EVOLUTION OF WHITE DWARFS: STARTING FROM PLANETARY NEBULAE

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1.INTRODUCTION

The ultimate aim in the study of White Dwarf (WD) evolution is to understand properly the observed Luminosity Function (LF) of WDs, that is the number of WDs observed per unit magnitude interval. The complicated route to the interpretation of this scarne quantity (12 fiducial points in the recent update of Liebert et al. 1988) is schematically summarized in figure 1. Clearly, the main input to the LF are the evolutionary (cooling) times, but it is necessary to consider their non trivial dependence on galactic evolutionary inputs, namely the initial mass function of disk stars, their age distribution with time (ultimately: the disk age), and their evolutionary properties. Stellar evolution enters in the problem of cooling by two main routes: first, by determining the mass of the WD as a function of the inital stellar mass and chemistry, second by fixing the internal constitution of the WD remnant for each given mass, and the initial physical conditions at the start of WD evolution (mainly the temperature distribution, which is important for the first phases of evolution). Of course, there is no need of good evolutionary inputs to study "theoretical" WDs. In fact, historically, the <u>first approach</u> in the study of "cooling" (Mestel 1952, Schwarzschild 1958)) has been directly related to the stimulating physical properties of these objects, in which neutrino losses at the beginning (Vila 1966, Savedoff et al. 1969) and, in late stages, liquification and crystallization of the plasma (Brush,Sahlin and Teller 1966, Hansen 1973) long recognized to be dominated by coulomb interactions, (Kirzhnits 1960, Abrikosov 1960, Salpeter 1961), are the main features to be investigated (Mestel and Ruderman 1967, Van Horn 1968, Kovetz and Shaviv 1970).

While also the theory of heat conduction by degenerate electrons went on improving in the course of years (Marshak 1940, Mestel 1950, Hubbard and Lampe 1969, Canuto 1970, to end, recently, with Itoh et al. 1983. 1984), we may regard as a <u>second approach</u>, the stage in which it has been recognized that full consideration should be given

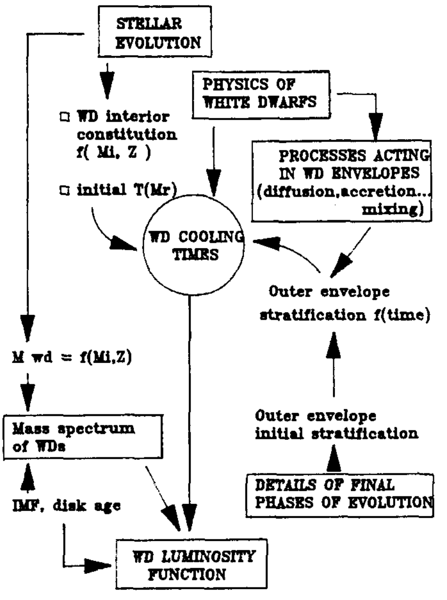

Figure 1: inputs for the determination of the luminosity function

to the envelope physics, (Bohm 1968, Van Horn 1971). The envelope is an incompletely ionized, strongly coupled, partially degenerate plasma, whose physics is much more complex than that of the interior, and this required long studies to assess properly the equation of state (Fontaine et al. 1974, Fontaine and Van Horn 1976, Fontaine, Graboske and Van Horn 1977, Magni and Mazzitelli 1979), while only one attempt has been made, as far as I am aware of, to compute opacities under these extreme conditions (Bohm et al 1977).

Together with the study of cooling properties through the central temperature - luminosity relations obtained by integration of envelope models (Koester 1972, Sweeney 1976), these years see the quantification of convection in the WD envelopes, and the study of possible transitions between hydrogen and helium dominated atmospheres by convective mixing (Baglin and Vauclair 1973, D'Antona and Mazzitelli 1974 and 1975, Vauclair and Reisse 1977) while the first insight is given to the role of additional problems like accretion of interstellar matter, gravitational settling, and radiative acceleration (Strittmatter and Wickramasinghe 1971, Koester 1976, Wesemael 1978, D'Antona and Mazzitelli 1979, Vauclair et al 1979).

In the seventies, two sets of complete models were evolved, by Lamb and Van Horn 1975 and Shaviv and Kovetz 1976. Only starting from 1984, a third approach to the study of WDs evolution begins, with a fundamental paper by Iben and Tutukov (1984): here and in the following computations (Koester and Schönberner 1986, Mazzitelli and D'Antona 1986, Iben and Mc Donald 1985 and 1986) consideration is given to the construction of the WD, following all the burning phases up to the building up of the degenerate carbon- oxygen core, and simulating mass loss by stellar wind and/or superwind until the WD phase is reached. The starting models for the WD cooling reflects the result of the previous evolution. Iben and Tutukov (1984) first found out that the "evolutionary" remnant hydrogen layer on DA WDs can appreciably contribute to the energy generation at very low luminosity by proton- proton burning.

