Canad. Math. Bull. Vol. **43** (1), 2000 pp. 79–86

Cyclotomic Schur Algebras and Blocks of Cyclic Defect

Steffen König

Abstract. An explicit classification is given of blocks of cyclic defect of cyclotomic Schur algebras and of cyclotomic Hecke algebras, over discrete valuation rings.

1 Introduction

A fundamental result of representation theory of finite groups is the classification of blocks of cyclic defect. The aim of this note is to use this classification as a tool for classifying blocks of cyclic defect of cyclotomic Hecke algebras and of the associated Schur algebras, over discrete valuation rings. This reproves and generalizes results of Xi and of Erdmann for classical Schur algebras over fields.

We define two classes of R-orders which will be the outcome of the classification. Here R is a discrete valuation ring with maximal ideal π, field of fractions K and residue field $k = R/\pi$. The notation $R - R$ (two tied copies of R) means the set $\{(a, b) \in R \times R : a - b \in \pi\}$.

Fix a natural number $n \in \mathbb{N}$. The R-order Λ_n is the following subset of $n - 1$ copies of 2×2-matrices over K and one copy of 1×1-matrices over K (with ties between the first and the second copy, the second and the third copy and so on):

$$\begin{pmatrix} R & R \\ \pi & R \end{pmatrix} \diagup \begin{pmatrix} R & R \\ \pi & R \end{pmatrix} \diagup \begin{pmatrix} R & R \\ \pi & R \end{pmatrix} \diagup \cdots \diagup \begin{pmatrix} R & R \\ \pi & R \end{pmatrix} \diagup (R)$$

The R-order Γ_n is the following subset of $n - 2$ copies of 2×2-matrices over K and two copies of 1×1-matrices over K (with ties between the first and the second copy, the second and the third copy and so on):

$$(R) \diagup \begin{pmatrix} R & R \\ \pi & R \end{pmatrix} \diagup \begin{pmatrix} R & R \\ \pi & R \end{pmatrix} \diagup \cdots \diagup \begin{pmatrix} R & R \\ \pi & R \end{pmatrix} \diagup (R)$$

For finite groups and also for Hecke algebras, there is a well-defined notion of blocks of cyclic defect. By Schur-Weyl duality, the blocks of a Hecke algebra and of the associated Schur algebra correspond to each other (where on both sides, 'full rank' algebras are considered). Thus it makes sense to talk about blocks of cyclic defect of cyclotomic Schur algebras as well (see Section 2 for definitions and references).

Received by the editors April 8, 1998.
AMS subject classification: Primary: 20G05, 20C20; secondary: 16G30, 17B37, 57M25.

Theorem 1.1 *Fix R as above.*

(a) Let Λ be an R-order which is a block of cyclic defect of a cyclotomic Schur algebra. Then Λ is Morita equivalent to Λ_n for some n.
(b) Let Γ be an R-order which is a block of cyclic defect of a cyclotomic Hecke algebra. Then Γ is Morita equivalent to Γ_n for some n.

The proof of the theorem will be based on Proposition 3.1, which gives the same classification in an abstract setup containing the situation of the theorem.

We note that the integer n can be read off from the group theoretic data (see [1], Section 17).

It is an important feature of cyclotomic Hecke and Schur algebras that they generalize both type A and type B situations. In the case of cyclic defect, the theorem however tells us that type B does not contribute any new examples.

Corollary 1.2 *The order Λ_n has infinite global dimension. The order Γ_n has global dimension $2n-1$.*

For each n, the order Γ_n contains an idempotent e such that $e\Gamma_n e$ is isomorphic to Λ_n. Moreover, the left Γ_n-lattice $\Gamma_n e$ is a full tilting module of the quasi-hereditary order Γ_n, and the Γ_n-Λ_n-bimodule $\Gamma_n e$ induces a Schur-Weyl duality

$$\Lambda_n \simeq \mathrm{End}_{\Gamma_n}(\Gamma_n e),$$

$$\Gamma_n \simeq \mathrm{End}_{\Lambda_n}(\Gamma_n e)$$

Corollary 1.3 *Let Λ and Γ be a cyclotomic Hecke algebra and a cyclotomic Schur algebra, respectively.*

(a) For each block of $k \otimes_R \Lambda$ there exists a natural number n such that this block is Morita equivalent to an algebra A_n which is the quotient of the path algebra of

$$\bullet \underset{\beta_1}{\overset{\alpha_1}{\rightleftarrows}} \bullet \underset{\beta_2}{\overset{\alpha_2}{\rightleftarrows}} \bullet \cdots \bullet \underset{\beta_n}{\overset{\alpha_n}{\rightleftarrows}} \bullet$$

modulo the ideal generated by the following relations: for each i with $1 \leq i \leq n-1$: $\alpha_i \cdot \alpha_{i+1} = 0$, $\beta_{i+1} \cdot \beta_i = 0$, $\beta_i \cdot \alpha_i = \alpha_{i+1} \cdot \beta_{i+1}$.

