CHAOTIC BEHAVIOR AND STATISTICAL ANALYSIS OF SOME MIRA AND SR STARS

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Abstract. Visual light curves of some Mira and semiregular variable stars are analyzed to search for their chaotic and statistical behaviour. Discussion are also given.

1. Data

We analyze light curves of some Mira and semiregular (SR) variable stars, which are listed in table I, to find out chaotic and statistical behavior. Light curves of these stars are obtained from the database of Variable Star Observers League in Japan (Saijo and Kiyota 1991). Periods of long-term observations are about 20000 days for these stars except for W Cyg, whose observation period is about 4000 days.

Before analysis, we average the same day's observations and smooth out the fluctuation with high frequency by averaged over 20 day's ones. After these arrangement, we get the time series with its unit time, 20 days.

TABLE I
Characteristics of Stars

Star	Type	Period	Magnitude	Sp.
o Cet	M	332	2.0–10.1	M5e
χ Cyg	M	408	3.3–14.2	S6e
R Leo	M	310	4.4–11.3	M6
V Boo	SRA	258	7.0–12.0	M6e
W Cyg	SRB	131	5.8–8.9	M4
RS Cnc	SRC	120	7.1–8.6	M6e

From GCVS 4th edition (1985)

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TABLE II
Period Obtained by AR Model

o Cet		χ Cyg		R Leo	
Period	Power	Period	Power	Period	Power
333.3	2.34	412.4	2.56	312.5	2.64
171.7	1.61	205.1	2.17	168.8	1.35
116.3	0.55	136.5	1.03	2105.3	1.21
		769.2	1.01		
V Boo		**W Cyg**		**RS Cnc**	
Period	Power	Period	Power	Period	Power
256.4	1.59	128.6	− 0.34	217.4	− 0.49
135.6	0.25	264.9	− 0.38	130.7	− 0.56
888.9	0.17			327.9	− 0.61

2. Statistical Analysis

To understand the statistical property of the time series, periodgram is a basic statistic and defined by Fourier transformation of time series. However, the periodgram analysis is difficult to determine the peak of spectrum. Therefore, we use Auto Regressive (AR) model to estimate the power spectrum (e.g. Akaike and Nakamura 1988). AR model expressed the value $x(t)$ as a linear combination of M past values, $x(t) = \sum_{m=1}^{M} a(m)x(t-m) + \varepsilon(t)$ (t=1,2,...), here $\varepsilon(t)$ is Gaussian noise. The parameters, $a(m)$ and $< \varepsilon\varepsilon >$, are estimated by Yule-Walker method. After estimating the parameter of AR model, we determine the power spectrum by

$$p(f) = \frac{\sigma^2}{|1 - \sum_{m=1}^{M} a(m)\exp(-i2\pi fm)|^2}$$

The main periods of the stars are easily determined because the power spectrum has a rational function form. The periods obtained from AR model are shown in table II.

3. Dynamical System Approach

The time series of Mira and SR stars behaves like a low dimensional chaotic dynamical system such as Lorenz system. Lorentz plot (return map of time interval between one local maximum and next local maximum) is useful to determine the low dimensional chaotic dynamics. However, in this case, this return map is difficult to plot because of missing observation. So we plot a stroboscopic return map, $x(t)$ vs $x(t+T)$. Stroboscopic return map of Mira stars has circular structure because of the periodic behavior, while that of SR stars shows more irregular map like a Brownian motion.

Correlation integral is one of a tool to determine the dimension of chaotic attractor (Grassberger and Procaccia 1983).

$$C_p(d) = \lim_{m\to\infty} \frac{1}{m^2} \sum_{i,j=1}^{m} H(d - |\mathbf{x}_i - \mathbf{x}_j|)$$

$$\mathbf{x}_i = (x(i), x(i+\tau), \ldots, x(i+(p-1)\tau))$$

where H is Heaviside function defined by $H(x) = 1$ for positive x, 0 otherwise. It is difficult to determine the dimension of Mira and SR stars, because of the limitation of observation number m and observational error. However, we conclude that some Mira stars' dimension is slightly above one and SR stars have more higher dimensions than those of Mira stars.

4. Discussions

Power spectrum of each star is obtained by AR analysis. Resultant periods and powers of each star are shown in table II, in which higher frequencies of main period of each stars are presented. Besides higher frequencies, we find another periods for R Leo and SR stars. Power spectrum of R Leo indicates probable second period of 2105 days, which is about 7 times of main period (312 days). For V Boo, we cannot find secondary period of 8–9 times main period, which Wood (1975) defined from the spectrum analysis. For W Cyg, we can find secondary period of 265 days, which is remarked in fourth edition of GCVS (1985). For RS Cnc, we find periods of 217 and 328 days besides main period of 131 days. But we cannot find period of 1700 days in GCVS. Period of 769 days for χ Cyg, which is about 1.9 times main period, cannot distinguish definitely from noise level.

From a dynamical system point of view, we cannot find evidence of chaos in the light curves of these stars. As mentioned above, however, SR stars show more irregular behavior and have more higher dimension than those of Mira stars. We cannot know the spatial structure of stars from light curve only. This lack of information play a role of noise. This effect induce the difficulty to understand a low dimensional dynamical system. We should recognize the variable star as a "spatio temporal chaos".

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