that the reader is aware that a healthy animal may be infected with tuberculosis by inoculating it with a portion of a diseased or tuberculous organ.

The facts on which the above statement regarding the direct transmission of tuberculosis rests, may be briefly stated under three heads:

1. Several tuberculoses, with the tubercle bacillus, have been found in a fœtus (unborn calf) of a cow suffering with pulmonary tuberculosis. In the human subject tuberculosis has been observed in an infant only fifteen days old. And the disease is stated to be commonest in infants between two and three years of age.
2. Animals become affected with tuberculosis when inoculated with portions of the apparently healthy organs of newly-born infants, the issue of mothers suffering with tuberculosis. This proves that the virus is present in the, as yet, apparently healthy offspring.
3. Children procreated while either parent is suffering with tuberculosis, often die at an early age of the same disease.

Other facts pointing to the same conclusion might be mentioned.

If once the possibility of the direct transmission of the tubercle bacillus from parent to offspring is admitted, the limit of age before it may manifest itself in the offspring cannot be defined. Many circumstances indicate that it may remain latent for years before it is carried into activity. Further, the risk of a proposer with an hereditary history of consumption or scrofula would appear, from this view, much more direct than if we suppose that he is merely subjected to a greater danger from contagion, owing to the hereditary susceptibility of his tissues.

In making this remark, I do not wish to under-estimate the danger of the acquirement of tuberculosis by contagion. This would be blind, when we remember that every consumptive patient sows the seed broadcast during the progress of the disease; and that an abundant harvest is reaped is shown by the number of deaths from tuberculosis in persons without hereditary taint.

I am,
Your obedient servant,

FREDERIC EVE, F.R.C.S.

15 *Finsbury Circus, E.C.,*
February 1888.

ON THE TRUE MEASURE OF THE PROBABILITIES OF SURVIVORSHIP BETWEEN TWO LIVES.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In the course of the discussion which followed the reading of Mr. Sunderland's paper on "Risk Premiums for Survivorship Assurances", Mr. Sutton called attention to the fact that an expression for the error involved in the use of the ordinary formula for the probabilities of survivorship between two lives, arising from the assumption of a uniform distribution of deaths throughout the year, had been deduced by Professor De Morgan in the third Appendix to his well-known "Essay on Probabilities." As it is not quite self-evident that the results arrived at independently by De Morgan and Mr. Sunderland are identical, I have thought that the following simple demonstration might be interesting to students of the Institute.

De Morgan's formula for the amount of the correction to be applied to the usual expression for the probability of (x) dying before (y) in the $(n+1)$th year following those ages, is

$$=\frac{(b-c)(p-q)-(a-b)(q-r)}{12xy} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where

$$xy=l_x l_y$$

$$a=l_{y+n} \qquad p=l_{x+n}$$

$$b=l_{y+n+1} \qquad q=l_{x+n+1}$$

$$c=l_{y+n+2} \qquad r=l_{x+n+2}$$

also,

$$(a-b)=d_{y+n} \qquad (p-q)=d_{x+n}$$

$$(b-c)=d_{y+n+1} \qquad (q-r)=d_{x+n+1}$$

Formula 1 thus becomes

$$=\frac{d_{y+n+1}d_{x+n}-d_{y+n}d_{x+n+1}}{12l_x l_y} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Mr. Sunderland's convenient expression for the amount of the correction (p. 87 of the present number of the *Journal*, formula 3) is

$$=\tfrac{1}{6}(a\beta-b\alpha) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where

$$a=-\tfrac{1}{2}(3d_{x+n}-d_{x+n+1})$$

$$b=\ \tfrac{1}{2}\ (d_{x+n}-d_{x+n+1})$$

$$\alpha=-\tfrac{1}{2}(3d_{y+n}-d_{y+n+1})$$

$$\beta=\ \tfrac{1}{2}\ (d_{y+n}-d_{y+n+1})$$

Formula 3 thus becomes, after multiplying out, reducing, and dividing by $l_x l_y$

$$=\frac{d_{y+n+1}d_{x+n}-d_{y+n}d_{x+n+1}}{12l_x l_y}$$

which is identical with formula 2 deduced by De Morgan.

This question has also been investigated by Mr. George King in the recently published *Institute of Actuaries' Text-Book*, Part II (Chap. iv, § 6-8, pp. 45-47), where an expression for the amount of the correction is deduced by Lubbock's formula of approximate summation. The result arrived at (formula 7, page 46), which represents the value of the correction in the nth year following ages x and y, is

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$$= \frac{(l_{x+n-1}-l_{x+n})(l_{y+n}-l_{y+n+1})-(l_{y+n-1}-l_{y+n})(l_{x+n}-l_{x+n+1})}{12l_xl_y}$$

or
$$\frac{d_{x+n-1}d_{y+n}-d_{y+n-1}d_{x+n}}{12l_xl_y} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Writing $(n+1)$ for n throughout in this formula, to apply the expression to the $(n+1)$th year, we obtain a formula identical with that deduced by De Morgan and Mr. Sunderland.

Formula 8 (Chap. iv, p. 47) in the *Text-Book*, gives an expression for the correction during the whole of life, in terms of the joint-life expectations and probabilities; while formula 2 (Chap. xiii, p. 222) gives the value of the correction to be applied to the single premium A^1_{xy}, in terms of the joint annuities and probabilities.

It is, however, to be noted that, in each of the three formulas in the *Text-Book* above cited, the expression for the correction is (doubtless through a misprint) preceded by a *minus* instead of a *plus* sign.

In an interesting paper by Mr. Peter Gray, upon "The True Measure of the Probabilities of Survivorship between Two Lives" (*J.I.A.*, i, 137) the expression for the correction is deduced by two independent processes, and numerical values are computed, upon the basis of the Carlisle Table of Mortality, for the total probability by the ordinary formula, and for the amount of the correction. It will be seen from these results that the correction is very small in amount, and that the ordinary expressions for the probabilities, and for the survivorship assurance, are abundantly correct for all practical purposes.

It should, perhaps, be noted that the problem investigated by Mr. Peter Gray is the probability of (y) dying before (x), and that in his resulting expression (*J.I.A.*, i, 149) the suffixes are reversed, as compared with formula 4 given above.

I am, Sir,
Your obedient servant,
THOMAS G. ACKLAND.

St. Mildred's House,
Poultry, E.C.,
9 *March* 1888.

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