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Exoplanets: The tip of the iceberg?

W. Benz[1], Y. Alibert[1], C. Mordasini[1] and D. Naef[2]

[1]Physikalisches Institut, University of Bern, Switzerland

[2]European Southern Observatory, Alonso de Cordova 3107, Casilla 19001 Santiago 19, Chile

Abstract. As the number of large scale ground- and space-bound planet detection and imaging projects is growing, the need for theoretical guidance in order to optimize instrumental design is rapidly mounting. In an effort to provide this required framework, we present the results of Monte Carlo simulations of the formation of giant planets and compare them with the current population of exoplanets. Our models show that due to the severe current observational detection bias only a small percentage (3.6 %) of the potentially existing planets can be detected. Indeed, a large number of planetary embryos never grow enough to become giant planets giving raise to a large populations of bodies with masses smaller than $\simeq$5 M_{jup}. In addition, this observational bias, coupled with the fact that systems enriched in heavy elements tend to form more massive planets, explains the currently observed correlation between stellar metallicity and likelihood to host planets. Finally, we show that the disk gas delivery rate during the late stages of formation actually determines the maximum planetary mass.

Keywords. Planet formation, core accretion, planetary mass distribution, metallicity.

1. Introduction

At the time of this Colloquium, roughly ten years after the first announcement by Mayor and Queloz (1995), 168 exoplanets have been discovered. From the beginning, they have shattered the understanding of the formation of planetary systems that had been patiently constructed based upon the study of a single example: our own solar system. The diversity amongst the new systems by baffling the earlier concepts has taught us how dangerous it can be to build theories on too small a data set. At a time when searching for life *as we know it* becomes possible, this comes as a serious reminder to keep our approaches as unbiased and open as possible.

The lack of good theoretical understanding creates two major problems. First, we cannot explain what is observed and second, we cannot predict what should be observed. The latter is especially severe at a time when major new ground- and space-based instrumentation is being discussed to detect and image smaller planets and possibly search for life. Progress in understanding has been difficult for many reasons. First and foremost, the formation and evolution of planet is a difficult subject involving large changes of scales, long timescales and many non-linearities and feed-back mechanisms. Furthermore, the topic is multi-disciplinary involving astronomy, cosmo-chemistry, material sciences, planetary sciences, climate physics, and, if one is looking for life, biology. Unfortunately, a tradition of exchange and collaboration between these fields is still lacking and more often than none, studies are carried out in one field ignoring the constraints from stemming from the others. Progress therefore relies on a better pooling of ressources, better multi-dsciplinary collaborations and, of course, more data and new bright ideas.

While over 168 exoplanets may seem a lot, the constraints they impose on formation theories are not easily usable in a quantitative fashion. Indeed, in most cases, these planets having been detected by indirect methods (radial velocity techniques essentially), we only know the orbital parameters and a minimum mass. In the few cases where the planet is actually transiting the star, more constraints can be derived to check structure models but since these planets may have been at least partially evaporated, their current characteristics are not necessarily directly related to their formation epoch. Short of direct imaging and the corresponding spectroscopic analysis, the only possibility of extracting constraints on formation models is a statistical approach in which the "observed" population is compared to synthetic populations computed under various assumptions. Differences can be quantified statistically and model parameters adjusted as to minimize differences.

We present here some first results of such a statistical approach and show that already promising results can be obtained. We present our model in section 2. Particular attention is given to the choice of initial conditions (section 2.2) which we chose based as much as possible on observations. We also compute the expected observational detection biases (section 2.3) and correct our synthetic planet populations before comparing them to the actual data. Finally, we present some of our results in section 3 with a particular focus on the differences between the underlying planet population and the fraction actually detectable for those are especially interesting for the design of the new generation of instruments. In particular, we show that the currently detected population of exoplanets may only be the very tip of the iceberg of a much larger populations of planets whose masses are too small to be detectable with the current radial velocity search techniques.

2. Theoretical approach

In this section we briefly explain how we generate various populations of synthetic planets which we then compare to actual observations. We discuss the formation model itself (section 2.1), the way in which we generate initial conditions (section 2.2) and finally the corrections involved in order to take into account the observational detection biases (section 2.3).