(b) For each block of $k \otimes_R \Gamma$ there is a natural number m such that the block is Morita equivalent to the following algebra B_m which is the quotient of the path algebra of

$$\bullet \underset{\beta_1}{\overset{\alpha_1}{\rightleftarrows}} \bullet \underset{\beta_2}{\overset{\alpha_2}{\rightleftarrows}} \bullet \cdots \bullet \underset{\beta_m}{\overset{\alpha_m}{\rightleftarrows}}$$

modulo the ideal generated by the following relations: $\beta_1 \cdot \alpha_1 = 0$ and for each i with $1 \leq i \leq m-1$: $\alpha_i \cdot \alpha_{i+1} = 0$, $\beta_{i+1} \cdot \beta_i = 0$, $\beta_i \cdot \alpha_i = \alpha_{i+1} \cdot \beta_{i+1}$.

The algebra A_n has infinite global dimension, whereas the global dimension of B_m is $2m-2$. Again there is a Schur-Weyl duality relating the two algebras (for $n = m$).

This corollary contains known results: The main result of Xi's paper [15] states the result of part (a) for blocks of the (type A) Schur algebras $S_p(p, p)$. Since p divides the

https://doi.org/10.4153/CMB-2000-012-0 Published online by Cambridge University Press

order of the symmetric group Σ_p just once, in this case every block has cyclic defect; thus the corollary applies to this case. Xi's proof (which is valid in any situation where p divides the group order just once) is based on quasi-hereditary structures and computations with Loewy lengths and composition series. More generally, Erdmann ([9], Proposition 4.1) observed that (a) is true for any block of cyclic defect of a type A Schur algebra, and similarly (b) for blocks of cyclic defect of symmetric groups. Her proof uses character theory.

2 Definitions and Notation

As before we fix a discrete valuation ring R with maximal ideal π, field of fractions K and residue field k. By an R-order we always mean an R-algebra Λ contained in the split semisimple K-algebra $K \otimes_R \Lambda$ such that K is already a splitting field.

Let us first briefly review some of the theory of blocks of cyclic defect. For more information, in particular for the definition of a defect group we refer to Alperin's book [1]. Let G be a finite group and k a field. Then the group algebra kG is a direct sum of two-sided ideals, called blocks. Assigned to each block is a p-subgroup (where p is the characteristic of k) which is called the defect group (and which is unique up to conjugation); the p-Sylow subgroup is the defect group of the block which contains the trivial module. If the block has cyclic defect (that is, the defect group is cyclic), then—by results of Dade, Janusz and Kupisch—it is Morita equivalent to a Brauer tree algebra whose structure is quite explicitly known. We will give two examples in the next section. If k is replaced by the discrete valuation ring R, then a block of cyclic defect is Morita equivalent to a Green order associated with the same Brauer tree (see Roggenkamp's paper [14]). The orders Λ_n in the theorem are Green orders having a line as a Brauer tree.

The proof of the theorem will be based on the two structures of cellular algebras and of quasi-hereditary algebras. Cellular algebras have been defined by Graham and Lehrer [10]. We use an equivalent definition given in [11].

Definition 2.1 **([11])** Let A be an R-algebra where R is a commutative Noetherian integral domain. Assume there is an antiautomorphism i on A with $i^2 = \mathrm{id}$. A two-sided ideal J in A is called a *cell ideal* if and only if $i(J) = J$ and there exists a left ideal $\Delta \subset J$ such that Δ is finitely generated and free over R and that there is an isomorphism of A-bimodules $\alpha : J \simeq \Delta \otimes_R i(\Delta)$ (where $i(\Delta) \subset J$ is the i-image of Δ) making the following diagram commutative:

$$\begin{array}{ccc} J & \xrightarrow{\alpha} & \Delta \otimes_R i(\Delta) \\ \downarrow{\scriptstyle i} & & \downarrow{\scriptstyle x\otimes y \mapsto i(y)\otimes i(x)} \\ J & \xrightarrow{\alpha} & \Delta \otimes_R i(\Delta) \end{array}$$

