

If, on the other hand, we adopt the second alternative together with the additional postulate that the δ 's are statistically independent and are all distributed with the same probability law (3), suggested by Dr. Primrose, then it can be proved $F(x)$ has the form

$$F\left(\frac{1}{r_1 + r_2 + \dots}\right) = \frac{1}{r_1} \left\{ 1 - \frac{1}{(r_1 + 1)r_2} \left\{ 1 - \frac{1}{(r_2 + 1)r_3} \left\{ 1 - \dots \right. \right. \right. \quad (5)$$

The function given by (5) is a very pathological one, as the following properties show:—

- (i) $F(x)$ is continuous and $F(0) = 0$ and $F(1) = 1$.
- (ii) For almost all values of x , $F(x)$ is differentiable and its derivative equals 0.
- (iii) $F(x)$ is rational if x is rational and also if x is a quadratic surd.
- (iv) If x is rational, $F(x)$ has a left-hand and a right-hand derivative at x ; and these two one-sided derivatives are equal if and only if the final denominator in the continued fraction form of x equals 3.

In (ii) the phrase (almost all x) means "all x which do not belong to a set of zero Lebesgue measure".

It can be shown, though I would not describe the proof as easy, that, for almost all values of Δ , the proportion of the first n denominators of Δ , which equal a prescribed integer r , tends to

$$\log_2 \left\{ \frac{(r+1)^2}{r(r+2)} \right\}$$

as $n \rightarrow \infty$. This and other results can be found in the following references:—

P. Lévy. *Théorie de l'Addition des Variables Aléatoires* (pp. 311–313) Paris: Gauthier-Villars (1937).

P. Lévy. "Fractions continues aléatoires" *Rend. Circ. Mat. Palermo* (2) 1 (1952) 1–39.

C. Ryll-Nardzewski. "Ergodic theory of continued fractions". *Studia Math.* 12 (1951) 74–79.

Yours etc., J. M. HAMMERSLEY

To the Editor of the *Mathematical Gazette*

DEAR SIR,

Further to Dr. H. Martyn Cundy's letter and postscript about the "Stroud" system, not only was it given by Prof. Everett in 1879, but it was also "proposed" by M. Hospitalier at the International Congress of Electricians of 1891. Stroud was anticipated more than once!

Yours etc., T. W. HALL