2.1. *Giant planet formation model*

We compute a population of synthetic planets within the framework of the core accretion scenario as it is currently the only formation model allowing quantitative comparisons to be made with observations. In this scenario a solid core surrounded by a tenuous gaseous envelope is first formed by the accretion of solid planetesimals which themselves were formed by sedimentation and coagulation of small dust grains (Wetherill & Steward 1989, Lissauer 1993). As the core grows beyond a critical mass (of the order of 15 $M_{\oplus}$ at 5 AU, but depending on different physical parameters), radiative losses can no longer be offset by planetesimal accretion and gas accretion runs away leading to the very rapid build up of a massive gaseous envelope (Pollack *et al.* 1996).

Recently, we have extended this model (Alibert *et al.* 2005a) to take into account both the evolution of the proto-planetary disk (gas and solid phase) and the migration of the forming planet (due to the momentum transfer between the planet and the gaseous disk). We showed that an important consequence of including these effects is to drastically reduce the formation timescale (Alibert *et al.* 2004) bringing it to an excellent agreement with the observationally derived lifetime of proto-planetary disks (Haisch *et al.* 2001). The numerical tools we have developed, as well as the tests we have performed to validate them can be found in Alibert *et al.* (2005a).

We have also used our approach to study the formation of the giant planets in our own solar system. The wealth of detailed *in situ* and remote measurements existing for Jupiter and Saturn makes them mandatory benchmarks for all formation models.

We have been able to show that our approach is capable of satisfying all the available relevant constraints for both planets within the same self-consistent formation model (Alibert, Mousis, & Benz 2005; Alibert *et al.* 2005b). To the best of our knowledge, this is the first time that a single formation model is capable of accounting quantitatively for all these characteristics.

2.2. *Initial conditions*

To allow a meaningful statistical comparison between our synthetic planet populations and the actually observed population, it is necessary to specify not only the initial conditions but also the probability of occurrence of each one. To keep the calculations tractable, we have limited the number of variables we vary to five: the proto-planetary disk mass, the metallicity (dust to gas ratio), the initial location of the planetary embryo ($0.6\,M_\oplus$ in mass), the rate of photo-evaporation and the age of the disk when the planetary embryo is created. In addition, we have also varied the gas mass accretion rate allowed through the gap. In one set of calculation we limit this accretion rate to the value derived by Veras & Armitage (2004) and in the other case, we allow for the maximum rate provided by the viscosity (α parameter) of the disk. As we shall see below, this gas accretion rate through the gap directly determines the maximum mass a planet can reach.

All other parameters (see Alibert *et al.* 2005a for a detailed description of our model) are kept constant, in particular the stellar mass ($1M_\odot$), the initial surface density distribution of the gas and solids (power law of index $-3/2$), the dissipation parameter ($\alpha = 0.002$), the reduction factor for type I migration ($f_I = 0.001$). Because the computation of a single model can take up to 2 to 3 hours of CPU time, we build a large 5 dimensional master table from $60'480$ simulations ($9 \times 4 \times 20 \times 21 \times 4$ respectively in the independent variables mentioned above). We then use a Monte Carlo approach to generate initial conditions as described below and interpolate the corresponding results linearly in our master table. Notice that in our calculations we always assume that one and only one planetary embryo forms in orbit around a given star.

Given a fixed initial surface density distribution $\Sigma(r, t = 0) = \Sigma_0 (a/a_0)^{-3/2}$, the mass of the initial gaseous disk in terms of Σ_0 is easily obtained once a_0, the inner (a_{in}) and outer (a_{out}) edge of the disk are specified. We chose $a_0 = 5$ AU, $a_{in} = 0.25$ AU, and $a_{out} = 50$ AU and generate disk masses at random varying Σ_0 from 100 g/cm^2 to 900 g/cm^2 with a probability of occurrence given by the mass distribution of circumstellar disks derived from observations (Beckwith & Sargent 1996). The disks obtained span a mass range going from about $0.008\,M_\odot$ to about $0.07\,M_\odot$. Following Murray *et al.* (2001), we relate the dust to gas ratio f_{dg} to the metallicity Fe/H of the disk (it is assumed that initially the star and the disk have the same composition) by the relation $Fe/H = log(f_{dg}/f_{dg\odot})$, where $f_{dg\odot} = 0.0167$ is the dust to gas ratio corresponding to solar composition. We vary f_{dg} from 0.00667 to 0.0333 using the metallicity distribution of stars in the solar neighborhood (Nordström *et al.* 2004) to define the probability of occurrence of a given disk. The resulting disks span a range of metallicity going from about $Fe/H = -0.4$ to $Fe/H = 0.3$.

