ON AXIOMS FOR SEMI-LATTICES

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By a semi-lattice we mean a system $<L, .>$ where L is a set and . is a binary operation in L that is idempotent, commutative and associative. In a recent article [2] D.H. Potts considers the problem of reducing the number of axioms for a semi-lattice. His result was that the following two axioms viz. (1) $xx = x$, (2) $(uv)((wx)(yz)) = ((uv)(xw))(zy)$ are sufficient to give a semi-lattice. But the second identity contains six elements instead of the original three. In the following we give a set of two simple identities with just three elements for a semi-lattice. This improves the above mentioned result.

THEOREM. A system $<L, .>$ is a semi-lattice if the following two identities hold in the system:

(1) $xx = x$

$$----(*)$$

(2') $(xy)z = (yz)x$

<u>Proof.</u> It is sufficient if we prove the commutative law.

We have	yz	$=$	$(yz)(yz)$	by (1)
		$=$	$(z(yz))y$	by (2')
		$=$	$((yz)y)z$	by (2')
		$=$	$((zy)y)z$	by (2')
		$=$	$((yy)z)z$	by (2')
		$=$	$(yz)z$	by (1)
		$=$	$(zz)y$	by (2')
		$=$	zy	by (1) .

We cannot simplify the axiom system (*) further with axioms involving fewer elements because due to a result of McKinsey and Diamond [1] it is known that it is not possible to give a set of postulates for semi-lattices where each postulate is a universal sentence with only two variables.

REFERENCES

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