THE BAADE-WESSELINK TECHNIQUE

Thomas J. Moffett
Department of Physics, Purdue University
West Lafayette, Indiana, 47907

INTRODUCTION

The historical foundations of the Baade-Wesselink (BW) method were established through the work of Baade (1926), Becker (1940), and Wesselink (1947). All modern versions of the BW-method are constructed upon these foundations. Since the radii of pulsating stars are fundamental to many areas of astronomical research (theories of stellar structure and evolution, galactic structure, and the cosmic distance scale), it is paramount that we strive to improve their observational determination. At the present time, the BW-technique is the only "direct" method for determining the radii of pulsating stars. Until a new technique is devised, better determinations of this important physical parameter will only occur through improvements in the BW-method.

By necessity, the scope of this review of the BW-technique will be limited in scope. The historical development along with the spectroscopic problems associated with the BW-technique are more fully discussed in the excellent review by Gautschy (1987).

THE BASIC IDEA

Using the definitions of effective temperature and apparent bolometric magnitude, the ratio of the radii at different phases in the pulsation cycle can be expressed as:

$$\log\left[\frac{R_2}{R_1}\right] = -0.2\left[(V_2 + B.C._2) - (V_1 + B.C._1) + 10\log\frac{T_{eff_2}}{T_{eff_1}}\right] \quad (1)$$

where the subscripts refer to the two phases, V, B.C., and T_{eff} refer to the apparent visual magnitude, the bolometric correction, and the effective temperature. This equation describes the behavior of the "photometric radii" over the pulsation cycle.

Integration of the observed radial velocity variation of the star throughout its cycle can be used to describe the behavior of the "spectroscopic radii":

$$R_2 - R_1 = -p\,T\int_{\Phi_1}^{\Phi_2}(V_r - \gamma)\,d\Phi \quad (2)$$

where p is the conversion factor from radial velocity to pulsation

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velocity, τ is the pulsation period in seconds, V_r is the observed radial velocity, and γ is the velocity of the star's center of mass. The center of mass velocity can be determined from the observed radial velocities by demanding that the following equation be satisfied:

$$\int_t^{t+\tau} (V_r - \gamma)\, dt = 0. \tag{3}$$

Equations (1) and (2) can now be solved, in principle, for the radius of the pulsating star.

$$R_1 = \frac{-p\,\tau \int_{\Phi_1}^{\Phi_2} (V_r - \gamma)\; d\Phi}{10^{-0.2\,[(V_2 + B.C._2) - (V_1 + B.C._1) + 10\,\log(T_{eff2}/T_{eff1})]} - 1} \tag{4}$$

Equation (4) represents the formal solution of the BW-method provided one knows the values of all the quantities on the right side. In practice, the values of the bolometric correction and effective temperature as functions of pulsational phase are difficult to determine from the observational data. Various schemes have been devised, which Gautschy (1987) refers to as "realizations" of the BW-method, to solve this equation. The fundamental problem faced in all realizations of the BW-method is that the flux emitted by the star is the result of both temperature (more correctly surface brightness) and radius changes during the pulsation cycle. One must devise means to separate these two effects.

Consider the idealized case of a pulsating star which maintains constant temperature and atmospheric structure throughout its cycle. This is the simplest realization since equation (4) would reduce to:

$$R_1 = \frac{-p\,\tau \int_{\Phi_1}^{\Phi_2} (V_r - \gamma)\, d\Phi}{10^{-0.2\,(V_2 - V_1)} - 1} \tag{5}$$

Since the surface brightness is constant, all of the quantities on the right side of the above equation are now easily determined from the observations. In the real world, this idealized case is approximated by working in the infrared region. The flux emitted by the star has a weaker coupling to the temperature in the infrared ($L_{IR} \propto T^{1.5}$) as compared to the optical region ($L_V \propto T^4$) of the spectrum. If the star is allowed to vary its temperature during the pulsation cycle but still required to be a blackbody, then the simple realization of equation (5) can still be used. By selecting pairs of phases with equal colors, one

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has selected pairs of equal surface brightness since the temperature and bolometric correction is uniquely determined by the color for a blackbody. In practice, this case is approximated by working in regions of the spectrum where the influence of non-blackbody effects (i.e. shock waves, etc.) is minimized.