The distribution of the starting position of planetary embryos is not constrained by observations and only theoretical arguments can be used. Here we follow Ida & Lin (2004a) and assume a uniform distribution in $log(a)$ between 1 AU and 20 AU for the starting positions. For a lack of better constraints, the distribution of the photo-evaporation rates is taken to be uniformly distributed between $3 \times 10^{-9}\,M_\odot$/y and $1.5 \times 10^{-8}\,M_\odot$/y (see Clarke *et al.* 2001; Armitage *et al.* 2003; Veras & Armitage 2004). These values together with our choice of the dissipation parameter α yield disk ages almost linearly decreasing between 0 and 8 Myr in good agreement with the observed age distribution (Haisch *et al.*

2001). Finally, the starting times of our embryos are taken uniformly distributed between 0 and 1 Myr after the start of disk evolution. Allowing embryos to start later results in lower mass planets thus further reducing the fraction of detectable planets. For example, starting times taken uniformly between 0 and 4 Myr, results in only 2 % of detectable planets, a number probably too small compared to actual detection rates.

2.3. *Detection biases*

A working planet formation model is not enough to statistically compare the computed population of synthetic planets with the one actually observed. To compare both populations it is also necessary to understand the various detection biases entering in the observational process and correct for them before comparison. This is not an easy task as these biases are technique and instrument (and probably also observer) dependent. Since the overwhelming majority of planets has been detected using radial velocity techniques, we shall focus here on biases affecting this detection technique.

To first order, in radial velocity searches the detection probability goes down with decreasing planetary mass and/or increasing distance to the star following Kepler's law and the instrumental accuracy determines whether the body can be detected or not. In fact, a large number of other quantities also play a role such as the magnitude of the star, its metallicity and rotation rate, orbital parameters of the planet, actual measurement schedule, jitter, etc. Using the method developed by Naef (2004, 2005) for the spectrograph Elodie, we computed for each planetary mass (in a grid ranging from 0.025 to 40 M_{jup}) and period (in a grid ranging from 35 to 38′000 days) 50′000 randomly chosen planetary orbits and determined from a χ^2 analysis the fraction of this number that can actually be detected. This fraction represents the detection probability corresponding to this planetary mass and orbital period.

We illustrate the results of these calculations in Fig. 1 in which we plot the detection probability assuming a instrumental accuracy of 5 m/s of a number of planets with masses ranging from 10 to 5000 $M_{\oplus}$. Note that with this instrumental accuracy we can expect that nearly all planets with masses larger than 1000 $M_{\oplus}$can be detected out to 4 AU while the discovery probability of a Jupiter mass planet drops below 50% beyond 3 AU.

We use these probabilities to determine which planets from a given synthetic populations are actually detectable and compare only those with the observed ones.

3. Results

In this section we present some results from our planet population simulations. In all cases, we have generated an ensemble of 250′000 planets. Note that our simulations always start with one and only one embryo with a mass of 0.6 $M_{\oplus}$. Hence, our simulations do not address multiple systems nor the question of whether a given system can grow embryos of this size. Here we simply assume that they can grow regardless of the mass, age or metallicity of the disk. We show the importance of the detection biases and compare the fraction of synthetic planets actually detectable to the one observed (section 3.1). We show that this detectable fraction is actually a function of the content in heavy elements. This dependance turns out to explain the correlation between planet detection probability and host star metallicity (section 3.2). Finally, we compare the mass distribution obtained with observation and illustrate the effect of limiting the gas accretion rate that flows through the gap on the characteristics of the final planets (section 3.3).

