F. Bagemihl
Nagoya Math. J.
Vol. 61 (1976), 203-204

THE THREE-SEPARATED-ARC PROPERTY OF THE MODULAR FUNCTION

FREDERICK BAGEMIHL

Let D be the open unit disk and Γ be the unit circle in the complex plane, and denote the Riemann sphere by Ω. If $f(z)$ is a function defined on D with values belonging to Ω, if $\zeta \in \Gamma$, and if Λ is an arc at ζ, then $C_\Lambda(f, \zeta)$ denotes the cluster set of f at ζ along Λ. If there exist three mutually exclusive arcs $\Lambda_1, \Lambda_2, \Lambda_3$ at ζ such that

$$C_{\Lambda_1}(f, \zeta) \cap C_{\Lambda_2}(f, \zeta) \cap C_{\Lambda_3}(f, \zeta) = \emptyset ,$$

then f is said to have the three-separated-arc property at ζ.

The following theorem answers a question raised by Belna [1, p. 220] concerning the modular function $\mu(z)$ that maps D onto the universal covering surface W of the extended w-plane punctured at the points $w = 0, 1, \infty$.

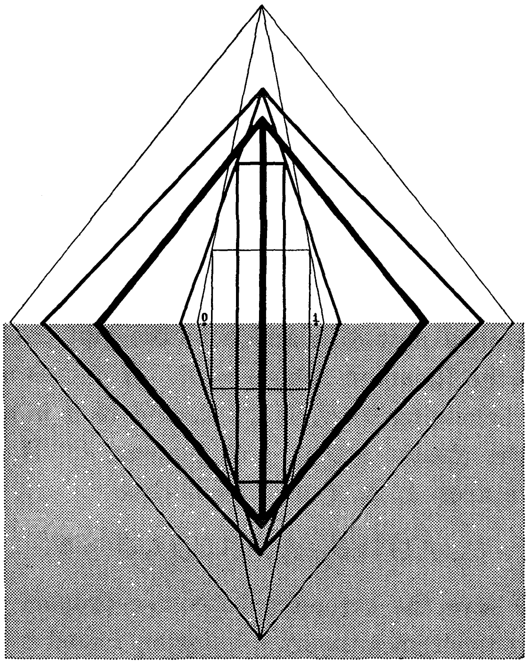
THEOREM. *The modular function $\mu(z)$ has the three-separated-arc property at every point of Γ.*

Proof. For convenience and clarity, we refer the reader to the Figure, which represents the w-plane. The shaded lower half is the lower half-plane, the unshaded upper half is the upper half-plane. We consider three graphs, g_1, g_2, g_3; g_1 is represented by the lightest lines, g_2 by the heavier lines, and g_3 by the heaviest lines.

For $j = 1, 2, 3$, let G_j denote the set of points on W that overlie the set g_j, and let γ_j be the preimage of G_j under the mapping $\mu(z)$. One readily infers from the Figure that if $\zeta \in \Gamma$, then there are in D three mutually exclusive arcs $\Lambda_1, \Lambda_2, \Lambda_3$ at ζ such that $\Lambda_j \subset \gamma_j$ $(j = 1, 2, 3)$. The cluster set $C_{\Lambda_j}(\mu, \zeta)$ is clearly a subset of g_j $(j = 1, 2, 3)$. Since it is evident that $g_1 \cap g_2 \cap g_3 = \emptyset$, the theorem is proved.

Received December 2, 1975.

https://doi.org/10.1017/S0027763000017396 Published online by Cambridge University Press



Figure

REFERENCE

[1] C. L. Belna, Intersections of arc-cluster sets for meromorphic functions, Nagoya Math. J. **40** (1970), 213–220.

University of Wisconsin-Milwaukee

https://doi.org/10.1017/S0027763000017396 Published online by Cambridge University Press