A COMPARISON OF EXTENDED AND SHORT METHODS IN THE CALCULATION OF A LIFE TABLE FOR MALES IN LONDON.

By K. C. K. RAJA,

L.R.C.P. & S. (Edinburgh), L.R.F.P.S. (Glasgow), D.T.M. & H., D.P.H. (Cambridge).

The last life table for the Administrative County of London was prepared by Mr King from the census population of 1911 and the deaths in 1910, 1911, 1912, and published in Part I of the Supplement to the *75th Annual Report of the Registrar-General*, 1914. The national life tables (No. 9), prepared by Sir Alfred Watson after the census of 1921, included one for "Greater London," the boundaries of which extend considerably beyond the County Council area and enclose the range of jurisdiction of the Metropolitan Police, so that this table is not geographically comparable with Mr King's 1911 table. The object of the present study was a comparison between the extended method of Mr King and the shorter ones of Drs Brownlee and Snow, to see whether the methods gave comparable results as regards the expectation of life, the question with which the *public health worker* is mainly concerned. As an example I selected the County of London (census population for 1921, deaths in 1920, 1921, and 1922) as affording, at the same time, an opportunity of making a table comparable with that which Mr King prepared for 1911. Drs Brownlee and Snow have already shown that their methods gave very satisfactory results as judged by the older life tables of this country. This paper will show that similar results have, in the main, been obtained as regards the 1921 experience.

The English life tables No. 8 (1911), prepared by Mr King, and No. 9 (1921), prepared by Sir Alfred Watson, were both based upon the population at the respective censuses and the deaths in the three years, centred on the year in which the census was taken. It was considered that this method gave a truer indication of the prevailing mortality experience of the community, in which the duration of life has been increasing continuously for several decades, than the earlier method of basing the table "upon two censuses and the deaths of the intervening ten years," which was followed in the preparation of No. 7 and the earlier life tables. For the same reason, as well as for the purpose of rendering the results comparable, it was desirable that the methods employed in the construction of the present life table should approximate, as far as possible, to those on which the 1911 and 1921 English tables were prepared.

For the extended table which I have constructed, I adopted Mr King's methods, a full explanation of which was given in the *Report of the Registrar-*

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General mentioned above. In view of the remarks of Sir Alfred Watson regarding the fluctuations of the birth-rate during, and immediately succeeding, the period of the war, it was thought necessary to adopt his method for working out the q_x* at the individual ages between 0 and 4 years. As the quarterly numbers of births for the separate sexes were not available for the County Council area, it was assumed that the sex ratio as given by the annual figures held good during the quarters. On this basis an estimate was made of the number of male births in each quarter. Subject to this limitation, the q_x has been worked out in accordance with the formulae given by Sir Alfred Watson for the ages 0, 1, 2, 3 and 4. He used the same method for age 5 also, but as separate figures for deaths at this age were not available for London, I have had to depend on the methods of graduation, to be mentioned later, for the q_x at this age. At the same time, another method was tested. Births from 1914 onwards were taken, and, by deducting the number of deaths among infants at the respective ages, populations aged 0, 1, 2, 3 and 4 at the beginning of 1920, 1921 and 1922 were arrived at. Thus, the arithmetical mean of the number of births in 1914 and 1915 was taken to represent the population of infants, aged 0, on January 1st, 1915 and, by subtracting from it the number of deaths of children, aged 0–1 in 1915, 1–2 in 1916, 2–3 in 1917, 3–4 in 1918, and 4–5 in 1919, the population entering on age 5 on January 1st, 1920 was calculated. By a similar process the populations entering on ages 0, 1, 2, 3 and 4 at the beginning of each of the three years, 1920, 1921 and 1922 were calculated. The total number of deaths of children at ages 0–1, 1–2, 2–3, 3–4 and 4–5 during the same years (1920, 1921, 1922), when expressed as a ratio of the populations calculated, gave the q_x for each of the respective ages. This method ignores immigration and emigration, but at these ages the error is probably too small to have any appreciable effect upon the results.

Two life tables were worked out, one utilising the q_x as found by the method just described and the other by Sir Alfred Watson's formulae. Table I shows the difference in expectation of life produced by the two methods. Sir Alfred Watson's method gives a very slightly longer mean length of life in each case. At birth it amounts to 0·12 of a year, nearly one month and thirteen days, but at the subsequent ages it is very much less. By the ninth year the difference has disappeared. From the fifth year onwards the deaths and the populations were grouped into quinquennial age periods up to age 100, and then in a single group of 100 and over. Utilising the methods described by Mr King these figures were graduated so as to secure central pivotal values of populations and deaths at the ages, 12, 17, etc., to 97. These population figures, on the addition of half the graduated deaths, gave l_x, the population at exact ages x, and the ratio of the graduated deaths to l_x gave the q_x at the respective quinquennial pivotal points, 12 to 97. By osculatory interpolation the value of q_x was obtained for the ages between 17 and 92. To complete the

* This and the similar terms are explained at the end of the paper.