Most realizations of the BW-technique are formulated in terms of surface brightness defined by Wesselink (1969) as:

$$S_V \equiv m_V + 5 \log \theta \tag{6}$$

or,

$$S_V \equiv M_V + 5 \log R + \text{const.} \tag{7}$$

where θ is the angular diameter, R the linear radius, and the constant is defined by the adopted zero points of S_V and M_V, plus the units used for the linear radius. The surface brightness can be expressed in a variety of forms. For example, since the surface brightness is the flux expressed in magnitudes per unit surface area of the star, one can write,

$$S_{bol} = -2.5 \log \left[\frac{4 \pi R^2 \sigma T_{eff}^4}{4 \pi R^2} \right] = -10 \log T_{eff} + \text{const.} \tag{8}$$

or, in terms of the visual magnitude of the star,

$$S_V = -10 \log T_{eff} - B.C. + \text{const.} \tag{9}$$

Barnes, Evans and Moffett (1978) express the surface brightness in terms of the visual surface brightness parameter, F_V, defined as,

$$F_V \equiv \log T_{eff} + 0.1\ B.C. \tag{10}$$

Comparing eqs. (9) and (10), we see that the relationship between the surface brightness and the visual surface brightness parameter is:

$$F_V = -0.1\ (S_V + \text{const.}) \tag{11}$$

Similar to eq. (6), the visual surface brightness parameter can also be expressed in terms of the angular diameter of the star:

$$F_V = 4.2207 - 0.1\ V_o - 0.5 \log \theta \tag{12}$$

where the constant was determined by adopting solar values, V_o is the un-reddened apparent visual magnitude, and θ the angular diameter expressed in arc milliseconds.

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Using the surface brightness approach is the most natural way to formulate a realization of the BW-technique since it separates the temperature dependent effects from the radius variations. In the classical BW-realization, pairs of phases with equal surface brightnesses are used to reduce eq. (4) to the simple form of eq. (5). Realizations which rely on pairs of points suffer from two problems. First, not all of the observed points are used in the final radius determination. Observations near the turning points in the light curve are not utilized, for practical reasons, in the pair selection process. Secondly, the selected pairs of phases are composed of inhomogeneous members. In general, one member of the pair will represent a point during the expansion portion of the pulsation cycle while the other member corresponds to a contraction portion (see Gautschy 1987).

On the other hand, if S_V or F_V is explicitly known, then data over the entire cycle can be used in the determination of the mean radius. The most popular approach is to parameterize the surface brightness in terms of a linear relation using some color index,

$$F_V = a + b\ (C.I.) \tag{13}$$

If the zero-point and the slope in the above equation are known, then measurements of the apparent magnitude and appropriate color index over the entire pulsation cycle allows one to determine the variation of the star's angular diameter over the cycle by means of eqs. (13) and (12). Integration of the radial velocity curve provides the changes in linear diameter over the cycle. The distance to a pulsating star, r (in parsecs), is related to its linear and angular diameter, θ, through the equation,

$$\Delta D + D_m = 10^{-3} r \theta \tag{14}$$

where ΔD is the instantaneous linear displacement from the mean diameter, D_m, both expressed in Astronomical Units. A regression analysis of θ against ΔD yields both the star's distance and mean radius. In this realization, the radius depends only on the value of the slope, b, but the distance depends on both the slope and the zero-point, a, in eq. (13).

MODERN REALIZATIONS

Since a major objective of radius determination via the BW-technique is to serve as a check on theory, one would like to maintain observational "purity". It is obvious that a radius determined without recourse to theory serves as the best observational check. In all realizations of the BW-method, theory enters through the calculation of the velocity conversion factor p, so absolute purity is never realized. The question which must be addressed is how much additional theoretical input is acceptable before observational purity is severely compromised?

