SYNCHROTRON THERMAL INSTABILITIES AND RADIO FILAMENTS IN THE LOBES OF CYGNUS A

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I. *INTRODUCTION*

Recent VLA observations of the lobes of Cygnus A exhibit complex "filamentary" structures, with typical scale width ~ 1 arcsec (Dreher, Carilli and Perley, 1987, Perley, 1987). The filaments appear aligned with the magnetic field, as results from polarization measures, suggesting that the field may play a fundamental role in the process of their formation.

We propose a mechanism for the possible formation of these filaments based upon a thermal instability connected with synchrotron emission from relativistic electrons. This type of instability was studied by Simon and Axford (1967), who discussed it in connection with the Crab Nebula filaments, and by Eilek and Caroff (1979), who generalized the previous study for application to quasar atmospheres.

The treatment followed here assumes in addition that the energetic equilibrium is maintained by continuous replenishment of relativistic electrons streaming from the "hot spot" region. In section II we outline the general picture of the stability process, in section III we derive the conditions and the growth rate of the instability, and in section IV we discuss the application to the formation of filaments in the lobes of Cygnus A.

II. *THE SYNCHROTRON THERMAL INSTABILITY*

Following the approach of Simon and Axford (1967) we consider a two components magnetized plasma, where the inertia is provided by protons and the internal energy by relativistic electrons. Therefore the system can be described by classical MHD equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \,, \qquad (1a)$$

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla P + \frac{1}{c}\mathbf{J} \times \mathbf{B} \,, \qquad (1b)$$

$$\frac{dP}{dt} - \gamma \frac{P}{\rho}\frac{d\rho}{dt} = (\gamma - 1)(\mathcal{Q} - \mathcal{L}) \,, \qquad (1c)$$

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$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) \quad , \tag{1d}$$

where the pressure is given by $P = N\bar{\epsilon}/3$, N is the number density of relativistic electrons, $\bar{\epsilon}$ the average energy per electron and ρ is the mass density of the protons; consistently with our assumptions $\gamma = 4/3$). In addition $\mathbf{B}$ and J are the magnetic field and current density respectively, and Q and $\mathcal{L}$ the absorbed and emitted power per unit volume. If the zeroth-order configuration is chosen in energy balance, the process of instability can be studied by linearizing the MHD equations, and assuming for the perturbed quantities a form $\propto \exp(-ik_x - ik_z + \omega t)$.

In the present astrophysical application, the energy losses are due to the synchrotron emission from relativistic electrons, while the energy input is given by continuous supply of particles into the instability region ("lobe region"). If we assume that relativistic electrons are accelerated in the "hot spot" (which is overpressured with respect to the whole lobe), they diffuse across the lobe region streaming along pressure gradients. In this case the energy balance at each point can be maintained by continuous advection of energetic particles. While in previous works a maxwellian distribution was assumed for electrons (Simon and Axford 1967, Eilek and Caroff 1979), here we discuss a power law distribution as expected in radio lobes.

We model this situation considering a rectangular domain, with uniform magnetic field directed along the z axis, where energetic electrons are injected at one end, and diffuse along the magnetic field with velocity v_0 (of the order of the Alfv̀en velocity V_A), while losing energy by synchrotron radiation. The system of equations (1) can be linearized; in agreement with the previous considerations, and assuming that the equilibrium variables vary only along the z direction, parallel to the magnetic field), one obtains:

$$\rho_0 v_0' = -v_0 \rho_0' \,, \qquad \rho_0 v_0 v_0' = -P_0' \,, \qquad v_0 P_0' - \gamma \frac{P_0}{\rho_0} v_0 \rho_0' = -(\gamma - 1)\mathcal{L}_0 \,, \tag{2}$$

where primes indicates d/dz derivatives, and where we set $Q_0 = 0$, as appropriate in the lobe region, and

$$\mathcal{L}_0 = \sigma_T c \frac{B_0^2}{8\pi} N_0 \bar{\epsilon}_0^2$$

For consistency the scale lengths of the variation of the equilibrium variables must be much larger than the perturbations wavelengths (local instability criterion), i.e. $k_z l_P \gg 1$, where $l_P = -P_0/P_0'$ and

$$l_P = l_{sync} \left(1 - \frac{c_s^2}{v_0^2} \right) \,,$$

with $c_s^2 = \gamma P_0/\rho_0$, and $l_{sync} = v_0 \tau_{sync}$ (τ_{sync} is the time scale of synchrotron losses). By introducing the nondimensional variables:

$$K_{x,z} = k_{x,z} c_S \tau_{sync}, \qquad V_{AS} = \frac{V_A}{c_S}, \qquad V_A^2 = \frac{B_0^2}{4\pi\rho_0},$$

$$L = k_x l_P, \qquad V_0 = \frac{v_0}{V_A}, \qquad V_0' = v_0' \tau_{sync} \,,$$

we get the following conditions for the validity of our approximation:

$$V_0 = \frac{L}{V_{AS}K_x}(1 + \gamma V_0'), \qquad V_0' = \frac{K_x}{LV_0V_{AS}}, \qquad L \gg \frac{K_x}{K_z}, \tag{3}$$

