models where it extends to depths considerably below the driving region. In fact, the red edge for our envelopes does occur at precisely those effective temperatures (9-10 x 10^3 K) where convection carries all the luminosity. This is an interesting result since it has been frequently proposed that very dominant convection was the cause of the red edge of the Cepheids. We have now demonstrated that it definitely causes the red edge in the ZZ Ceti variables.

III. RESULTS

Our LNA analyses of the radial modes of 0.6 $M_\odot$ white dwarf envelopes with hydrogen rich surface layers show that there is an instability strip which lies at effective temperatures between ~ 9×10^3 K and 1.4×10^3 K

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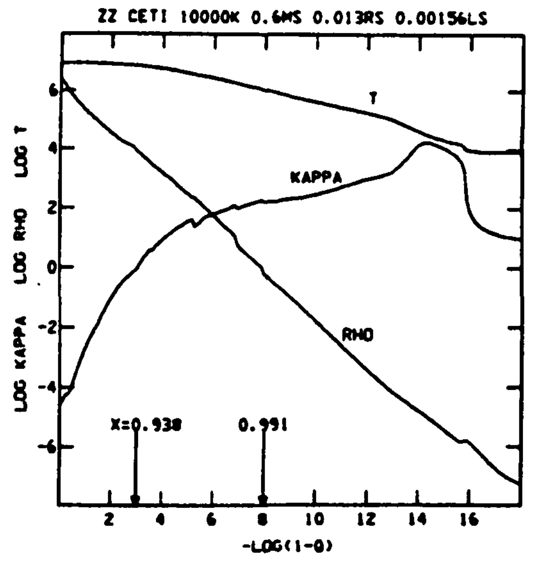

Fig. 1

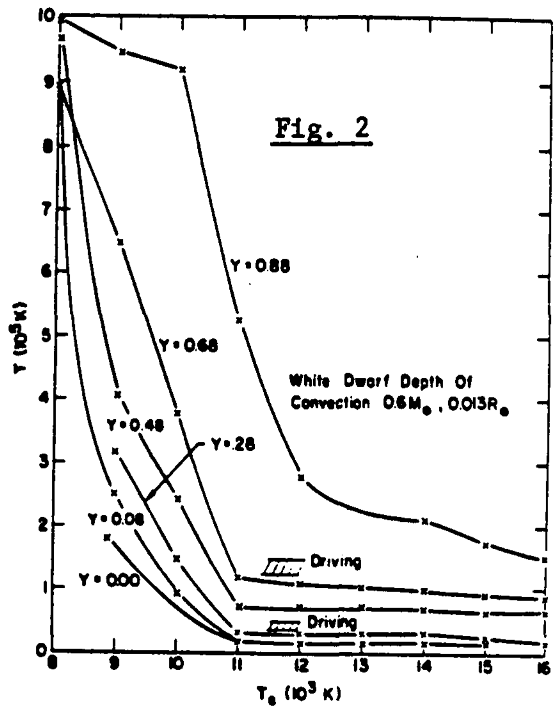

Fig. 1 - The temperature, opacity, and density as a function of surface mass fraction. Two values of x = r/R are marked with arrows.

Fig. 2 - The depth in temperature to which the surface convection zone reaches as a function of effective temperature. Each "x" marks a particular model and "Y" is the mass fraction of helium.

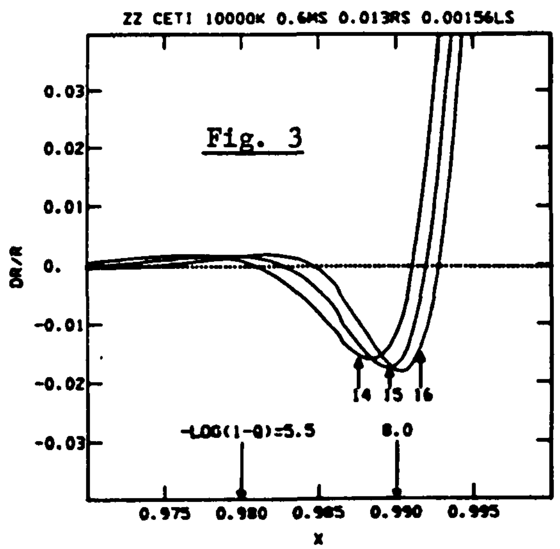

Fig. 3

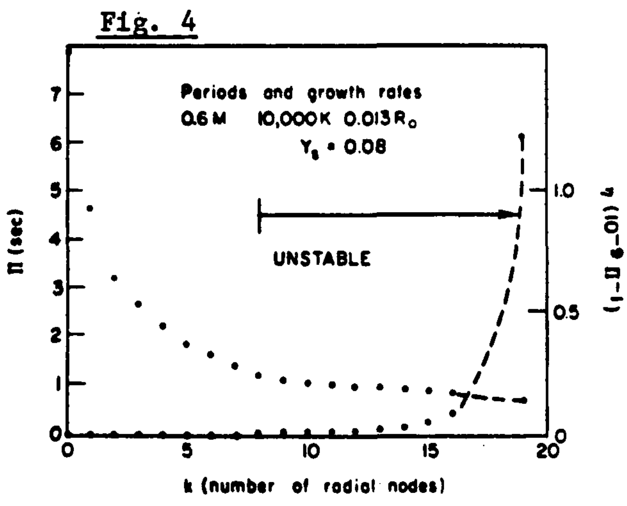

Fig. 4

Fig. 3 - The Lagrangian eigenfunction as a function of x = r/R. Two values of the surface mass fraction are noted with arrows. "14", "15", and "16" is the number of nodes for that model.