Two philosophies have evolved in formulating modern realizations. In the empirical approach, one tries to find a region of the spectrum where the effects of surface gravity, abundance, shocks, temperature etc. are minimized. An example of this approach is the use of the infrared region to minimize the effect of temperature changes to approximate the simple realization of eq.(5). The advantages of the empirical approach are simplicity and observational purity. The major disadvantage is that one intentionally avoids some of the interesting physics (e.g. shock waves, surface gravity effects, etc.). A concern is the compromising of physical reality in favor of observational purity.

In the modeling approach, one tries to handle some of the non-blackbody effects by invoking theory (model atmospheres) to some degree. An example of the modeling approach is the CORS method of Caccin et al. (1981) in which two color indices, calibrated by the model atmosphere grids of Kurucz (1979), are used to track variations in T_{eff}, B.C., and g_{eff} over the pulsation cycle. The advantages are a more realistic realization and the inclusion of interesting physics. The disadvantages are the realization is more complex and observational purity is compromised to some degree. This last point is often overlooked or not fully appreciated. Simon (1988) and Fernley et al. (1988) both stress that model atmospheres used to track surface gravity effects etc. may contain inaccurate or missing physics which could lead to systematic errors in the radii determined in these realizations.

THE EMPIRICAL APPROACH

The realizations of the BW-technique used by Gieren (1986) and Moffett and Barnes (1987) are identical in that they both use the visual surface brightness method. The method used by Balona (1977) and Balona and Martin (1978a,b) is essentially the same realization except that the method of linear least squares is not used in the solution.

In the realizations of Moffett and Barnes (1987) and Gieren (1986), the Thompson (1975) method is used to determined the slope of the visual surface brightness relation, eq.(13), for each Cepheid. These individual values are then used to determine a mean slope relation which is then used in a linear least squares solution for the mean radius. In Simon's (1988) nomenclature, this is a "consecutive" realization since the mean radius and slope of the visual surface brightness are not found in a "concurrent" manner.

Balona (1977) made a major contribution by emphasizing that the method of linear least squares does not correctly take into account the effects of errors, in all the observational quantities, on the final mean radius determination. Balona's formulation of the surface brightness method uses the following working equation:

$$V_o = a + b\ (B-V)_o - 5\ \log(R_o+\Delta r) \qquad (15)$$

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This gives the star's apparent magnitude as a function of the surface brightness variation (first two terms) and the surface area variation (third term) at each phase during the pulsation cycle. The constant α contains a distance term which is different for each star. The last term is linearized under the assumption that Δr is small compared to the mean radius R_0. Coulson, Caldwell and Gieren (1986) later improved this realization by developing an iterative procedure which no longer required that Δr be small. Since Balona's realization is a concurrent solution, the slope of the surface brightness relation, b, is allowed to vary star by star.

Balona states that the Principle of Maximum Likelihood is used to solve eq. (15). Strictly speaking, this is incorrect since the Principle of Maximum Likelihood requires that both the errors in all observables and the functional form of their distribution be known. If one assumes, as Balona does, that the errors follow a Gaussian distribution, then the Principle of Maximum Likelihood reduces to the method of nonlinear least squares.

The above realizations, and in fact all realizations both empirical and modeling, have a common unresolved problem. The spectroscopic and photometric radii curves do not agree in shape and/or phase for most stars. The correct phasing between the photometric and spectroscopic curves is very difficult to disentangle because there are two different sources of phase mismatch. The first is a "real" phase shift due to the fact that the photometric and spectroscopic data were acquired at difference epochs. Inaccuracies in the period or real changes in the period of the star produce a real phase shift. The second source is an "artificial" phase shift introduced by an incorrect slope in the surface brightness relation, eq.(13), as pointed out by Barnes et al. (1977) and Balona and Martin (1978a). The real phase shifts can only be eliminated by having simultaneous photometric and spectroscopic data which is usually not the case.