Nevertheless, the recent results also showed a large spread in the total evolutionary time of WDs according to the different authors: in practice, we (Mazzitelli and D'Antona 1986) had obtained cooling times down to $\log L/L_\odot \sim -4.5$ of the order of 5-7 10^9 yr, others had obtained times longer than 10^{10} yr. These differences have obvious consequences on the weight to be given to the interpretation of the luminosity function of WDs in terms of disk age (D'Antona and Mazzitelli 1978, Winget et al. 1987), and must be clarified.

Winget and Van Horn (1987) have shown that most of the differences between the results of different researchers can be attributed to the different physical and chemical inputs adopted. While we are still comparing our computations in the effort to understand whether this is certain, let us take advantage of this conclusion and realize that, although on the "physics" of WDs there is today broad agreement, the uncertainty in the inputs to be used (see Mazzitelli and D'Antona 1987 for an overview, and Mazzitelli 1988a for an update), is such that the cooling times are "evolutionary" uncertain by at least a factor two!

In practice, our times were shorter mainly for two reasons: i) our models are very oxygen rich, having been obtained by adopting the new reaction rates suggested by Kettner et al. (1982) for the $^{12}C+{}^4He$, and ii) we have chosen envelopes which at low T_{eff} are helium dominated and mostly deprived of metals down to the end of the evolution, adopting very low surface opacities (Cox and Tabor 1976 for $Z=10^{-5}$). Even if 3/4 of WDs show hydrogen dominated spectra, it seems (Greenstein 1986) that most of them are mixed at low T_{eff}, leading to helium dominated atmospheres and fast cooling. Unfortunately, how much Hydrogen is left on the surface (if any) and also how large is the helium layer remnant depend on the <u>details</u> of the final phases of evolution, as it has been shown both by stellar evolution computations (Schönberner 1983, 1987, Iben 1984, Mazzitelli and D'Antona 1986, Wood and Faulkner 1986) and by the theoretical inferences bases also on the observations (Iben et al. 1983, Renzini 1987). This conclusion is however frustrating, as the final phases of pre-WD evolution, linked to the complex hydrodynamical problem of Planetary Nebula ejection, can be solved up today only parametrically, and the unknowns leave shadows on our global understanding of WD evolution. The fourth approach to the study of WDs is therefore the combination between what we may infer on the external layer composition by the predictions of stellar evolution, and what information we may extract from the observational evidences on WDs, interpreted through the knowledge of the processes acting on WD envelopes. In recent years, this has been the tentative approach of the Canadian group (e.g. Fontaine and Wesemael 1987). The ongoing discussion on the evolution of WDs is another tentative to follow this route.

I will concentrate on:

-the main phases of WD evolution;

-the outer envelope composition, its links to previous evolution, and its influence on the final fate of the WD;

-comparison of theoretical and observational luminosity functions;

-problems with building of cool WD models.

2.THE MAIN EVOLUTIONARY STAGES OF WHITE DWARFS

WD evolution can be divided into five main regimes according mainly to the stellar luminosity. Into these regimes the physical processes which play a major role are different, so as the information which we may derive from observations. These phases are illustrated mainly with the results of our models of $0.68M_\odot$ (Mazzitelli and D'Antona 1986) and of $0.564M_\odot$ (D'Antona and Mazzitelli 1989).

1st stage: $\log L/L_\odot > 0$; $\log T_{eff} > 4.7$: "mixed approach to WDs".

This phase includes the late CNO burning -if the remnant hydrogen layer is the maximum possible), the onset of diffusion of CNO elements, the main period in which neutrino losses are dominant and are counteracted by the residual gravitational contraction of the outer layers. Helium burning is never dominant in these stars, although it may have been playing a major role before.

The effect of diffusion of CNO in the interior has been selfconsistently investigated by Iben and McDonald 1985, and has two main effects: it lengthens this phase of evolution, as the reactions $^{12}C \rightarrow {}^{14}N$ contribute energy for a longer time, but it leaves a smaller

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hydrogen mass on top of the WD so that proton-proton burning at a later stage is less important. In any case, Iben and McDonald showed that diffusion induced nuclear burning can not reduce the mass of the evolutionary remnant by more than a factor two. If there are reasons to believe that in many cases the hydrogen remnant is orders of magnitude smaller, this can not be inputed to this mechanism.

The external layers are fully radiative, but convection in pure helium envelopes begins to appear just at log $T_{eff} \sim 4.70$, with scarce dependence on the gravity and on the treatment of convection.

