

DIFFUSION IN WHITE DWARFS

David Muchmore

Astronomy Department
University of Washington

Two decades ago it was realized (Schatzman, 1958) that metals should be rapidly removed from the atmospheres of white dwarfs by diffusion through the steep pressure gradients present. Yet metals are observed. Their presence thus prompts us to consider the diffusion process in greater detail and to seek mechanisms which might deposit metals in WD atmospheres.

The description of diffusion in white dwarfs is facilitated by the use of the moment equation formulation of the kinetic theory. This approach can treat such effects as a) the multicomponent nature of the plasma, b) electron degeneracy, and c) ionic correlations - all of which are at best awkward to handle with the conventional treatment of the diffusion process (e.g. Chapman and Cowling, 1970). If one forms velocity moments of a transport equation - the Boltzmann equation for example - one obtains for each species of particle in the gas a diffusion equation from the second order moments and a heat flow equation from the third order moments. With fairly unrestrictive assumptions one obtains for a gas which has a nearly Maxwellian velocity distribution the equations in the forms

$$-\nabla p_s + \rho_s \vec{g} + q_s \vec{E} = \sum_t K_{st} (\vec{w}_s - \vec{w}_t) + \sum_t K'_{st} \left(\frac{m_s \vec{r}_t - m_t \vec{r}_s}{m_s + m_t} \right) \quad (1)$$

$$2.5 N_s kVT = -2.5 \sum_t K'_{st} \frac{m_t}{m_s + m_t} (\vec{w}_t - \vec{w}_s) - 0.5 K''_{ss} \vec{r}_s - \sum_t K'''_{st} \vec{r}_s + \sum_t K''''_{st} \vec{r}_t \quad (2)$$

(Burgers, 1969, Eqns 18.1 & 18.2) in which subscripts denote individual components of the gas - as p_s , ρ_s , and q_s are the partial pressure, mass density and charge density of species s . K_{st} , K'_{st} , etc., the resistance coefficients, are roughly inversely related to diffusion coefficients. In the case of binary diffusion, for example, the relation is simply

$$D_{st} = \frac{kTN_s N_t}{(N_s + N_t) K_{st}} \quad (3)$$

The w 's are flow velocities with respect to the mean mass flow of the gas and the r 's are residual heat flow vectors which are contractions of the third order moments of the distribution function. Together with the

conditions

$$\nabla \cdot \sum_s \rho_s \vec{w}_s = \nabla \cdot \sum_s q_s \vec{w}_s = 0, \quad (4)$$

these form a system of linear equations for the w 's, r 's and E which is amenable to machine solution.

In stellar diffusion problems one frequently encounters cases in which the gas cannot be properly regarded as a single medium through which trace elements diffuse. For most stellar material, H and He are both significant constituents. And often one is interested in diffusion through a medium of electrons and one or more species of ion. As an example in point, we will consider the case in a model for a $\lambda 4670$ star like those of Muchmore and Böhm (1977). This model has $T_{\text{eff}} = 7000\text{K}$, $\log g = 8$, and $C/\text{He} = 10^{-3}$. The outer convection zone has a depth of $z = 11$ km. The material at the base of this convection zone (CZ) has $\text{HeI} \approx \text{HeII} \gg \text{HeIII}$ and C is ionized 1, 2 or 3 times. At this density (180 g cm^{-3}) the mean interionic distance $a \approx 2D$, where D is the Debye radius. So Debye shielding does not occur and furthermore a itself is only about half of the Bohr radius. Accordingly, the scale for collisions with ions is about a while that for collisions with neutral atoms will be larger. If particle interactions are described as a) induced dipoles (polarizability $\alpha = 3 \cdot 10^{-25}$) for neutral atom - ion collisions and b) Coulomb collisions truncated in the usual way at the interionic distance, then at the base of this CZ, each HeI atom contributes about five times as much resistance to the flow of C as does a HeII ion. Thus, unlike the case in dilute gases, the resistance to diffusion due to atoms is not negligible and one is compelled to face the multicomponent condition of the gas in describing the diffusion of trace elements.