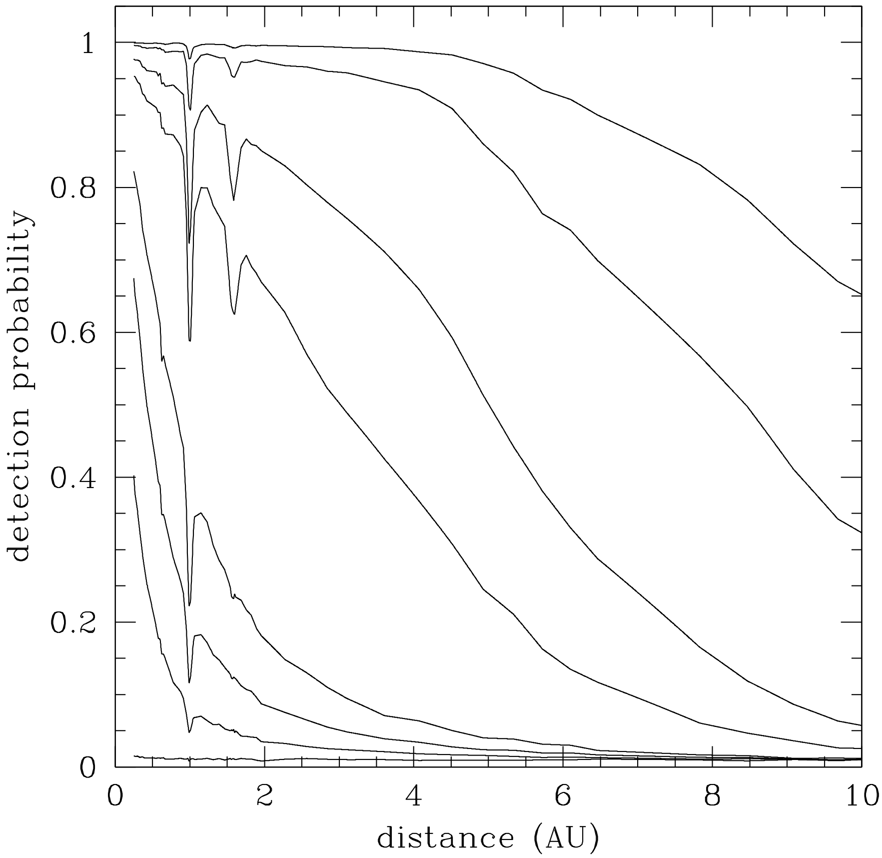

Figure 1. Detection probability as a function of distance to the star assuming an instrumental accuracy of 5 m/s. The various curves are for different planet masses: $10, 50, 100, 200, 300, 500, 1000, and 5000\ M_\oplus$ from bottom to top.

3.1. *Existing and detectable planets*

Using the method described in section 2.2, we have generated a population of 250′000 planets using the maximum gas delivery rate provided by the disk's viscosity (no limitations due to the gap). We have then determined which of these planets are potentially detectable using the probabilities calculated in section 2.2. The resulting distributions are show in Fig. 2.

The most striking feature in this figure is clearly that the detectable planets represent only a small fraction of the underlying planet population since of the 250′000 bodies actually only 8′948 or 3.6% are detectable. Note that this fraction depends on the starting time distribution of the embryos. Those which are starting late generally remain small and are therefore, on average, not observable thus reducing the fraction of detectable planets. The numbers here correspond to embryos starting within the first million year of the disk. Allowing for embryos to start within the first 4 Myr of disk evolution leads to a fraction of detectable planets of the order of 2%.

A further inspection of Fig. 2 reveals that although our models reproduce reasonably well the overall planetary mass distribution (see section 3.3 below), they seem unable to reproduce the proper radial distribution. The paucity of synthetic planets between

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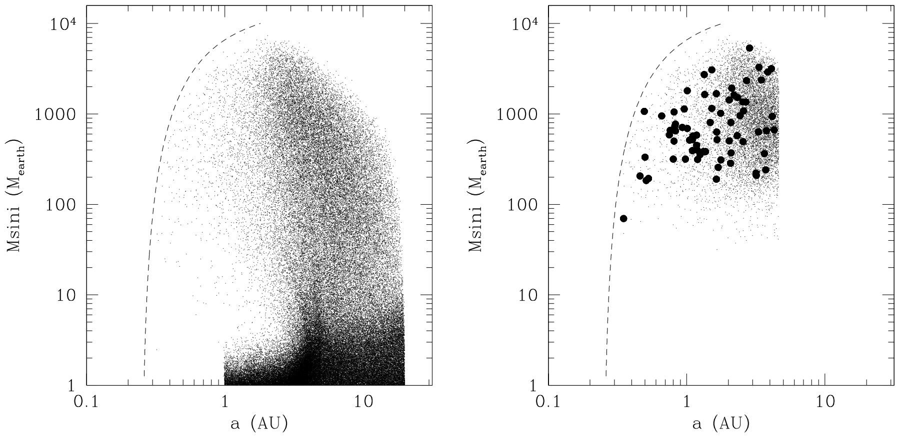

Figure 2. Actual population of synthetic planets (left panel) compared to the observable population assuming a 5 m/s instrumental accuracy (right panel). The actually observed population is shown as big dots. The sharp cutoff at about 5 AU corresponds to about a 10 year period, the actual observation baseline. To avoid inner boundary problems, we only considered planets with feeding zones (four times the Hill radius on either side) not reaching the inner edge of the disk located at 0.25 AU).The dashed line indicates the loci where the planet's feeding zone reaches this inner boundary.