Observationally, the hottest WDs appear at $T_{eff} > 10^5$K (the PG 1159 class (Wesemael, Green and Liebert 1985) followed by the hottest DAs (the DAOs) and by the DOs. The hottest WDs are helium dominated: while Fontaine and Wesemael (1987) suggest that most DAs appear at $T_{eff} \lesssim 80000$K, where gravitational separation had the time to bring to the surface the amount of hydrogen necessary to form the hydrogen atmosphere, we also know that the WDs of low mass with <u>thick</u> hydrogen layers have radii considerably larger than the corresponding one of helium atmosphere WDs, and, at large T_{eff}, the time of evolution of non DAs can be considerably longer than that of DAs (e.g. Iben and Tutukov 1984 and figure 2). If we consider, on the other hand, an already stratified hydrogen layer of much smaller mass (say $10^{-7}M_\odot$), the radius does not differ significantly from that of the WD in which no hydrogen is present at the surface (e.g. Koester and Schönberner 1986). In practice, PG 1159 WDs may be progenitors of some DA WDs with very thin hydrogen envelopes, as suggested by Fontaine and Wesemael 1987, but the lack of DAs at large T_{eff} may also mean that many DAs have thick ($\sim 10^{-4}M_\odot$) remnant hydrogen layers.

Several features conspire against a meaningfully simple assessment in this first stage:

i) The time of evolution in the PG1159 stage (at $T_{eff} \sim 10^5$yr) is of the order of 10^5yr from all <u>evolutionary</u> computations. Observers must take care in adopting <u>these</u> timescales when trying to derive information on the number of white dwarfs expected in the following evolutionary phases. Models built from <u>non evolutionary</u> starting models do not necessarily give the correct times at these first stages, although later on differences of a million year in the total cooling time becomes negligible. Furthermore, the evolutionary times show a somewhat large dependence on the evolutionary inputs (compare, in figure 2, the timescale for $0.68M_\odot$ and for $0.56M_\odot$). As an entire set of computations for the whole set of parameters to be explored is not yet available, it is difficult at present to say how many selection biases weigh on the interpretation of the hottest WDs;

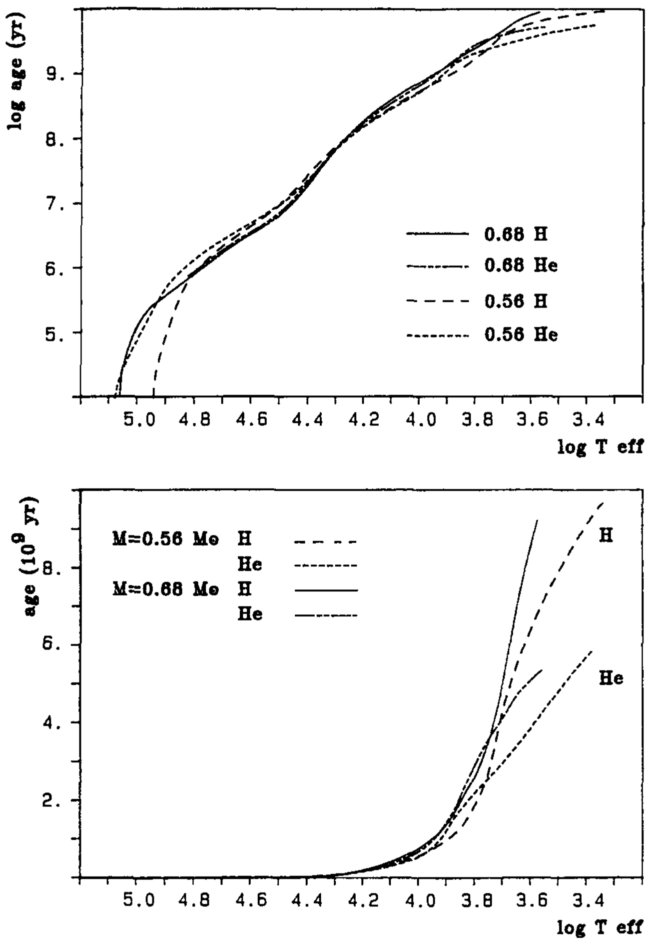

Figure 2: Logaritmic (upper part) and linear (lower part) behavior of evolutionary times with T_{eff} for the models of Mazzitelli and D'Antona 1986 and of D'Antona and Mazzitelli (1989).

ii) it is not easy to derive correct T_{eff} and gravities;

iii) the atmospheric chemical composition is not simply linked to the previous evolution, as radiation pressure, operating on the lines, is the dominant process for establishing a given pattern of abundances, regardless the initial compositions (Vauclair et al 1979, Vauclair 1987);

iv) mass loss is still operating: it is not easy to predict what happens to a given spectral type in the subsequent evolution: for instance, DAOs may become DAs when diffusion of helium sets in, or DBs if the remnant hydrogen is completely lost by wind.