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values for the period 5 to 16 the known values of q_x at 3, 4, 12, 17 and 18 were taken and Lagrange's formula was used for calculating q_x's at ages 5, 6 and 7. Next, taking values at ages 4, 5, 6, 7 and 12 as u_1, u_2, u_3, u_4 and u_9 the intervening values of u_x were worked out on the assumption that the fifth and subsequent differences were negligible. Similarly, with the known values of q_x at ages 9, 10, 11, 12 and 17 the q_x's at ages 13, 14, 15 and 16 were calculated, the same process also being utilised for supplying these values at ages beyond 92. At 104 it was found that the value of q_x became negative and hence the life table has stopped with age 103.

Table I. *Comparison of expectation of life as obtained by Watson's method and an alternative method. Males. Administrative County of London.*

Age	Watson's Method (*a*)	Alternative method (described in the text) (*b*)	(*a*) − (*b*)
0	54·28	54·16	Nearly 1 month and 13 days
1	58·35	58·33	,, 7 days
2	58·98	58·96	,, 7 ,,
3	58·77	58·76	,, 3·5 ,,
4	58·25	58·22	,, 11 ,,
5	57·59	57·56	,, 11 ,,
6	56·83	56·80	,, 11 ,,
7	56·00	55·98	,, 7 ,,
8	55·12	55·11	,, 3·5 ,,
9	54·22	54·22	—
10	53·31	53·31	—

Mr King's extended method gives a smooth graduation and the process of construction has been so fully explained by him that there is little or no difficulty in understanding and applying it. It must be admitted, however, that it is a laborious process and, in this respect, the shorter methods devised by Drs Brownlee and Snow have their advantage. It is proposed, first, to indicate briefly the basis of these two methods and then to illustrate by tabular presentation the close relationship between the results deduced from them and those obtained by the extended method.

There are some points in common between these two short methods. The authors utilised the data given by the life tables, prepared by the extended method, as their basis and constructed empirical formulae which yield the expectation of life at the definite age periods with only a small margin of error from the figures in the extended tables. Dr Snow devised two functions, ${}_np_x$ and ${}_nk_x$. The former expresses the probability of living for a period of n years after attaining age x, namely, $\frac{l_{x+n}}{l_x}$ in terms of the death-rate for the age group. He evolved functional relationships, by the method of least squares, between the death-rate and ${}_np_x$, which, in some parts of the life table, were linear and in others parabolic of the second order. Towards the end of the life table, as the population rapidly decreased to extinction, the relationship was best expressed by a hyperbolic function of the form

$$p = ae^{-br},$$

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so that the curve was made to asymptote to the x-axis. ${}_nk_x$ is defined by the relation

$$ {}_nk_x = \frac{l_x + l_{x+1} + \dots l_{x+n-1}}{l_x}. $$

Dr Snow established, again by the method of least squares, a relationship between ${}_np_x$ and ${}_nk_x$. Thus, from known values of the former, the latter could be deduced. Tables have been prepared, which cover the entire field of death-rates likely to be met with, and have been published along with Dr Snow's lucid description of the whole process in Part II of the supplement to the *75th Annual Report of the Registrar-General*, 1914. The construction of life tables by this method has, therefore, been made sufficiently easy to be safe in the hands of public health workers.

Dr Brownlee first established that there existed a linear relationship between life table death-rate and standardised death-rate at the respective age periods. He worked at first with twelve tables which, "being constructed on different plans, probably represented varying degrees of accuracy." Mr Finch of the General Register Office subsequently repeated this work, utilising data from more recent life tables and evolved equations of relationship, which gave closer approximation. In his later work Dr Brownlee made use of this material and, at the same time, discovered that the use of a standard population, dying out in arithmetical progression, gave results which were eminently satisfactory.

For a full discussion of this method reference may be made to Dr Brownlee's paper in the *Journal of Hygiene*, XIII, 178 (July, 1913) on "Studies in the meaning and relationships of birth- and death-rates," and to his report to the Medical Research Council on "The use of death-rates as a measure of hygienic conditions" (*Special Report Series*, No. 60).