The algebra A (with the involution i) is called *cellular* if and only if there is an R-module decomposition $A = J_1' \oplus J_2' \oplus \cdots \oplus J_n'$ (for some n) with $i(J_j') = J_j'$ for each j and such that setting $J_j = \bigoplus_{l=1}^{j} J_l'$ gives a chain of two-sided ideals of A: $0 = J_0 \subset J_1 \subset J_2 \subset \cdots \subset J_n = A$ (each of them fixed by i) and for each j ($j = 1, \ldots, n$) the quotient $J_j' = J_j / J_{j-1}$ is a cell ideal (with respect to the involution induced by i on the quotient) of A/J_{j-1}.

If Λ is an R-order and there is a cell chain such that after tensoring both with K and with k this chain becomes a cell chain of a cellular algebra, $K \otimes_R \Lambda$ and $k \otimes_R \Lambda$ respectively, then

https://doi.org/10.4153/CMB-2000-012-0 Published online by Cambridge University Press

A is called *integral cellular*.

Quasi-hereditary algebras over fields have been defined by Cline, Parshall and Scott [5]. In [6], the same authors extended their definition to an integral setup suitable for modular representation theory. We are going to use special cases of these definitions only. In particular, we will define quasi-hereditary algebras only inside the class of cellular algebras.

Definition 2.2 Let A be a finite dimensional algebra over a field. Let J be a cell ideal in A (with respect to some anti-automorphism i as above). Then J is called a *heredity ideal* if and only if there is a primitive idempotent e in A generating J as a two-sided ideal and such that the left A-module Δ is isomorphic to the projective module Ae.

If A is cellular with all sections in the defining chain being heredity ideals, then A is called *quasi-hereditary*.

Let R be a discrete valuation ring as above and let Λ be an R-order. Then Λ is called a *quasi-hereditary order* if and only if there is a finite chain $0 = J_0 \subset J_1 \subset J_2 \subset \cdots \subset J_n = \Lambda$ of two-sided ideals such that tensoring with both K and k makes this chain into the defining chain of a quasi-hereditary algebra.

Our objects of study are the following:

Definition 2.3 **(Ariki and Koike [3])** Let S be the ring $\mathbb{Z}[q, q^{-1}, u_1, \ldots, u_r]$ for some natural number r. Then the Hecke algebra $\mathfrak{H}_{n,r}$ (called a *cyclotomic (or Ariki-Koike) Hecke algebra*) is defined over S by the following generators and relations:

Generators: $t = a_1, a_2, \ldots, a_n$

Relations:
$$(t - u_1)(t - u_2)\cdots(t - u_r) = 0$$
$$a_i^2 = (q-1)a_i + q \ (i = 2, \ldots, n)$$
$$ta_2ta_2 = a_2ta_2t$$
$$a_ia_j = a_ja_i(|i-j| \geq 2)$$
$$a_ia_{i+1}a_i = a_{i+1}a_ia_{i+1}(i = 2, 3, \ldots, n-1)$$

Specializing $r = 1$ and $u_1 = 1$ respectively $r = 2$ and $u_1 = -1$ one gets back the classical Hecke algebras of types A and B, respectively.

We note that a more general class of cyclotomic Hecke algebras appears in [4].

This Hecke algebra can be seen as deformation of the group algebra of the wreath product $(\mathbb{Z}/r\mathbb{Z}) \wr \Sigma_n$. Ariki and Koike show in [3] that $\mathfrak{H}_{n,r}$ is a free S-module of rank $n!\, r^n$, and they classify the simple representations of the semisimple algebra frac $(S) \otimes_S \mathfrak{H}_{n,r}$. An important application of Ariki-Koike Hecke algebras is Ariki's proof of (a generalization of) the LLT-conjecture which gives an explicit way of computing characters of Hecke algebras (see [2]).

In [8] Schur algebras for these Hecke algebras have been constructed; these generalize the q-Schur algebras defined by Dipper and James [7]. The construction involves a Schur-Weyl duality between these Schur algebras and the above Hecke algebras.