In this model, the electrons are weakly degenerate, $\eta = 2$ to 7 for the layers below the outer CZ. The electrons couple to the ions mainly through the overall radial electric field rather than through binary collisions, thereby supporting much of the weight of the star. Since the pressure gradient for degenerate electrons is greater than that for a like number of ions, there is an additional charge separation beyond that which occurs in an ideal gas. Consequently, the equilibrium electric field strength is increased by a factor of as much as $(1 + 1/Z)$ for a single species of ion with charge Z (Muchmore, 1979).

The magnitude of the electric field is also influenced by the non-equilibrium character of the gas, that is the fact that it maintains gradients in pressure and temperature. The right hand sides of the diffusion equations are thus necessarily non-zero (Eq. 1) and affect the magnitude of the electric field. The primary effect here is the thermal diffusion of the electrons which increases the electric field. Fig. 1 shows the ratio of the actual electric field strength calculated from Eqns. 1, 2 and 4 compared with that obtained from the equilibrium equations, i.e. Eq. 1 with the r.h.s. set to zero. It is seen that the field is substantially stronger near the surface of the star than is predicted by the equilibrium approach. These are the layers of the star in which resistance coefficients are smallest and drift velocities largest.

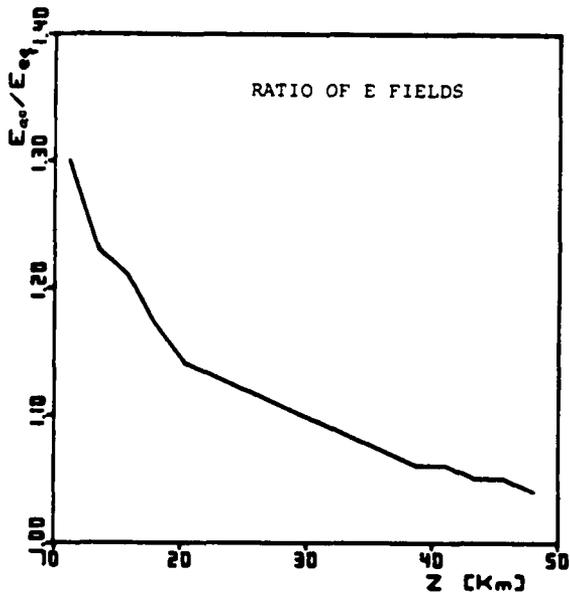


Fig. 1 - Ratio of the actual E field to that obtained from the equilibrium equations as a function of depth in a $\lambda 4670$ star with $T_{\text{eff}} = 7000\text{K}$.

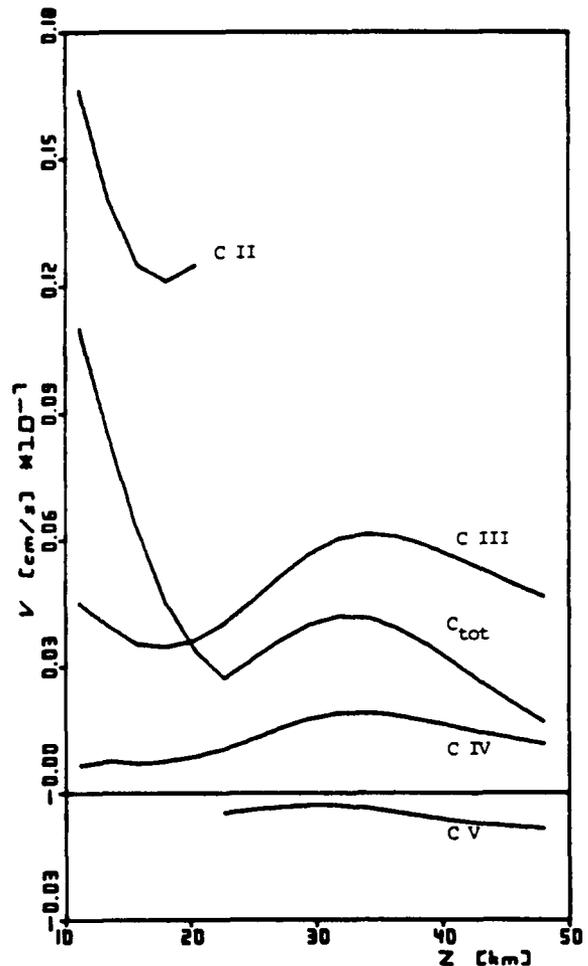


Fig. 2 - Diffusion velocities for individual C ions and for the mean flow of the 3 most abundant C ions (C_{tot}) as a function of depth.

