BULL. AUSTRAL. MATH. SOC. 12A10
VOL. 25 (1982), 303-304.

# PROBLEMS ON THE DISTRIBUTION OF CONJUGATES OF ALGEBRAIC NUMBERS

C.W. LLOYD-SMITH

This thesis concerns several problems related to the distribution of the conjugates of an algebraic number. It is divided into two parts.

In Part 1, we work mainly with the diameter function

$$\operatorname{diam}(\alpha) = \max_{1 \le i, j \le n} |\alpha_i - \alpha_j|$$

for an algebraic integer $\alpha$ of degree $n$, with conjugates $\alpha = \alpha_1, \alpha_2, \ldots, \alpha_n$. We consider the problem of finding lower bounds for $\operatorname{diam}(\alpha)$ when $n > 1$. We approach this problem by using $D(\alpha)$, the circumdiameter of the set of conjugates of $\alpha$.

Chapter 1 contains an introduction to this topic. Our results all explicitly assume that $\alpha$ is of degree greater than or equal to $2$.

In Chapter 2, it is shown that $D(\alpha) \ge \sqrt{3}$ and, if $\alpha$ has zero trace, $D(\alpha) \ge 2$. These results are best possible. It follows that $\operatorname{diam}(\alpha) > 3/2$ and, if $\alpha$ has zero trace, $\operatorname{diam}(\alpha) > \sqrt{3}$. This improves an earlier result of Favard [1] that $\operatorname{diam}(\alpha) > \sqrt{3/2}$. Also, we obtain the best possible result that $\operatorname{diam}(\alpha) \ge \sqrt{3}$ when $\alpha$ is reciprocal.

In Chapter 3 best possible results are obtained for $D(\alpha)$ and $\operatorname{diam}(\alpha)$ when $\alpha$ is totally real or belongs to a CM-field. Indeed, all such $\alpha$ for which $D(\alpha) < 4\cos(\pi/9) - 1$ or

$$\operatorname{diam}(\alpha) \le \sqrt{3}\left(2\cos(\pi/9) - (1/2)\right)$$

Received 30 October 1981. Thesis submitted to the University of Adelaide, December 1980. Degree approved: August 1981. Supervisor Dr P.E. Blanksby.

In Chapter 4, all algebraic integers of degrees 4 and 5 with diameter at most 2 are found. Corresponding results for $D(\alpha)$ are also obtained. This extends similar results of Favard for degrees 2 and 3 .

In Part 2, we consider some problems for algebraic numbers of denominator $q$ , which are the analogues of known results for algebraic integers. (The denominator of an algebraic number $\alpha$ is the leading coefficient in the (primitive) minimal polynomial of $\alpha$ .) This work is introduced in Chapter 5.

In Chapter 6, a generalization of a result of Smyth [2] is proved. If $\alpha$ has degree $n$ and conjugates $\alpha = \alpha_1, \alpha_2, \ldots, \alpha_n$ satisfying $|\alpha_1 \ldots \alpha_n| \geq 1$ and if $\alpha$ is non-reciprocal, it is shown that

$$\Lambda(\alpha) = \prod_{j=1}^{n} \max\{1, |\alpha_j|\} \geq \psi_q$$

where $\psi_q > 1$ is a zero of a certain quartic polynomial. The condition $|\alpha_1 \ldots \alpha_n| \geq 1$ is shown, in the next chapter, to be necessary. It is conjectured that $\psi_q$ can be replaced by the unique real zero of $qx^3 + (q-1)x^2 - qx - q$ .

In Chapter 7 there are a number of results which give lower bounds (in terms of the degree and denominator) on the maximum moduli, and also on $\Lambda(\alpha)$ , for various classes of algebraic numbers.

## References

[1] J. Favard, "Sur les formes décomposables et les nombres algébriques", *Bull. Soc. Math. France* 57 (1929), 50-71.

[2] C.J. Smyth, "On the product of the conjugates outside the unit circle of an algebraic integer", *Bull. London Math. Soc.* 3 (1971), 169-175.

Australian Bureau of Statistics,
PO Box 10,
Belconnen, ACT 2616,
Australia.

https://doi.org/10.1017/S000497270000530X Published online by Cambridge University Press