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2nd stage: $0 > \log L/L_\odot > -1.5$, $4.7 > \log T_{eff} > 4.3$: "end of neutrino cooling".

In this phase the deviations of the radius from a constant value begin to be negligible. Neutrino cooling decreases, while, possibly, nuclear burning by the p-p chain becomes important. Helium convection begins and becomes progressively more important when decreasing T_{eff}.

In the range $4.65 \lesssim \log T_{eff} \lesssim 4.5$ the HeII lines should be already visible, but there are no DBs (Wesemael et al 1985). The latter appear only at $\log T_{eff} = 4.5$. Pelletier et al (1986) suggest that DO evolve into DAs and, later on, the onset of He convection below the H-layer is able to mix this layer and leads to the appearence of DB stars. Hydrogen masses up to 10^{-15} of the total mass of the star can be mixed at $\log T_{eff} < 4.5$ (Pelletier et al 1986, Liebert, Fontaine and Wesemael 1987, D'Antona and Mazzitelli 1975). Actually, the statistic significance of the DB gap is given by the relative population of the range. The ratio of expected numbers of WDs is equal to the ratio of the relevant cooling times multiplied by the ratio of discovery probability. The ratio of the time spent in the range where DBs are predicted but are not seen ($45000 \gtrsim T_{eff} \gtrsim 30000$K) to the time spent in the DB phase (say $30000 \gtrsim T_{eff} \gtrsim 21000$K and $21000 \gtrsim T_{eff} \gtrsim 12000$K) in our models ranges from 1:9:60 for the $0.68 M_\odot$ evolution (Mazzitelli and D'Antona 1986) to 1:5.3:33 for the evolution of 0.56 and $0.6 M_\odot$. The latter values are consistent also with the ratios derived from Koester and Schönberner (1986) models (1:5:27). As the relative probability of discovery is 1:0.8:0.2 (Wesemael et al. 1985) we should expect from 10 to 17 DBs at $30000 \gtrsim T_{eff} \gtrsim 12000$K for each DB at larger T_{eff}. As in the Palomar Green survey there are 39 DBs, only 2 to 4 hot DBs are missing, and the DB gap can be considered at least partially due to observational selection effects. Wesemael et al (1985) expected from 6 to 7 hot DBs, based on Winget et. al. evolutionary times: this is a further indication that observers must be very careful in the use of evolutionary times at large T_{eff}, as they depend critically on the starting models. I conclude that not necessarily most of DBs come out from mixing of a very thin outer hydrogen layer.

3rd stage: $-1.5 > \log L/L_\odot > -3$ $4.3 > \log T_{eff} > 4.0$: "cooling".

The inner structure is dominated by "cooling", possibly with residual p-p burning if the hydrogen layer is thick. Helium convection in He-envelopes reaches its maximum depth around 10000K. In hydrogen envelopes convection sets in around $\log T_{eff} = 4.2$.

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According to the available computations, this is the region where the evolutionary times do not differ too much for "details" of evolution and so this is the best place where to normalize the luminosity function.

From an observational point two interesting classes appear: the DBAs (DBs with hydrogen abundances of 10^{-3} - 10^{-5} -Shipman et al 1987) and the DQs (WDs with C_2 abundances from 10^{-3} to 10^{-7} -Koester et al. 1982, Wegner and Yackovich 1983, 1984). While DBAs are still not well explained -the trace hydrogen can be due to accretion of interstellar matter, and however can not be due to mixing of the hydrogen layer, unless our prediction on convection efficiency are wrong by about two orders of magnitude (see the full discussion by Shipman et al. 1987 and by Koester 1987)- DQs are tracers of previous evolution: carbon appears in the envelope as it is dredged up by the sinking helium convection.

The case of DQs is the best occasion we have to get information on the previous evolution from the WD surface composition. It was obvious even ten years ago that this carbon should come out from the regions of the WD where triple alpha processes had occurred. The first attempts to see whether this carbon could be picked up directly from the core failed: if the helium envelope was so small that helium convection could reach its base, also carbon would have been convective, and a pure carbon composition would have resulted at the surface (D'Antona and Mazzitelli 1979, Fontaine and Michaud 1979). Subsequently, Koester et al (1982) suggested that the appearence of carbon at the surface could be due to the encounter of the convective region with the region in which finite but small abundances of carbon are present due to the effect of diffusion from the core. Relevant computations have been done by Muchmore (1982, 1984), Fontaine et al (1984) and Pelletier et al (1986). As shown by Wegner and Yackovich (1983) the correlation between carbon abundance and T_{eff} predicted by this theory is successfully consistent with the observations. Unfortunately, the best fit is achieved for very small He-buffer layers ($\log M_{he}/M_t$ =-3.5 -4.0), but stellar evolution predicts helium intershell remnants from 10^{-3} to 10^{-2} $M_\odot$ (e.g. Mazzitelli and D'Antona 1987). In fact, although there are several phases in which the sudden loss of the hydrogen envelope could keep the pure helium layer at a minimum, on top of the star remains a massive layer in which the carbon abundance is very large. D'Antona and Mazzitelli 1979 suggested that, instead of coming from the core itself of the WD, the Carbon could have been picked up by the helium convection from this region of the He intershell enriched in carbon during the thermal pulse phase.