The dispersion relation, obtained annulling the determinant of the linearized MHD equations (1), is a 6^{th} order polynomial:

$$\sigma^6 + a_1\sigma^5 + a_2\sigma^4 + a_3\sigma^3 + a_4\sigma^2 + a_5\sigma + a_6 = 0, \tag{4}$$

with $\sigma = (\omega - ik_z v_0)\tau_{sync}$. The coefficients of the polynomial (see the Bodo *et al.*, (1988) for their general expressions) depend upon the zeroth order configuration, and have the following functional form:

$$a_i(K_x, K_z, V_{AS}, \alpha_N, \alpha_\epsilon, \alpha_B, L)$$

where we have defined

$$\alpha_{N,\epsilon,B} \propto \left[\frac{(N,\epsilon,B)}{\mathcal{L}}\frac{\partial\mathcal{L}}{\partial(N,\epsilon,B)}\right]_o, \tag{5}$$

For a homogeneous plasma ($L \to \infty$) $a_6 = 0$, and Eq. (4) is reduced to a 5^{th} degree polynomial, while, if $K_z = 0$ also, the dispersion relation becomes a third degree polynomial that, for the particular case of $K_x \gg 1$, leads to the following stability condition:

$$\alpha_N - \alpha_\epsilon\left(1 + \gamma V_{AS}^2\right) + \alpha_B > 0, \tag{6}$$

(Simon and Axford 1967).

In this case the unstable mode is a purely growing filamentary perturbation aligned with the magnetic field. In the following Section we shall discuss some numerical roots of Eq. (4) by choosing for the parameters values consistent with the physical conditions in the lobes of Cygnus A.

III. *STABILITY ANALYSIS*

The growth rate of unstable perturbations has been evaluated numerically by means of a NAG routine for the search of the roots of a high order polynomial, and are presented in function of the values of the physical parameters. Concerning the values of α, it is found for synchrotron losses $\alpha_N = 1$, and $\alpha_B = 2$, while the value of α_ϵ depends from the spectrum of relativistic electrons. For a maxwellian distribution with an average energy $\bar{\epsilon}$, is $\alpha_\epsilon = 2$. For a power law distribution $\bar{\epsilon}$ depends from the spectral parameters, namely the maximum (ϵ_{max}) and minimum (ϵ_{min}) energy cut-offs, and the spectral index Γ. The values of α are then evaluated form Eq.(5) by assuming that in the instability process the spectrum is varying only one of the three parameters ϵ_{max}, ϵ_{min}, and Γ. If we assume a typical spectral index $\Gamma = 2.5$, and $\epsilon_{max}/\epsilon_{min} = 10^3$, we have from Eq.(5) (see also Bodo *et al.* 1988) $\alpha_\epsilon \approx 4$ (varying Γ), $\alpha_\epsilon \approx 1.5$ (varying ϵ_{min}) and $\alpha_\epsilon \approx 0$ (varying ϵ_{max}).

The nondimensional wavenumbers K_x and K_z, can be expressed in terms of physical variables:

$$K_{x,z} \approx \frac{10^6}{\lambda_{pc}^{\perp,\parallel}}(\nu_{10} B_{-5}, n_{th_{-4}})^{-1/2},$$

where ν_{10} is the frequency of radio emitting electrons in units of 10 GHz, B_{-5} is the magnetic field in units of 10^{-5} gauss, $n_{th_{-4}}$ is the number density of thermal particles in units of 10^{-4} cm^{-3}, and $\lambda_{pc}^{\perp,\parallel}$ are the components of the perturbation wavelengths, in units of 1 parsec, perpendicular and parallel to the magnetic field direction. From the typical values of the physical parameters of the extended radio lobes, and for perturbations much smaller than the typical sizes of radio components, we must have in general $K_x, K_z \leq 10^{5-6}$. If furthermore is $V_{AS} \sim 1$ (as from equipartition conditions) conditions (3) provide a rough estimate of L: for the results presented here it will be assumed $L = 10^5$