The electric field strength is large under these conditions, with eE taking values from 3.3 to 4.5 m g, so that the field strength substantially influences the relative^p diffusion of various ions. For the reasons cited above, it is necessary to solve the full set of flow equations for all the major species in the gas in order to obtain the correct value for E . E is particularly interesting in light of the suggestion by Michaud and Fontaine(1979) that it may drive H out of He dominated atmospheres.

Fig. 2 presents diffusion velocities for C in this model. The most significant points which appear include the fact that the different ions of C diffuse at vastly different rates. In fact, CV diffuses upward rather than downward relative to the center of mass of the gas. Furthermore, the overall diffusion velocity for C does not decrease rapidly inward and in fact even increases inward in some layers. This behavior arises from the change in the ionization of He. The reduction in the fractional abundance of HeI which occurs in the layers reduces the overall resistance to diffusion and is readily handled only with the multi-component flow equations. These points emphasize that it is not possible to calculate reliable diffusion velocities without knowing the ionization state of the gas. The present results arise from the treatment of pressure and degeneracy effects on ionization as described by Böhmer and Cassinelli(1971) and Muchmore and Böhmer(1977). This equation of

state is known not to be thermodynamically consistent (Fontaine, et.al., 1977). One expects that since the diffusion velocities are highly sensitive to the ionization of the gas, the use of a better equation of state will give results which differ substantially from these quantitatively but probably not qualitatively.

Use of the moment equation formalism should also prove fruitful in another way. At high particle densities one must worry about the validity of the Boltzmann equation, in particular the assumption that collisions are isolated events - that multiple body collisions are rare and negligible. By taking moments of the first equation of the BBGKY hierarchy rather than the Boltzmann equation, one obtains diffusion equations in which the usual Maxwell-Boltzmann transfer integrals are replaced by integrals over the pair distribution function of the gas. And we have considerable information about this function from Monte Carlo studies (Brush, et.al., 1966, Hansen, 1973).

If the model we have been discussing evolves due to diffusion with no replenishment of C in the CZ, the C concentration develops in time as shown in Fig. 3. If on the other hand, one has a steady state in a He rich envelope, the C concentration beneath the outer CZ attains the equilibrium values shown in Fig. 4 in which C/He is maintained at 10^{-3} in the CZ. The steady state requires a flux of $3 \cdot 10^{14} \text{ cm}^{-2} \text{ s}^{-1}$ of C. The layers beneath the CZ are clearly not homogeneous in composition. Does it happen that the CZ penetrates to a mean molecular weight barrier with the C - O core and dredges material upward (see Vauclair and Reisse, 1977)? To consider such possibilities in detail, we still need better, self-consistent models and a better quantitative knowledge of the microscopic behavior of dense plasmas.