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This would solve the discrepancy between the evolutionary theory and the prediction of the diffusion-convection model, but is contradicted by the results by Muchmore (1984), which imply, at large T_{eff}, very fast settling of carbon even in the regions where it is not a trace element. The whole intershell region is never very small, and fast gravitational settling leaves about 70% of it as a pure helium layer at the top of the WD. This mass can not be smaller than $10^{-3}M_{\odot}$.

We must therefore look for mechanisms which skip from the WD the most of its helium, to reduce the layer to the $2x10^{-4}M_{\odot}$ or less indicated by the interpretation of DQs. We can think about:

i) winds in the post planetary nebula phase: if the hot temperature domain is crossed during the stationary helium burning stage, a reasonable rate of $10^{-8}M_{\odot}$/yr acting for $5x10^{4}$yr is able to do the job. But, if the helium mass is reduced below a percent of solar mass, the 3α reactions can no longer be sustained, the evolutionary times shorten, and the phase of large luminosity, where reasonably strong winds may act, finishes. A further argument against winds is the following: we should probably expect a much larger spread in carbon abundances in the DQs than actually observed.

ii) when a He-shell flash is ignited in the blue, it is very probable that at the peak of the pulse convection reaches the bottom of the hydrogen layer, bringing protons in the region of helium burning , with consequences which up today are predictable only by speculations. The explosive burning of hydrogen, occurring mostly at the base of the convective envelope, could be sufficient to expell the entire helium layer! This occurrence, foreseen by Renzini (1982) was tentatively investigated by Iben et al, 1983 and by Iben 1984 (see also Iben 1987). If any hydrogen is left, (according to Iben et al. 1983, $4x10^{-5}M_{\odot}$, less than $10^{-6}M_{\odot}$ according to Iben 1984), it can easily be lost by wind, leading to expose helium and carbon rich layers. It is very easy that a last He-shell flash occurs in the blue mainly for low mass stars. It has been found for instance also in the computations by Caloi (1989) regarding the evolution of very blue horizontal branch stars. The indication that DQs have space velocities larger than the average sample of other spectral types (McMullin et al. 1987, Sion et al. 1988) may be in favour of the interpretation of DQs as a subclass of WDs having suffered a late He-shell flash with hydrogen mixing in their pre-WD life. In the framework of this interpretation, we may regard the extremely carbon rich nucleus of the planetary nebula NGC 246 (Husfeld 1987) as a possible progenitor.

I conclude with a caveat: remember that the results by Pelletier et al (1986) depend on many other parameters: they show that a turbulent diffusion could modify substantially the situation, by allowing helium shell masses about a factor ten larger ($2\times10^{-3}M_\odot$), in substantial agreement with the stellar evolution predictions. Although Pelletier et al. (1986) are not in favour of this interpretation, as the turbulence parameter should be adjusted with the effective temperature, let us remember that also the lithium depletion during main sequence evolution requires the same type of adjustment of turbulent diffusion with T_{eff}.

<u>4th</u> <u>stage</u>: $-3>\log L/L_\odot>-4.5$ $4.0>\log T_{eff}>3.6$: "crystallization".

In the interior crystallization sets in. In the outer hydrogen layer convection reaches its maximum depth at $\log T_{eff}\sim 3.7$. The WD with hydrogen or helium atmospheres become considerably different in their internal structure. If mixing of a "massive" ($10^{-5}M_\odot$) H-layer occurs, the evolution is delayed until the extra-thermal content of the WD is lost (D'Antona and Mazzitelli 1987).

This is probably the most important stage for our understanding of white dwarfs. The input physics of crystallization is assumed as more or less "standard" by all authors. The latest years have seen the interesting development of the idea that oxygen and carbon are not miscible in the solid phase (Stevenson 1980), and that "oxygen snow" settles at the center liberating gravitational energy which contributes to substantially lengthen the evolution. This idea, first developed by Mochkovitch (1983), has been carefully explored recently by Garcia-Barro et al. (1988a and 1988b). A recent new investigation of the crystallization properties of carbon oxygen mixtures indicates however that disordered crystallization, as first assumed by Kovetz and Shaviv 1970 is probably the best approach to the reality (Barrat, Hansen and Mochovitch 1988).

While the interior suffers the transformations which will ultimately lead to the reduction of its thermal energy like in a common crystal, the external layers begin to play a major role.