Definition 2.4 **(Dipper, James and Mathas [8])** Let S be the ring $\mathbb{Z}[q, q^{-1}, u_1, \ldots, u_r]$ for some natural number r. For each multipartition λ, let $M^\lambda := m_\lambda \mathfrak{H}_{n,r}$ be a Young module over the cyclotomic Hecke algebra $\mathfrak{H}_{n,r}$. Then the *cyclotomic Schur algebra* $\mathfrak{S}_{n,r}$ is defined to be the endomorphism ring

$$\mathfrak{S}_{n,r} := \mathrm{End}_{\mathfrak{H}_{n,r}}\Big(\bigoplus_{\lambda} M_\lambda\Big)$$

where λ runs through all multipartitions.

We will consider all these algebras over discrete valuation rings only, which means that they have to be localized and completed in a suitable way.

3 Proofs

Throughout, we fix a discrete valuation ring R. By order we always mean an R-order. For simplicity, algebras are assumed to be indecomposable as rings.

Technically, our main result is as follows:

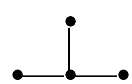
Proposition 3.1 *(a) Let Λ be a quasi-hereditary order which is in Schur-Weyl duality with a cellular Green order Γ; that is, there exists a bimodule ${}_\Lambda M_\Gamma$ such that $\mathrm{End}_\Lambda(M) \simeq \Gamma$ and $\mathrm{End}_\Gamma(M) \simeq \Lambda$. Then there is a natural number n such that Λ is Morita equivalent to Λ_n.*

(b) Let Γ be a Green order and suppose Γ is integral cellular. Then there is a natural number n such that Γ is Morita equivalent to Γ_n.

Let us first verify that the orders in the theorem satisfy the assumptions of the proposition.

A cyclotomic Hecke algebra is integral cellular by [10] and also by [8]. A cyclotomic Schur algebra is integral quasi-hereditary by [8]. Schur-Weyl duality is built in the definition given in [8]. A block of cyclic defect is a Green order by [14].

Next we give two examples of Brauer tree algebras (over a field) which will be used in the proof below.

The *first example* is that of a Brauer tree algebra of the graph  . This algebra has three simple modules (associated with the edges of the tree, say a, b, and c), and as a left module over itself it has the following composition series:

$$ {}_A A = \begin{array}{c} a \\ b \\ c \\ a \end{array} \oplus \begin{array}{c} b \\ c \\ a \\ b \end{array} \oplus \begin{array}{c} c \\ a \\ b \\ c \end{array} $$

From this information, it is easy to write down the periodic projective resolutions of the simple modules. For instance, a minimal projective resolution of the simple of type a looks as follows:

$$ \cdots \to \begin{array}{c} a \\ b \\ c \\ a \end{array} \to \begin{array}{c} c \\ a \\ b \\ c \end{array} \to \begin{array}{c} b \\ c \\ a \\ b \end{array} \to \begin{array}{c} a \\ b \\ c \\ a \end{array} \to a \to 0 $$

https://doi.org/10.4153/CMB-2000-012-0 Published online by Cambridge University Press

This determines extensions between simple modules as follows: $\mathrm{Ext}^1_A(b,a) = \mathrm{Ext}^1_A(c,b) = \mathrm{Ext}^1_A(a,c)$ is zero, whereas $\mathrm{Ext}^1_A(a,b) = \mathrm{Ext}^1_A(b,c) = \mathrm{Ext}^1_A(c,a)$ has dimension one. In particular $\mathrm{Ext}^1_A(b,a)$ is not isomorphic to $\mathrm{Ext}^1_A(a,b)$.

This algebra is not cellular (see below).

The *second example* is that of a Brauer tree algebra having tree •——•——⋆ where the vertex denoted by $\star$ is exceptional with multiplicity two.

This algebra has two simple modules, say a and b, and the following composition series as left module over itself:

$$ {}_AA = \begin{array}{c} a \\ b \\ a \end{array} \oplus \begin{array}{ccc} & b & \\ a & & b \\ & b & \end{array} $$

This algebra is cellular (with respect to an involution fixing the vertices and turning around the arrows in the quiver). In fact, there is a one-dimensional cell ideal, which as left module is simple of type b. Factoring it out, there is another one-dimensional cell ideal, again simple of type b. The next section has dimension four and consists of two copies of a uniserial module with socle b and top a. Factoring out this ideal, one arrives at the field (a simple module of type a). The length of this cell chain is four, and there is no shorter one.

The proof of Proposition 3.1 is split up into two lemmas.