Balona and Martin (1978a,b) clearly demonstrate that introducing an arbitrary phase shift between the velocity and light curves is equivalent to changing the value of the slope in eq.(13). The converse is also true. If the adopted value of the slope of the surface brightness relation is slightly off, then an artificial phase shift is required to bring the spectroscopic and photometric curves into phase alignment. In a consecutive approach using a mean value for the slope, artificial phase shifts are inevitable since the individual slopes will scatter about the mean value. Leaving the slope as a free parameter, which forces phase alignment, is claimed to be one of the virtues of concurrent solutions, but this virtue is more aesthetic than substantive. As Balona and Martin (1978b) show, these two approaches are equivalent ways of obtaining phase alignment. Since real phase shifts can also be present, it could be argued that a final phase adjustment between the spectroscopic and photometric curves might be a better approach.

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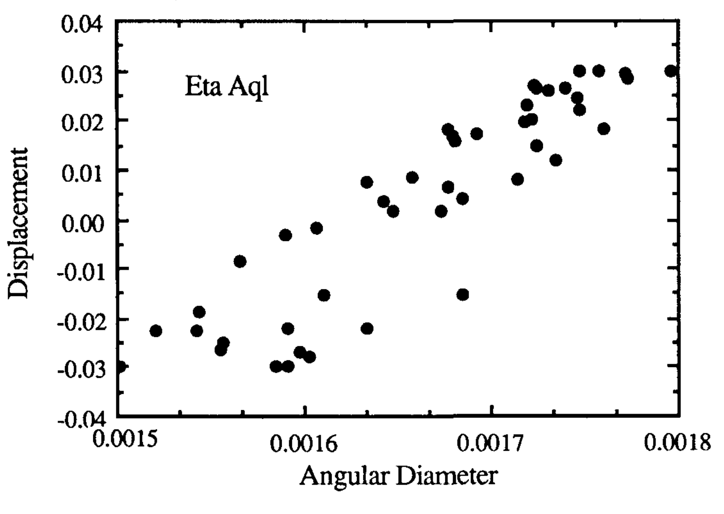
The real problem lies in the parameterization of the surface brightness by a linear relation of a single color index, eq.(13), which is an oversimplification of physical reality. This problem is present in all realizations, both empirical and modeling. The breakdown of this simple assumption is clearly seen in stars with very dynamic atmospheres, e.g. RR Lyrae stars. In these stars, the photometric curves are in very poor agreement with the spectroscopic curves near maximum light. In most applications, these problem phases are excluded in the final solution for the mean radius. The same shortcoming is present, but not as pronounced, in the Classical Cepheids. If the surface brightness relation is correct, then the photometric and spectroscopic curves should be in both phase and shape agreement. A plot of photometric versus spectroscopic displacement should show a one-to-one correspondence i.e. a straight line. Figure 1 shows such a plot where the equivalence is approximately, but not strictly, true.

Since the photometric radii, or angular diameters, are very sensitive to errors in the photometry, it is difficult to decide if the scatter from a straight line in Figure 1 is real, or merely due to observational uncertainty. In order to differentiate between the two possibilities, we solved eq.(15), using the $(V - R)_0$ index, for a, b, and R_0 for each of the 63 Cepheids in our sample (Moffett & Barnes 1987). The residuals of the observed value minus the calculated visual magnitude, as given by eq. (15), were determined for each Cepheid. The residuals for all the Cepheids were then combined into small phases bins (Barnes et al. 1987). Combining the residuals in this manner reduces the influence of random

Figure 1. The angular diameter as determined from the surface brightness relation versus the linear displacement from integration of the velocity curve.