First of all, we begin to reach critical conditions at the surface. I discuss now these facts on the basis of our latest WD models (D'Antona and Mazzitelli 1989) referring to the evolution of a $1M_\odot$ pop.I star which becomes a white dwarf of $0.564M_\odot$ after losing mass simply by stellar wind during the first and second giant branch evolution. Our previous computations were done down to $\log L/L_\odot\sim -4.5$, as the opacity and e.o.s. employed were not extended enough to study lower luminosities. The e.o.s. has been recently updated by Mazzitelli (1988b) and for the opacities we decided to make extrapolations which

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could allow us to perform wider computations, although we must still keep in mind that these must be considered only educated guesses.

If the metals are scarce, as they observationally are in WDs, the densities reached in the atmospheres at low temperature become so large that the photospere is out of the normal available radiative opacity tables (Cox and Stewart 1970, or analogous). At low temperature, large density, the helium is mostly atomic, and we must not worry for electron conduction. We selected one set of radiative opacities (the $Z=10^{-5}$ mixtures by Cox and Tabor 1976), and extrapolated them up to the pressure ionization boundary. At densities large enough that pressure ionization has set in, but at low temperature, the conductive opacities can be computed by the formulation by Itoh et al (1983 and 1984). The result is shown in figure 3.

Inspection of our models shows that, while the T_{eff} declines below 10000K, the structures begin to enter the pressure ionization domain: this has the effect of changing the slope of the relation $T_{eff} - T_c$ in a way which depends on the assumed envelope composition (He, H, or H with large metal content, see as an example figure 10 in Mazzitelli and D'Antona 1986).

<u>5th stage</u>: log $L/L_\odot < -4.5$, $\log T_{eff} < 3.6$: "Debye cooling".

What happens in the interior at this phase depends critically on the external opacities. After crystallization is completed, Debye cooling will sure set in, but at which luminosity it is still in the phase of debate, further it is not clear whether any stars had enough time to reach the stage of Debye cooling! In order to describe this stage we must therefore rely on models which are able to reach it in a time shorter than the age of the Universe. In our models, actually this happens.

The reasons why we had chosen for these models very low opacities was precisely to understand how short evolutionary times of "typical" WDs could be. Also the fact that these WDs are mainly composed of oxygen conspires to have them get an early crystallization in the interior, so that, by the time the stars are at $\log L/L_\odot=-4.5$ Debye cooling is already very efficient. The total evolutionary time down to $\log L/L_\odot=-5.3$ is 5.8×10^9 yr for the helium envelope models, 9.3×10^9 yr for the hydrogen atmosphere models (figure 2). The central temperature is 3×10^5 K at $\log L/L_\odot=-5.3$, while the relevant Debye temperature:

$$\vartheta_D = 3.48\times10^3 (Z/A) \rho^{1/2}$$

at center ($\rho_c = 3\times10^6$ g cm^{-3}) is a factor ten larger, and the specific heat is reduced to $6x10^6$ cgs, while it is $2x10^8$ cgs in normal conditions. However, the temporal evolution of the surface luminosity does not show the signs of acceleration we would have primarily expected. The reasons for this behaviour are examined in the following sections.

The lower T_{eff} models have their photosphere at the boundary of pressure ionization! This interesting feature is by no means new: it had been found in the pure helium models computed by Bohm et al 1977.

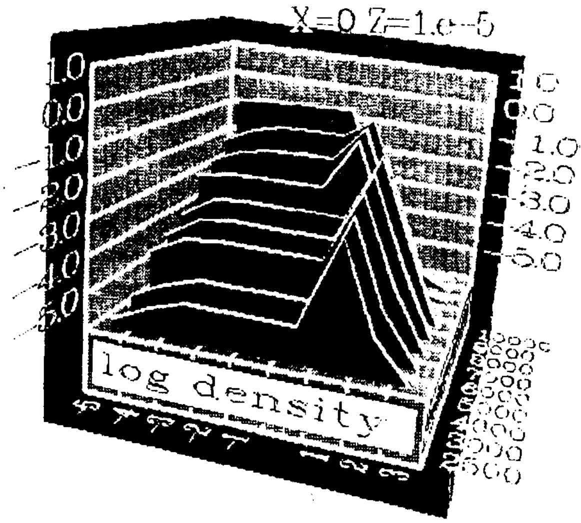

Figure 3: logarithm of the opacities adopted for the helium composition, as function of log ϱ (cgs) and T (K). The pressure ionization is assumed to occur at $0 < \log \varrho < 1$

If we look at the opacities adopted for the helium composition (figure 3), we see that these are so low when helium is not ionized that even the quite low electron conduction opacities which are adopted at log ϱ =1 are two orders of magnitude larger, a plausible explanation of the reason why the photosphere is reached just at this interface. The model having helium surface at log T_{eff}=3.46 resembles quite closely the model plotted by Böhm et al (1977) at logT_{eff}=3.48, even in the central temperature reached. It is interesting to notice that also in the coolest models with hydrogen envelopes the photosphere is reached at the boundary of pressure ionization. We can conclude that the effect of Debye cooling appears at the surface through the opacity of free electrons at the boundary of pressure ionization. This clearly deserves further, much more physically appropriate investigation.