$0.5 - 2$ AU can have different origins. First, it is possible that the detection biases are worse than what has been computed here and that the probability of detecting planets of several Jupiter masses located at $2 - 5$ AU from their parent star is actually significantly lower than the $20 - 40\%$ used here (for example, some of the early instruments did not have 5 m/s accuracy). The second possibility is that the orbital migration of the growing planets is underestimated in our models. This hypothesis which is currently being examined can be used to illustrate the potential power of such a Monte Carlo approach in discussing the different processes governing planet formation.

3.2. *Correlation with metallicity*

We have seen in the previous section that the overall percentage of detectable planet is quite small. Here we show that this fraction is actually a strong function of the assumed heavy elements content (metallicity) in the system at the beginning of planet formation. We recall that our simulations were started with a metallicity distribution corresponding to the one for F and G stars in the solar neighborhood. Hence, we can compute the detection probability as a function of metallicity by simply computing the ratio between detectable to existing planets in fixed metallicity bins and compare the results directly with observations. The results of this computation are show in Fig. 3 in which we also plotted the observationally determined probability as given by Marcy *et al.* (2005).

The explanation of this correlation is straightforward and is due to two effects. First, we find that, within the framework of our models, metal rich systems favor the formation of massive planets. This is quite intuitive as for these systems the core reaches the critical mass earlier therefore allowing more gas to be accreted before the disappearance of the disk. The second effect is simply related to the fact that the detection of massive planets is actually easier than the detection of smaller ones. We recall that in our models we assumed that all systems regardless of their metallicity can form embryos of 0.6 $M_{\oplus}$. It is therefore the subsequent evolution of these embryos coupled to the detection biases that is at the origin of the correlation. We cannot exclude that the formation of these

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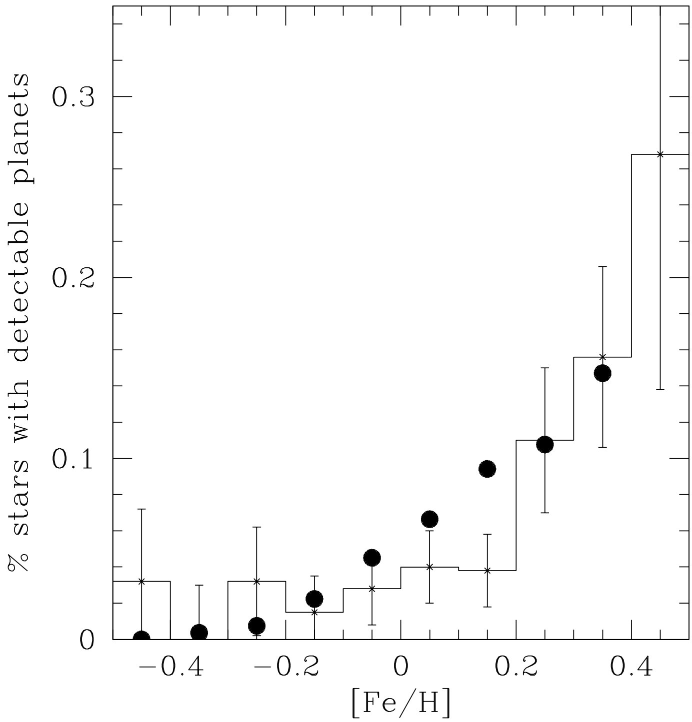

Figure 3. Percentage of stars with detectable planets as a function of the metallicity of the host star. Histogram: Observations (from Marcy *et al.* 2005); Black dots: Present models.

embryos is in itself a function of metallicity. However, we note that if this were the case, the correlation should even be stronger than the one observed. The future detection of smaller and smaller mass objects will allow us to address this issue more quantitatively.

3.3. *Mass distribution of planets*

The actual mass distribution of the planets shown in Fig. 2 is displayed in Fig 4. We again separate the existing planet mass distribution (left panel) from the detectable planet distribution (right panel). In the latter case, we also plotted the distribution of the "observed" planets to allow for comparison. In addition we also computed the mass distribution obtained using a different formation model in which the gas accretion rate is limited in the presence of a gap to a maximum value which is a function of the planet's mass (Veras & Armitage 2004).