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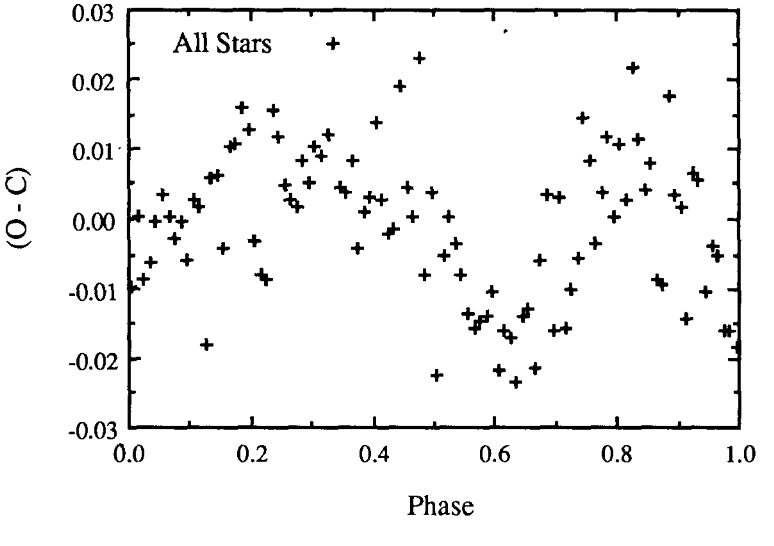
observational errors. The results of this comparison are shown in Figure 2 which clearly indicates that there are systematic effects present.

Despite the trend shown in Figure 2, the residuals produced by the simple linear version of the surface brightness relation are not bad. This simple empirical approach produces residuals with a peak-to-peak spread of less than ± 0.02 magnitudes. As a point of comparison, Hindsley and Bell (1987) used static model atmospheres to produce synthetic infrared magnitudes of Cepheids which were systematically different from the observed values by 0.15 magnitudes. The empirical approach using the simple linear surface brightness relation appears to be a good, but not perfect, approximation for Cepheids.

The empirical calibration of the surface brightness, F_V, in eq.(13) using $(V-R)_o$ is very good for non-variable stars (see Barnes et al. 1978). This is confirmed by the theoretical work of Bell and Gustafsson (1980) in which static model atmospheres were used to show that the combined effects of metal abundance and gravity are smallest in the (V-R) index. This empirical calibration starts to breakdown for variable stars because of their dynamic nature. Several researchers have shown that the influence of dynamic effects can be reduced by the judicious selection of both the magnitude and color index employed. Jones et al. (1987a), working with RR Lyrae stars, found that the size of the required artificial phase shift is smallest for the (V-K) index when used in combination with the V-mag. They found even better

Figure 2. The combined residuals for all stars as a function of pulsation phase.



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agreement between the photometric and spectroscopic curves when the K-mag. replaced the V-mag. Fernley et al. (1987) pointed out the advantages of the infrared over the optical region for radius determination. For RR Lyrae stars, the optical region is near the peak of their flux distribution so $L_V \propto T^4$ but in the infrared one is on the Rayleigh-Jeans tail of the Planck function, where $L_K \propto T^{1.5}$. In other words, the luminosity variation in the infrared is dominated by the changes in surface area rather than temperature, but the opposite is true in the optical region. Errors in the temperature description have a lesser effect in the infrared than in the optical region. In addition, the effects of metallicity and gravity are an order of magnitude less sensitive in the infrared than in the optical.

Coulson, Caldwell, and Gieren (1986) used the method of Balona (1977) to find the mean radius of the Cepheid TT Aql employing a variety of magnitude and color index combinations. Their study gave a range of 68 to 107 $R_\odot$ for the mean radius depending on the particular combination used in the solution. Their mean period-radius relation for Cepheids, based on a nonlinear least squares solution using the V-mag. and Cousins (V-I) index, is in excellent agreement with the mean relation of Moffett and Barnes (1987), based on a linear least squares solution using the V-mag. and Johnson (V-R) index. For example, the mean period-radius relation of Coulson et al. (1986) predicts a 68.2 $R_\odot$ radius for a 10 day Cepheid and the corresponding relation of Moffett and Barnes (1987) predicts 70.8 $R_\odot$. The particular color index used has a much greater effect on the radius determination than does the particular mathematical technique employed in the realization.

All realizations of the BW-technique will benefit from the high precision radial velocity measurements being made of pulsating stars. The CORAVEL velocities, for example, have uncertainties of ±1 $km \cdot sec^{-1}$ or less for a single velocity measurement. Integrations of radial velocity curves of this quality yield linear displacements of very high precision.

