in *pounds-weight* per square inch, or more likely in dynes cm^{-2} Many of us remember our school boy days when, after working out a problem in Dynamics, we looked up the answers to see whether we had to divide by g to make our result agree with the answer.

Yours etc., JOHN SATTERLY

To the Editor of the *Mathematical Gazette*

DEAR SIR,

Mr. Whitfield in his review of my book on "Three-Dimensional Dynamics", which appeared in Vol. XLIII, No. 346, of the Mathematical Gazette, after congratulating me on giving a correct proof of the variational principles in impulse theory (Kelvin's and Robin's Theorems) goes on to say that my statement that "Bertrand's Theorem involves no stationary property" is false. This is a statement which I think needs clarifying.

Bertrand's Theorem states that the kinetic energy of any free system when set in motion by a set of impulses is greater than that of the same system when subject to frictionless constraints and set in motion by the same impulses. If frictionless constraints can be imposed on a system, the constraints being such that they can be so continuously varied that the resulting motion differs by as little as one pleases from the actual motion of the free system, then Bertrand's Theorem can certainly be associated with a stationary property, since in the result

$$\tfrac{1}{2}\Sigma m(\mathbf{v}^2 \;\; \mathbf{v}^2) = \tfrac{1}{2}\Sigma m(\mathbf{v}_2' \;\; \mathbf{v}_2') - \tfrac{1}{2}\Sigma m(\mathbf{v}_2 - \mathbf{v}_2') \;\; (\mathbf{v}_2 - \mathbf{v}_2'),$$

where $\mathbf{v}_2$ corresponds to the free system and $\mathbf{v}_2'$ to the constrained system, we can replace $\mathbf{v}_2'$ by $\mathbf{v}_2 + \delta\mathbf{v}_2$ giving

$$\Sigma m(\mathbf{v}_2 \;\; \delta\mathbf{v}_2) = 0,$$

i.e.

$$\delta\Sigma m(\mathbf{v}_2 \;\; \mathbf{v}_2) = 0,$$

so that the actual motion corresponds to a stationary value of the kinetic energy The actual motion in this case corresponds to the constrained motion which has the maximum kinetic energy. Thus, for instance, suppose we consider a uniform rod AB of mass M and length $2a$ set in motion by an impulse J applied at A at right-angles to AB. Let the motion be defined in terms of $\mathbf{v}$, the velocity of G the centre of mass, together with $\boldsymbol{\omega}$, the angular velocity of the rod. The direction of $\mathbf{v}$ will clearly be at right-angles to AB in the direction of J Hence, taking $\boldsymbol{\omega}$ to have the appropriate direction, the equations to determine the motion are

$$Mv = J, \qquad I\omega = aJ,$$

I being the moment of inertia of the rod with respect to an axis through G perpendicular to the rod. We thus have

$$v = J/M, \quad \omega = aJ/I = 3J/Ma.$$

Now we can clearly apply a frictionless constraint to the system by fixing a point of the rod by means of a smooth pin, and the motion as

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given above can be obtained by making the kinetic energy of this constrained motion a maximum for variations in the position of the pin. The constrained motion which corresponds to the motion of the free rod is that with the pin placed at the instantaneous centre. The same result applies, of course, whatever the point at which the impulse is applied. If, however, the impulse is applied at a point distant from the centre $>a/3$, then the instantaneous centre is a point of the rod, but if the distance of the point from the centre is $<a/3$, then the instantaneous centre of the motion is a point outside the rod, which makes the nature of the constraint applied in this case somewhat unreal. To say that a stationary property can be associated with Bertrand's Theorem as a general result, then one should be able to say that, no matter what the system, one can always impose *real* frictionless constraints on the system of such a nature that they may be so varied that the resulting constrained motion differs by as little as one pleases from the motion of the free system. Clearly this is not always possible. Thus, while Bertrand's Theorem can be associated with a stationary property in particular cases, one cannot say that it can be so associated as a general principle. This is the point of the statement in my book that Bertrand's Theorem involves no stationary property "since it does not follow that frictionless constraints can be imposed on the system in such a way that the $\mathbf{v}_2$'s and the $\mathbf{v}_2''$s corresponding to the different systems differ from one another by infinitesimal amounts" I feel, however, that this statement does not in itself make the position completely clear, and I am pleased to have this opportunity, arising out of the review of my book by Mr. Whitfield and correspondence with him, to clarify more exactly the variational implications of Bertrand's Theorem.

Yours etc., C. E. EASTHOPE

To the Editor of the *Mathematical Gazette*

DEAR SIR,

QUERY:—

A common form of crossword frame consists of a square, divided into rows and columns each containing 15 small squares, some of which are blacked out to form a centrally symmetrical pattern. Assuming that

(i) No row or column is completely blacked out;
(ii) No 'word' consists of more than 13 or less than 3 letters;
(iii) At least one letter of every 'word' is shared by a 'word' in the other direction;

is it possible to determine just how many frames can be constructed?

Yours etc., B. A. SWINDEN

SAMUEL PEPYS AND JOHN WALLIS

1952. *From Samuel Pepys to Sir Godfrey Kneller, March,* 26, 1702.

I have long, with great pleasure, determined, and no less frequently declared it to my friend Dr. Charlett, upon providing as far as I could by your hand, towards immortalizing the memory of the person,* for

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