Fig. 4 - The period in seconds and growth rate (ΔKE/KE per period) as a function of overtone. k is the number of nodes.

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depending upon the composition. We find unstable modes in all envelopes lying in the strip and for all compositions (Y = 0.0 to Y = 0.88). The fact that the pure hydrogen (Z = .02) models also pulsate is extremely important and we shall discuss its significance in the next section. As is well understood for the Cepheids (Cox, King, and Tabor 1973) the blue edge is a strong function of composition. For Y = 0.08 it lies at $\sim 1.4 \times 10^4$ K, while for Y = .88 it extends past 1.6×10^4 K.

The radial eigenfunctions ($\delta r/r$) for the 13th, 14th, and 15th overtones are shown in Figure 3 as a function of radius fraction, x = r/R. As one expects for such high order overtones, the motion is largest in the surface layers and damps out quickly as one goes deeper through numerous nodes. Figure 4 shows both the period (in seconds) and growth rates (ΔKE/KE) for one model as a function of k. All of the low order overtones are stable and it is not until k equals 7 that we find an instability. Nevertheless, the growth rates remain small until k has reached 15 and then grow rapidly to a maximum of 1.25×10^{-9} per 0.5 sec. This means that the pulsation will e-fold in kinetic energy every 4×10^8 sec or 13 yr. In fact, the growth rate is still increasing at the last value of k, and it could continue to grow for even higher overtones. Our limit to k is set by the number of zones in the mesh (195) and we are currently redoing our analysis for some envelopes with 400 zones. This will allow us to study values of k up to $\sim$ 40. The integral of the work function (ΔKE/KE per period) is given in Figure 5, and it shows where both the driving (positive values) and damping (negative values) occur in the envelope as a function of zone number. The jagged appearance of these curves is caused by interpolation in a real physics table; no analytic fit is used.

The instability in these envelopes is caused by the κ and γ mechanisms. The driving in zones 145 to 155 and 170 to 180 is caused by helium ionization at temperatures of 2×10^4 K to 5×10^4 K and 10^5 K to 2×10^5 K. An analogous graph for the pure hydrogen model would show driving at temperatures $\sim 4 \times 10^4$ K. This particular envelope (T = 10^4 K) was unstable. For larger effective temperatures, we find that the driving regions are located much closer to the surface at much smaller values of the surface mass fraction. At $T_e \sim 1.5 \times 10^4$ K there is no longer enough mass in the driving region to cause an instability.

IV. SUMMARY AND DISCUSSION

The LNA radial pulsation analysis of DA dwarfs in the observed instability strip has shown that they are all unstable in their higher order overtones. However, the modes that we predict are unstable all have periods that are 2 to 3 orders of magnitude shorter than those observed in the ZZ Ceti variables. This is perplexing since even the models calculated with a nearly pure hydrogen composition (X = .98, Z = .02) have unstable modes so that even if all of the helium has had time to settle beyond the reach of a surface convection zone, the stars should still pulsate with short periods. Here we suggest a number of solutions to this dilemma.

First, it is still possible that a proper search for these periods has not yet been done. It is difficult to obtain and reduce data at the

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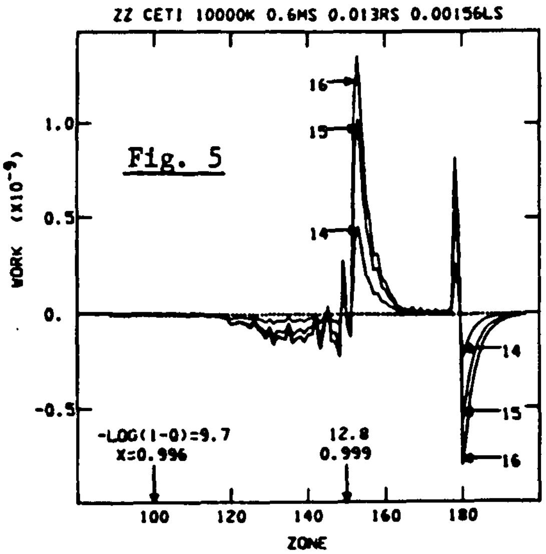

Fig. 5 - The plot of the work function over one pulsation period as a function of zone number. Two values of the surface mass fraction and the radius fraction are denoted by arrows. Note that this region occurs much closer to the surface than that plotted in Figure 3.

0.1 sec. rates necessary to discover these periods and only Richer and Ulrich (1974) have reported on data taken with integration times shorter than 1 sec. Unfortunately, their power spectrum analysis gave incorrect results (McGraw 1979, private communication). Therefore, it could be possible that these periods are there but have not been looked for with short enough integration times. Recently, a search was begun that will be able to detect periods as short as 0.2 sec. (McGraw and Starrfield 1979, in preparation). At the present time, only the results for G117-B15A ($T_e \approx 13{,}500$) have been analyzed and they have proved to be negative. However, this variable is a known ZZ Ceti star which is already pulsating at a period of 216 sec. (McGraw 1977) and it seems quite likely that once a variable star is trapped in one particular mode it cannot easily change and begin pulsating in another mode. Nonradial growth rates (Dziembowski 1979) may be larger than our radial growth rates. If this is true, then the stars in which to search for short period oscillations may be the current nonvariables in the instability strip. Another possible explanation for this discrepancy is that this star has zero helium content to below 150,000 K and is bluer than the pure hydrogen case blue edge.

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