Simon (1987) has proposed a realization which takes full advantage of these improved velocities by inverting the classical BW-method. Instead of trying to pick pairs of phases at which the surface brightnesses are equal, Simon picks pairs of phases at which the linear displacements are equal, since these can now be determined with high precision. Once phase pairs of equal radii are known, one can determine the values of the coefficients of the adopted surface brightness relation by noting that any difference in the flux levels is due to the surface brightness. The formulation of this realization is very general, since the adopted parameterization of the surface brightness can be of any form: linear or nonlinear, single color index, multiple color indices, dynamic parameters (kinetic energy, acceleration), etc. In his method, both the radial velocity and photometric curves are represented by a Fourier series. This realization requires photometric data of equivalent precision (±0.002 mag.) as the radial velocities. In addition, rather complete phase coverage over the pulsation cycle is needed to determine an accurate Fourier series representation of the velocity and photometric curves. The use of pairs of points has the same disadvantage as mentioned earlier, inhomogeneous pair members. However, Simon (1988) described a consecutive solution version of his method which removes the phase pair problem. Since this is a new realization, it has not been fully tested, but it does show great promise for improvements in the empirical approach.

THE MODELING APPROACH

All of these realizations use model atmospheres in some fashion to determine the photometric displacement curve. Realizations of this class suffer from the same problems encountered in the empirical approaches: poor phase and shape agreement between the photometric and spectroscopic displacement curves, strong sensitivity to the spectral region used, and difficulty in assigning realistic error estimates to the final mean radius.

Fernley et al. (1988) have used a very direct method which matches the observed flux to the predicted model atmosphere flux in order to determine the mean radius of the RR Lyrae star X Arietis. The total flux of the star at any phase, $L(\Phi)$, can be determined from the observed de-reddened flux at the top of the earth's atmosphere, ℓ_λ, from:

$$L(\Phi) = \int \ell_\lambda(\Phi)\, d\lambda \tag{16}$$

The effective temperature can be determined from the standard relation:

$$L(\Phi) = \theta^2(\Phi)\, \sigma\, T_{eff}^4(\Phi) \tag{17}$$

if an initial guess of the angular diameter, θ, is made. Using this estimate of the effective temperature, an improved determination of the angular diameter can be obtained by using the predicted flux from a model atmosphere via the equation:

$$L_{IR}(\Phi) = \theta^2(\Phi)\, F_{IR}\{T_{eff}(\Phi)\} \tag{18}$$

where L_{IR} is the observed value and F_{IR} is the model atmosphere prediction. Iteration between eqs. (17) and (18) yields the final values of $T_{eff}(\Phi)$, and $\theta(\Phi)$. In evaluating the integral in eq. (16), observations over the range of 2,000 to 22,000 Å were used. This gives temperature determination based on a wide spectral range encompassing over 95% of the total flux rather than a temperature estimate based on the rather narrow spectral region defined by a color index. Equation (18) makes use of the fact that most of the variation in the infrared magnitudes, H and K bands, is due to radius (surface area) changes with reduced sensitivity to temperature, metallicity, and gravity effects.

Cohen and Gordon (1987) used a similar realization to study RR Lyrae stars in the globular cluster M5. The major difference is that they use two narrow regions in the spectrum, one in the optical and the other in

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the infrared. The observed fluxes in these two regions are fitted to Kurucz (1979) model fluxes to find the angular diameter as a function of phase.

Burki and Benz (1982) and Burki and Meylan (1986) adopt a quadratic surface brightness relation calibrated by means of model atmospheres. The shape mismatch between the two displacement curves is still present, and the final mean radius excludes the phase interval of poor fit.

The most complex modeling realization is the CORS method developed by Caccin et al. (1981), Sollazzo et al. (1981),and Onnembo et al. (1985). They assume that the atmosphere of the pulsating star can be described by the QSA approximation (quasi-static approximation), the plane-parallel approximation for the radiative transfer and radiative and hydrostatic equilibrium. The simple assumption that the surface brightness can be described by a linear relation of a single color index is abandoned. The effective temperatures and gravities are described by two color indices calibrated via model atmospheres and the physical calibration of the Walraven photometric system. The CORS realization is currently the only one which uses all observations over the entire cycle to calculate the radius at one particular phase. In principle, the CORS method treats the BW-problem in a realistic rather than oversimplified fashion, but in practice, it is only as reliable as the model atmospheres and calibration of the photometric system used in its application. The fundamental problem remains; a dynamic situation is described by a series of static models.