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3. THE LUMINOSITY FUNCTION OF WHITE DWARFS

The work on the luminosity function of WDs has seen the efforts of many careful researchers, starting from Luyten (1958) and Weidemann (1967), and ending with Fleming et al (1986), but I think every theoretician should be grateful to Liebert Dahn and Monet 1988 to have made two non simple efforts:

- to take the responsibility to assert that the number density at $\log L/L_\odot \simeq -4.5$ is no longer a lower limit, but a significant point with estimated error bars;

- to convert the observed magnitudes into theoretical values.

The different M_v - M_{bol} relations adopted by Liebert et al. 1988 give also a clear hint of how much uncertain we must consider the very low luminosity points.

Let me take advantage of all the previous discussion on evolutionary times to make clear one point: if we want to compare the theoretical and observed LFs, we must find a reasonable way of normalization. It is clear that we must avoid normalization at large luminosities, where the evolutionary times are somewhat dependent on the previous evolution. Probably the best choice for normalization is the region of pure "cooling" at $-1.5 > \log L/L_\odot > -3$, which is safe from dramatic problems, at least if p-p nuclear burning does not play a very important role (as it seems from Iben and McDonald 1985). Further, probably this is also the region in which we may trust the observational points without entering in difficult problems as the drastic decrease of discovery probability (Lamb and Van Horn 1975, Iben and Tutukov 1985).

Let me further define the theoretical LF simply as

$$\log \varphi = \log (dt/ d\log L/L_\odot) + \text{constant}$$

Fit with observations is thus simply a vertical shift which determines the value of the constant. This approach is valid until we may consider the birth rate of WDs constant with time, namely, until the proper WD evolutionary times are not longer than -say- $5 \div 8 \times 10^9$ yr, otherwhise, proper account must be taken of the finite age of the disk in which WDs have been searched (D'Antona and Mazzitelli 1978).

In figure 4 I compare the LF obtained by our latest computed models of $0.56 M_\odot$ with the observational LF. I further show the comparison with the previous LF obtained by the evolution of $0.68 M_\odot$ WDs with helium or hydrogen envelopes (Mazzitelli and D'Antona 1986, D'Antona and Mazzitelli 1986). I show also the LF from the models of Winget and Van Horn for a disk age of 10^{10} yr, adopted by Liebert et al. 1988 as a comparison. In figure 6b this LF is shifted, to achieve a better fit of the observational points in the "secure" region at logL/L⊙=-2. The overall shape of the LF is very reasonably fit by our models, particularly by those having helium envelopes, apart from the crucial point at log L/L⊙=-4.5. (The models used in D'Antona and Mazzitelli 1986 were extended only to logL/L⊙ =-4.3, and this problem was not so evident). In particular, there is a good agreement between the theoretical curve and the flattening shown by the observational