We notice again that the population of planet that are actually detectable represents only a small fraction of all the planets forming in our models. We point out that these models start with seed embryos of $0.6\,M_{\oplus}$ and therefore cannot be considered as reliable Earth-like planet formation models. However, the presence of low mass planets can be taken as an indication that our formation model predicts that a large fraction of these embryos will not be able to grow substantially even less become giant planets. Hence, it is an indication that terrestrial like planets should be in fact quite more frequent than giant planets.

Fig. 4 also shows that the rate at which the disk can feed the planet during the late stages of formation determines directly the maximum mass that can be reached. The maximum mass is reached using the gas delivery rate set by the disk's internal dissipation

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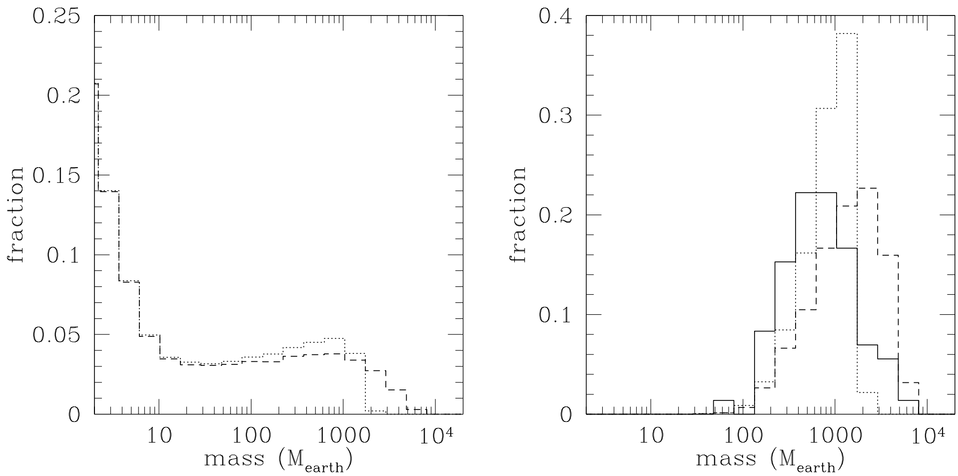

Figure 4. Mass distribution of planets. The mass distribution of the actual planets is shown on the left panel while the one of detectable planets is shown on the right (with a different normalization). In both panels, the dashed line represents the models in which the maximum gas delivery rate to the planet is given by the disk while models where this rate is limited by the gap (Veras & Armitage 2004) are shown by a dotted line. Finally, the solid line in the right panel shows the distribution of the actually detected panels that fall in the domain in which models have been calculated.

mechanism (dashed line). If the formation of a gap is limiting the amount of gas the disk can deliver to the planet, the maximum planetary mass will be correspondingly reduced. To show this, we computed also models in which the gas delivery rate is reduced due to the presence of a gap (dotted line) as given by Veras & Armitage (2004). We notice that there is a difference in the maximum mass reached of almost a factor 5.

Finally we note that when comparing our mass distribution to the “observed” one, the agreement is not too bad even though our models seem to predict systematically more massive planets. More work will be required to pinpoint the origin of this discrepancy.

4. Conclusions

Extended core accretion models allow for a quantitative comparison with observations. While there are still important discrepancies between computed and observed planet populations, this statistical approach is promising and allows already to explain some of the observed trends. The correlation between stellar metallicity and likelihood to host planets can be explained in terms of an observational bias connected to the detection method (radial velocity techniques are most sensitive to large mass objects) and the fact that systems with more heavy elements tend to form more massive planets. Finally, our models show that the maximum mass a giant planet can reach is determined by the amount of gas the disk can deliver to the planet during the runaway growth phase. This delivery rate is set by the flow of gas through the gap as well as by possible disk instabilities occurring during these late stages.

Our calculations also indicate that a large fraction of planetary embryos do not grow and become giant planets detectable by the current radial velocity searches. Hence, for every currently detected exoplanet there should exist of order $20-30$ times more “failed” giant planet cores with masses smaller than $\simeq 5\ M_{\oplus}$. The currently known population of exoplanets appears therefore to be truly only the tip of the iceberg.

Acknowledgements

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