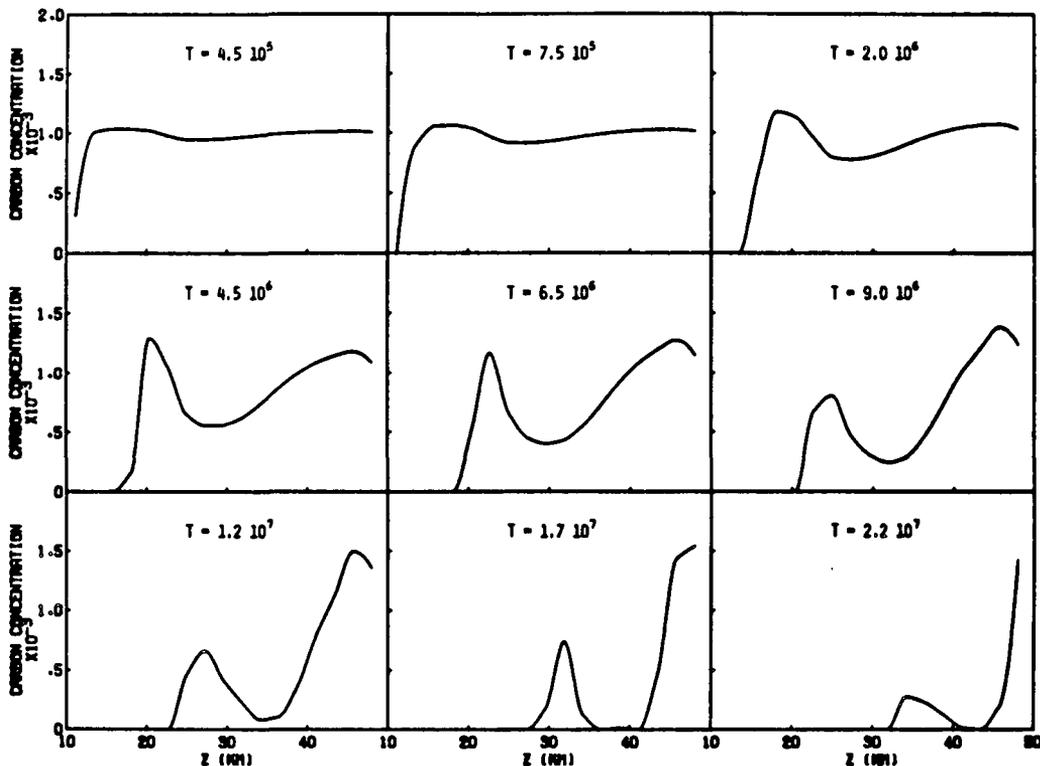
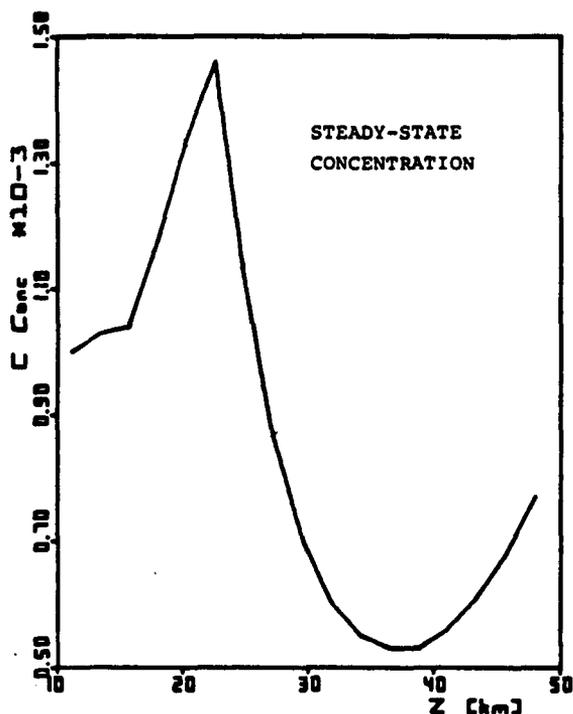


Fig. 3 - Carbon concentration vs. depth. Times are in years.

Fig. 4 - Carbon distribution in a steady-state He envelope with a surface concentration $C/He = 10^{-3}$.



This research has been supported by NSF grant AST 78-20542.

References

- Böhm, K.H., Cassinelli, J.C. 1971, *Astron. Astrophys.*, 12, 21.
 Brush, S.G., Sahlin, H.L., Teller, E. 1966, *J. Chem. Phys.*, 45, 2102.
 Burgers, J.M. 1969, *Flow Equations for Composite Gases*, Academic Press, New York.
 Chapman, S., Cowling, T.G. 1970, *The Mathematical Theory of Non-Uniform Gases*, 3rd ed., Cambridge Univ. Press, Cambridge.
 Fontaine, G., Graboske, H.C., Van Horn, H.M. 1977, *Ap. J. Suppl.*, 35, 293.
 Hansen, J.P. 1973, *Phys. Rev. A*, 8, 3096.
 Michaud, G., Fontaine, G. 1979, *Ap. J.*, 229, 649.
 Muchmore, D.O., Böhm, K.H. 1977, *Astron. Astrophys.*, 54, 499.
 Muchmore, D.O. 1979, in preparation.
 Schatzmann, E. 1958, *White Dwarfs*, North Holland Publishing Co., Amsterdam.
 Vauclair, G., Reisse, C. 1977, *Astron. Astrophys.*, 61, 415.