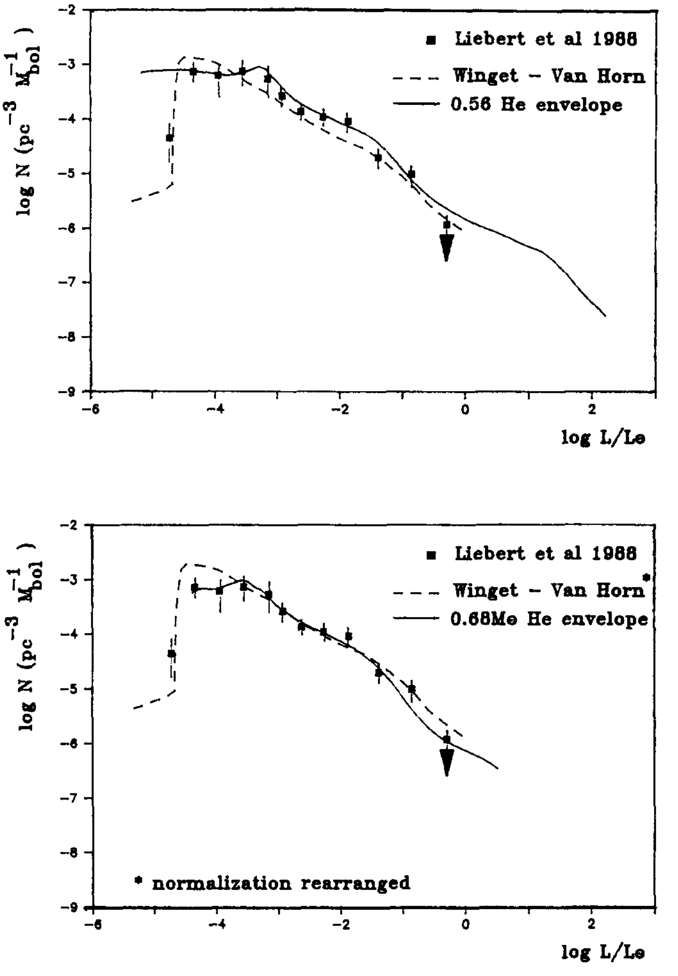

Figure 4:Comparison of observed and theoretical luminosity functions.

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LF at $\log L/L_{\odot}<-3$.

Although the following exercize has already been done in many forms, let me derive the dependence of the luminosity function on the luminosity as a function of two quantities: the functional relation between the specific heat and the WD average (central) temperature:

$$C_p \propto T^k{}_c$$

where k=0 in the case of a gas, and k=3 in the the Debye phase; the second is the relation between central temperature and T_{eff}:

$$T_c \propto T_{eff}{}^n$$

Simple algebra and the fundamental relations:

$$L = 4 \pi R^2 \sigma T_{eff}{}^4$$

and

$$L = d\ E_{th}\ /\ dt$$

where the thermal energy is

$$E_{th} \propto T_c{}^{k+1}$$

provide

$$\varphi = dN/\ d\log(L/L_{\odot}) \propto dt/d\log(L/L_{\odot}) \propto L^{(n(k+1)/4-1)}$$

in the case of k=0:

$$\varphi \propto L^{(n/4-1)}$$

in the Debye phase:

$$\varphi \propto L^{(n-1)}$$

We can derive the index n from our models. In the first phases, we have n 1.6, but, at $\log L/L_{\odot}=-3$, n suddenly increases to n=2.7 due to the discussed effects of e.o.s. and atmospheric opacities. Here the LF behaviour should change from $L^{-0.6}$ to $L^{-0.3}$. For this reason the helium atmosphere models, where the change of slope of the $T_c - T_{eff}$ relation occurs earlier, seem more appropriate to reproduce the flattening of the LF. In the meantime k becomes larger than zero and the LF shows some decline. Had the n index remained the same, during Debye cooling the LF would have shown a <u>decrease</u> with the power 1.7 of the luminosity. In our models, however, n <u>decreases</u> to $n \simeq 1$, and <u>the LF remains flat</u>.

We see therefore that the results we have obtained are related in a crucial way to the input opacities adopted, which determine the dependence of T_{eff} on T_c. If we want that Debye cooling appears at the

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surface with a sharp decrease of the LF, we need a steeper relation, which our (maybe naif) extrapolations do not indicate. It is clear that further exam of the physics in this difficult region begins to be really worthwhile.

As present models seem to reproduce very well the global shape of the LF, but are not able to explain the factor 10 deficit in the WDs at very low luminosity, although they reach Debye cooling very early, we must then investigate the other possibilities.

Both our models and the models by the Texas group (Winget and Van Horn 1987, Wood et al 1987) agree that simple increase of the low temperature opacities is able to increase substantially the evolutionary times so that the drop in the LF acquires the meaning that all WDs formed from the birth of the Universe (or at least of the disk) are still visible (Winget et al. 1987, Liebert et al 1988). To increase the ages we can think either that the metal abundances in the envelopes of cool WDs, although not perfectly determined, are somewhat larger than those we have assumed (see for a summary Koester 1987). Accretion of metals from interstellar clouds may be more and more relevant at later stages of evolution. It is also possible that hydrogen (although not seen in most of cool WDs) is present in cool WDs. Interestingly enough, if this is the case, the very drastic conditions we meet at the photospheric boundary are avoided, and we may also trust more the results (a similar situation is encountered when dealing with brown dwarfs, e.g. D'Antona and Mazzitelli 1985, D'Antona 1987). Finally, maybe that the interior composition of WDs may be not so substantially dominated by oxygen (although for the small masses it is difficult to believe that carbon is more than 20%).

Remember also that, although it helps in solving the factor ten discrepancy in the LFs at $\log L/L_\odot=-4.5$, prolonged cooling may create a problem with the space densities around $\log L/L_\odot=-4.3$, if the numbers given by Liebert et al. 1988 are to be taken at face value.

I conclude that, if the observational luminosity function is correct, we are left with two interesting alternatives: are the metals or hydrogen in the atmospheres of WDs sufficient to considerably prolongate the lifetime at low luminosity? Or is Debye cooling responsible for the drop in the luminosity function, and we are simply missing the correct $T_{eff} - T_c$ relation due to our poor understanding of high density atmospheres and envelope physics?

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