Lemma 3.2 *Let Γ be an R-order which is both an integral cellular algebra, say with involution i, and a Green order. Then Γ is Morita equivalent to an order Γ_n for some n.*

Proof Fix a cell chain $J_1 \subset J_2 \subset \cdots \subset J_n = \Lambda$ for some natural number n. Then $K \otimes_R J_1 \subset K \otimes_R J_2 \subset \cdots \subset K \otimes_R J_n = K \otimes_R \Lambda$ is a cell chain of the split semisimple K-algebra $A(K) = K \otimes_R \Lambda$. The subquotients in this cell chain are just the simple Wedderburn components of $A(K)$. Over $k = R/\pi$ we get a cell chain $k \otimes_R J_1 \subset k \otimes_R J_2 \subset \cdots \subset k \otimes_R J_n = k \otimes_R \Lambda$ of the finite dimensional k-algebra $A(k) = k \otimes_R \Lambda$.

By Proposition 5.1 of [11], the induced involution i on the cellular algebra $A(k)$ must fix isomorphism classes of simple modules. Thus for any two simple $A(k)$-modules, say L and L' there must be an isomorphism $\mathrm{Ext}^1_{A(k)}(L,L') \simeq \mathrm{Ext}^1_{A(k)}(L',L)$. This implies that the Brauer tree of $A(p)$ is a line (see Proposition 5.3 of [11] for more detail). See the first example for an illustration.

Assume the Brauer tree of $A(k)$ has an exceptional vertex. Then we are in a situation as in the second example. The cell chain has length equal to the number of Wedderburn components of $A(K)$, which equals the number of edges of the Brauer tree. In the example this number is two, whereas we have seen that for $A(k)$ (given in the example) one needs a chain of length four.

In general, the existence of an exceptional vertex forces the cell length of $A(k)$ to be strictly bigger than that of $A(K)$. In fact, there is an indecomposable projective module P

having a composition series of one of the following three types:

$$\begin{array}{c} a \\ b \\ a \\ \cdots \\ b \\ a \end{array} \quad \text{or} \quad \begin{array}{ccc} & a & \\ b & & \\ a & & \\ \cdots & & c \\ b & & \\ & a & \end{array} \quad \text{or} \quad \begin{array}{ccc} & a & \\ & & c \\ b & & \cdots \\ & & c \\ & a & \end{array}$$

The number of $A(k)$-simples up to isomorphism is $n-1$ whereas the length of the given cell chain is n. By Proposition 4.1 of [11], a cell ideal either does not contain any idempotent or it is a heredity ideal generated as two-sided ideal by a primitive idempotent whose equivalence class is uniquely determined. Thus all but one subquotient in the cell chain must contain a primitive idempotent (not contained in a smaller ideal in the cell chain) and hence be a heredity ideal in the respective quotient algebra. Clearly, the algebra $A(k)$ itself does not contain a heredity ideal, hence it must contain a cell ideal J such that $A(k)/J$ is quasi-hereditary. The intersection of J with P is either zero or the socle of P. Therefore, in the quotient $A(k)/J$ there is an indecomposable projective module, say Q, which is either P or $P/\operatorname{soc}(P)$.

Let J' be a heredity ideal of $A(k)/J$. No other indecomposable projective module maps injectively into Q. Hence the intersection of J' (which is, by definition, a direct sum of copies of an indecomposable projective module) with Q must be trivial. Continuing by induction it follows that no section in the heredity chain of $A(k)/J$ can intersect non-trivially with Q.

This gives the desired contradiction. Therefore there is no exceptional vertex. ■

This implies part (b) of the theorem.

Lemma 3.3 *Let Γ be a Green order and Λ be any order such that there exists an idempotent $e = e^2 \in \Lambda$ with $e\Lambda e = \Gamma$ and $\Lambda \simeq Hom_\Gamma(\Lambda e)$. Suppose that Λ is integral quasi-hereditary.*

Then the idempotent $1-e$ is a sum of primitive idempotents which all are equivalent and all of which lie in the unique heredity ideal of Λ.

The situation in the lemma is usually called a 'double centralizer property between Λ and Γ' or a 'Schur-Weyl duality', and this also provides us with a 'Schur functor from Λ to Γ'.

Note that the lemma proves a more general statement than what is needed for the proposition.