The growth rate ($Re(\sigma)$) is plotted *vs.* K_z for a fixed vale of K_x ($= 10^5$), for some values of V_{AS}, and for three values of α_ϵ: $\alpha_\epsilon = 4$ (Fig. 1), $\alpha_\epsilon = 1.5$ (Fig. 2) and $\alpha_\epsilon = 0$ (Fig. 3). First of all we notice as a general feature for all unstable perturbations, that they are always travelling modes ($Im(\sigma) \neq 0$), as expected from the assumption of a travelling plasma, therefore the ratio K_x/K_z is related with the propagation direction of the perturbation with respect to the magnetic field direction. Three unstable modes are found: the perturbation analyzed by Simon and Axford (1967), modified by the streaming plasma (we call it the condensation mode), and two slow MHD modes (with opposite phase velocities). The former mode has large growth rate for small values of K_z, but it is sharply damped by increasing K_z; conversely the two slow MHD modes have a constant growth rate for a large range of values of K_z, and are damped for K_z approaching K_x. This stabilization is expected taking into account that for $K_x/K_z \rightarrow 0$ the magnetic field perturbation, which drives the instability, tends to vanish. In addition we see that increasing the magnetic field strength tends to stabilize the perturbations since it inhibits the pressure build-up associated with them, more precisely, in the present case, stability is found for $V_{AS} \approx 1$. Finally, the general behavior of the instability is not very sensitive to the spectral parameters, even though the exact value of the growth rate can vary.

III. *SUMMARY*

We can summarize our results as follows:
1) The thermal instability related to the synchrotron emission can be present in the lobes of Cygnus A under a quite wide range of conditions.
2) The typical time scale of development of instability is $\sim \tau_{sync}$.
3) The most unstable modes are found for small values of K_z, i.e. for modes propagating almost perpendicular to the magnetic field direction, which is in accord with the observational data.

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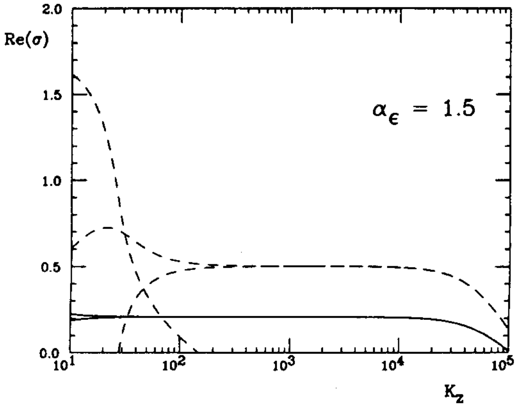

Figure 1: Growth rate ($Re(\sigma)$) as a function of K_z for $K_x = 10^5$, $V_{AS} = 0.1$ (dashed lines), and $V_{AS} = 0.5$ (solid lines), with $\alpha_\epsilon = 4$.

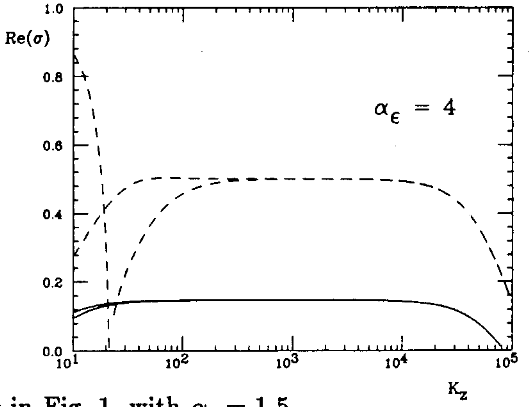

Figure 2: Same as in Fig. 1, with $\alpha_\epsilon = 1.5$.

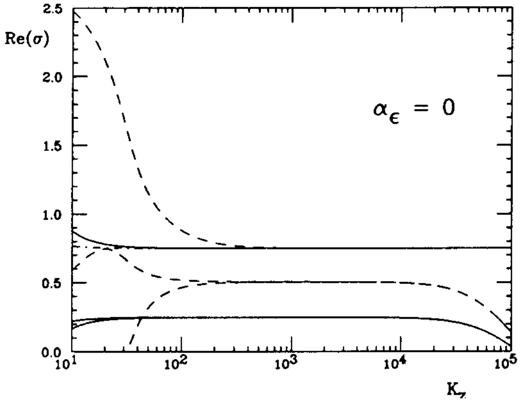

Figure 3: Same as in Fig. 1, the dot–dashed line is for $V_{AS} = 1$, and $\alpha_\epsilon = 0$.

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