The most popular version of modeling realizations uses synthetic colors produced from model atmospheres to effectively determine a surface brightness relation. Examples of this approach are the papers by: McNamara and Feltz (1977), Manduca et al. (1981), Carney and Latham (1984), Liu and Janes (1988), Jones (1988), and Jones et al. (1987a,b, 1988).

Most researchers using the modeling approach have pointed out the major limitation of this technique. The final mean radii are only as good as the theoretical model input parameters. The only available model atmospheres are static models which do not satisfactorily reproduce the dynamic effects present in real stars. Faced with describing a dynamic atmosphere with a static model, one is forced to either pick a spectral region where the dynamic effects (shock waves, rapid changes in effective gravity, etc.) are minimized or exclude certain phase regions, near minimum radius in the case of the RR Lyrae stars, or do both. The fundamental requirement for better mean radii via the modeling approach is better model atmospheres which include dynamic effects.

QUALITY OF RESULTS

Assigning realistic uncertainties to the final mean radius has been a problem for all of the realizations. Balona (1977) correctly points out that the method of linear least squares is not strictly valid

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when all of the variables are subject to observational errors. He recommends that the Principle of Maximum Likelihood be used in place of linear least squares since the observational errors are treated in the correct manner. However, the full advantage of the Maximum Likelihood method is only realized when both the standard errors and the form of their distributions are known for all observational quantities. This latter requirement has not been included in any of the so called Maximum Likelihood solutions. Hawley et al. (1987) and Fernley et al. (1987) recommend an iterative nonlinear least squares with outlier rejection as a better mathematical technique in BW-solutions. It is clear that linear least squares is not the best mathematical method to use. Nonlinear least squares is better and the full application of the Principle of Maximum Likelihood may be the best.

Simon (1988) has the best procedure for estimating the uncertainty due to the methodology of the particular realization. One would like to have a good estimate of the true uncertainty of the mean radius due to all sources (observational, methodological, physical aptness of the adopted surface brightness relation), but these are difficult to estimate. Error estimates for the modeling approaches are even more complex. In addition to the uncertainties referred to above, one has to include errors due to incorrect or missing physics in the model atmospheres, errors in constructing synthetic colors and magnitudes, and errors due to interpolation between grids in the model atmospheres.

Using current BW-realizations, the true uncertainty of individual mean radii are probably not better than ±10% (Gautschy 1987). If a large sample of stars is used to determine a mean period-radius relation, the result may be better than ±10%. However, as Gautschy (1987) points out, it is very difficult to compare the various period-radius relations since the distribution of stars, both in terms of the number of stars and their period distribution, makes direct comparisons hard to interpret.

CONCLUSIONS

We must concur with Gautschy's statement that, **THE** BW-method does not yet exist. Of the two general approaches, empirical and modeling, there are no compelling reasons to pick one as superior to the other. Since the two approaches actually complement each other, we strongly recommend that improvements to both be vigorously pursued. Knowledge of mean radii at the 10% level is simply not good enough to address current problems in pulsating stars. For example, the current precision in the radii implies an uncertainty of 30% in the mass determination.

On the observational side, several things are needed to improve all types of BW-realizations. More high quality radial velocities with good phase coverage and, ideally, with simultaneous photometry to eliminate the phasing problem would be a great help. Photometric data, again with good phase coverage and with a precision to match the radial velocities,

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are needed. Additional, high precision infrared observations are needed to realize the advantages offered in the infrared region of the spectrum.

On the theoretical side, better model atmospheres which include dynamic effects represent the most urgent need. More work on the projection factor for converting observed radial velocities to pulsational velocities would benefit all realizations.

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