Proof An indecomposable Γ-lattice is either irreducible or projective. Decomposing the right Γ-lattice Λe into a direct sum of indecomposables, say $\Lambda e = \oplus M_i$ we can (by applying a Morita equivalence, if necessary) assume without loss of generality that for $i \neq j$ the summands M_i and M_j are not isomorphic. Clearly, Λe also decomposes as $\Lambda e \simeq X \oplus Y$ where X collects the Γ-projective direct summands and all the indecomposable direct summands of Y are irreducible (and not projective) over Γ.

The proof is based on comparing the following natural numbers: The number of (non-isomorphic) direct summands of X and of Y is denoted by $n(X)$ and $n(Y)$, respectively. The

https://doi.org/10.4153/CMB-2000-012-0 Published online by Cambridge University Press

number n counts the simple Wedderburn components of $A(K) = K \otimes_R \Gamma$ which equals the number of simple components of $B(K) = K \otimes_R \Lambda$. This number in turn equals the number of equivalence classes of primitive idempotents in the integral quasi-hereditary algebra Λ, whereas the Green order Γ has $n - 1$ equivalence classes of primitive idempotents, that is, $n(X) = n - 1$. Schur-Weyl duality tells us that $n(X) + n(Y)$ equals n. Hence $n(Y)$ is one, that is, Y is irreducible. The idempotent $1 - e$ by construction is the identity morphism on Y.

It remains to show that $1 - e$ lies in the heredity ideal J of Λ and that J is unique. By definition, J must contain some primitive idempotent, say e'. If e' is not equivalent to e, it must be equivalent to the identity morphism on some direct summand, say X_1 of X. From the structure of a Green order it follows that there are two non-equivalent central idempotents in Γ, which do not annihilate X_1. Hence e' cannot be equivalent to e, which lies in one simple component of $B(K)$. Thus e generates J, and by the same argument, e is (up to equivalence) the only primitive idempotent in Λ which lies in a simple component of $A(K)$. Also, J is the unique heredity ideal of Λ. ■

This implies part (a) of the theorem.

The corollaries follow easily by direct computations, which we skip.

References

[1] J. L. Alperin, *Local representation theory*. Cambridge University Press, 1986.
[2] S. Ariki, *On the decomposition numbers of the Hecke algebra of* $\mathfrak{G}(m, 1, n)$. J. Math. Kyoto Univ. **36**(1996), 789–808.
[3] S. Ariki and K. Koike, *A Hecke algebra of* $(\mathbb{Z}/r\mathbb{Z}) \wr \Sigma_n$ *and construction of its irreducible representations*. Adv. in Math. **106**(1994), 216–243.
[4] M. Broué and G. Malle, *Zyklotomische Heckealgebren*. Astérisque **212**(1993), 119–189.
[5] E. Cline, B. Parshall and L. Scott, *Finite dimensional algebras and highest weight categories*. J. Reine Angew. Math. **391**(1988), 85–99.
[6] ———, *Integral and graded quasi-hereditary algebras*. I. J. Algebra **131**(1990), 126–160.
[7] R. Dipper and G. James, *The q–Schur algebra*. Proc. London Math. Soc. **59**(1989), 23–50.
[8] R. Dipper, G. James and A. Mathas, *Cyclotomic q-Schur algebras*. Math. Z. **229**(1998), 385–416.
[9] K. Erdmann, *Schur algebras of finite type*. Quart. J. Math. Oxford Ser. (2) **44**(1993), 17–41.
[10] J. J. Graham and G. I. Lehrer, *Cellular algebras*. Invent. Math. **123**(1996), 1–34.
[11] S. König and C. C. Xi, *On the structure of cellular algebras*. Algebras and modules II, Geiranger, 1996, 365–386, CMS Conf. Proc. **24**(1998), Amer. Math. Soc.
[12] ———, *Cellular algebras: inflations and Morita equivalences*. J. London Math. Soc., to appear.
[13] B. Parshall and L. L. Scott, *Derived categories, quasi–hereditary algebras and algebraic groups*. Proc. of the Ottawa-Moosonee Workshop in Algebra 1987, Math. Lect. Note Series, Carleton University and Université d'Ottawa, 1988.
[14] K. W. Roggenkamp, *Blocks of cyclic defect and Green-orders*. Comm. Algebra **20**(1992), 1715–1734.
[15] C. C. Xi, *The structure of Schur algebras* $S_k(n, p)$ *for* $n \geq p$. Canad. J. Math. **44**(1992), 665–672.

Fakultät für Mathematik
Universität Bielefeld
Postfach 10 01 31
D–33501 Bielefeld
Germany
e-mail: koenig@mathematik.uni-bielefeld.de