By applying either of these short methods the calculation of the expectation of life at different age periods will not take more than a couple of hours. However, it must be pointed out that there is a difference between the two short methods in that Dr Snow's method gives not only the expectation of life but also the number of survivors at quinquennial and decennial periods. Dr Brownlee's formulae apply solely to one particular section of a life table, *e.g.* the evaluation of the life table death-rate; knowing this function we can obtain the expectation of life from the relationship

$$ E_x = \frac{1000}{\text{Life table death-rate}}. $$

For the sake of comparability the q_0 of Dr Snow's method has been taken to be the same as that calculated for the extended table by Sir Alfred Watson's formula. In Dr Brownlee's method it is the death-rate for the age group 0–4 that is used. The figure has been obtained from the same formulae. It may be seen that, on the whole, the expectation of life is given with a sufficient degree of accuracy for most public health purposes, except perhaps at birth and beyond 65. It has been stated by Dr Snow that, for early ages, even the extended method gave only results of doubtful accuracy.

To estimate the accuracy of the short methods when applied to the mortality of the whole country Brownlee's and Snow's equations were applied to the requisite mortality data for males in England and Wales during 1920–22. The results as will be observed in Table III are in close accord with those obtained by Sir Alfred Watson in his extended life table for the same period.

Table II. *Comparison of expectation of life obtained by Watson's extended method and the short methods of Brownlee and Snow. Males. Administrative County of London*, 1921.

Age	Extended method	Dr Snow's method	Dr Brownlee's method	Snow's deviation from extended method	Brownlee's deviation from extended method
0	54·28	53·47	53·08	−0·81	−1·20
5	57·59	57·25	57·25	−0·34	−0·34
10	53·31	53·25	53·33	−0·06	+0·02
15	48·81	48·76	48·82	−0·05	+0·01
20	44·52	44·46	44·46	−0·06	−0·06
25	40·30	40·24	40·25	−0·06	−0·05
35	31·91	31·83	31·89	−0·08	−0·02
45	24·08	24·01	24·07	−0·07	−0·01
55	16·93	16·98	17·05	+0·05	+0·12
65	10·94	10·98	11·07	+0·04	+0·13
75	6·38	6·31	6·52	−0·07	+0·14
85 and upwards	3·60	3·33	—	−0·27	—

Table III. *Comparison of expectation of life obtained by Watson's extended method and the short methods of Brownlee and Snow. Males. England and Wales*, 1921.

Age	Extended method	Dr Snow's method	Dr Brownlee's method	Snow's deviation from extended method	Brownlee's deviation from extended method
0	55·62	56·07	55·04	+0·45	−0·58
5	58·81	59·28	58·89	+0·47	+0·08
10	54·64	55·17	54·95	+0·53	+0·31
15	50·12	50·21	50·33	+0·09	+0·21
20	45·78	45·89	45·96	+0·11	+0·18
25	41·60	41·63	41·72	+0·03	+0·12
35	33·25	33·28	33·38	+0·03	+0·13
45	25·22	25·26	25·34	+0·04	+0·12
55	17·73	17·77	17·86	+0·04	+0·13
65	11·36	11·39	11·50	+0·03	+0·14
75	6·59	6·55	6·77	−0·04	+0·18
85 and upwards	3·72	3·55	—	−0·17	—

In both tables Dr Snow's method gives a better fit for some age periods and Dr Brownlee's for others.

It is interesting to compare the increase in expectation of life that the County of London attained between the years 1911 and 1921. The 1911 figures have been taken from Mr King's table. It starts with age 12 and a comparison of the earlier years is, therefore, impossible. It may be seen that there has been a definite increase at every age except 85, when there is a slight decrease. It has been said that the accuracy of the mortality figures at the later age periods is questionable and this reason may explain the decrease.

Table IV. *Comparison of the expectation of life of Males, Administrative County of London, in* 1911 *and* 1921.

Age	1911	1921	Increase of expectation 1921 – 1911
12	49·43	51·49	2 years
15	46·74	48·81	2 years
20	43·35	44·52	1¼ years
25	38·06	40·30	2¼ years
35	29·84	31·91	2 years
45	22·45	24·08	1½ years
55	15·95	16·93	1 year
65	10·51	10·94	5 months
75	6·28	6·38	1 month
85	3·71	3·60	Decrease of 1 month
95	2·19	2·38	Increase of 2 months

Lastly, I must acknowledge the help and guidance which Mr W. T. Russell, of the Epidemiological Section of the London School of Hygiene and Tropical Medicine, has so generously given me throughout the preparation of this life table.

In Table IV the values of the expectation of life as obtained by the extended method have been given only for specific ages. In case the values are required for individual ages Table V gives a detailed presentation of the facts.

Table V. *Life table. London Administrative Area. Males*, 1920–22.

Age	l_x	d_x	Complete expectation of life	Age	l_x	d_x	Complete expectation of life
0	1,000,000	86,122	54·28	29	804,606	3,398	36·91
1	913,878	25,008	58·35	30	801,208	3,541	36·06
2	888,870	11,842	58·98	31	797,667	3,711	35·22
3	877,028	7,157	58·77	32	793,956	3,909	34·38
4	869,871	5,081	58·25	33	790,047	4,151	33·55
5	864,790	3,603	57·59	34	785,896	4,432	32·73
6	861,187	2,589	56·83	35	781,464	4,731	31·91
7	858,598	1,937	56·00	36	776,733	5,029	31·10
8	856,661	1,563	55·12	37	771,704	5,305	30·30
9	855,098	1,400	54·22	38	766,399	5,546	29·50
10	853,698	1,390	53·31	39	760,853	5,763	28·71
11	852,308	1,485	52·40	40	755,090	5,980	27·93
12	850,823	1,645	51·49	41	749,110	6,401	27·15
13	849,178	1,838	50·58	42	742,709	6,485	26·38
14	847,340	2,040	49·69	43	736,224	6,784	25·61
15	845,300	2,233	48·81	44	729,440	7,096	24·84
16	843,067	2,407	47·94	45	722,344	7,434	24·08
17	840,660	2,560	47·08	46	714,910	7,809	23·32
18	838,100	2,697	46·22	47	707,101	8,232	22·58
19	835,403	2,819	45,37	48	698,869	8,699	21·84
20	832,584	2,924	44·52	49	690,170	9,197	21·10
21	829,660	3,013	43·67	50	680,973	9,730	20·38
22	826,647	3,084	42·83	51	671,243	10,298	19·67
23	823,563	3,120	41·99	52	660,945	10,902	18·97
24	820,443	3,123	41·15	53	650,043	11,536	18·23
25	817,320	3,118	40·30	54	638,507	12,192	17·60
26	814,202	3,131	39·45	55	626,315	12,871	16·93
27	811,071	3,185	38·60	56	613,444	13,573	16·28
28	807,886	3,280	37·75	57	599,871	14,296	15·64

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Age	l_x	d_x	Complete expectation of life	Age	l_x	d_x	Complete expectation of life
58	585,575	15,029	15·01	82	92,488	15,524	4·27
59	570,546	15,761	14·39	83	76,964	13,842	4·03
60	554,785	16,494	13·78	84	63,122	12,156	3·81
61	538,291	17,227	13·19	85	50,966	10,495	3·60
62	521,064	17,959	12·61	86	40,471	8,894	3·40
63	503,105	18,641	12·04	87	31,577	7,387	3·22
64	484,464	19,259	11·48	88	24,190	6,029	3·05
65	465,205	19,864	10·94	89	18,161	4,828	2·90
66	445,341	20,496	10·40	90	13,333	3,768	2·76
67	424,845	21,191	9·88	91	9,565	2,852	2·65
68	403,654	21,954	9·38	92	6,713	2,086	2·57
69	381,700	22,716	8·89	93	4,627	1,480	2·50
70	358,984	23,395	8·42	94	3,147	1,023	2·44
71	335,589	23,914	7·97	95	2,124	696	2·38
72	311,675	24,207	7·54	96	1,428	470	2·29
73	287,468	24,276	7·13	97	958	319	2·18
74	263,192	24,141	6·75	98	639	219	2·01
75	239,051	23,782	6·38	99	420	153	1·80
76	215,269	23,188	6·03	100	267	108	1·54
77	192,081	22,356	5·69	101	159	76	1·25
78	169,725	21,311	5·38	102	83	51	0·95
79	148,414	20,082	5·08	103	32	27	0·66
80	128,332	18,688	4·79	104	5	—	—
81	109,644	17,156	4·53				

KEY TO THE NOTATION.

q_x: The probability of dying in a year. The ratio of the total number of deaths in the year, x, to the population entering on that year.

l_x: The population attaining age x at the beginning of the year.

${}_np_x$: The probability of living n years for the population entering on age x.

${}_nk_x$: $\dfrac{l_x + l_{x+1} + \dots l_{x+n-1}}{l_x}$.

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