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A MATHEMATICAL AND COMPUTER SIMULATION MODEL OF THE RUNNING ATHLETE

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This thesis describes the construction, computerisation and simulation of a mathematical model of the running athlete. The model is in part biomechanical and in part biochemical; in part theoretical and in part empirical. The simulated output closely resembles results from human subjects.

A three variable force, velocity and distance, Newtonian biomechanical model of Keller [1] is examined and extended to include a fourth variable, mechanical power developed. The runner can control the force exerted through the legs (unless overcome by fatigue as is discussed below) and so affect acceleration and velocity, and hence also distance covered. A resistance to motion, proportional to velocity, and a maximal force limit the attainable velocity. Power is derived from force and velocity. This model segment is feed-forward linked by equating mechanical power demand to biochemical power supply.

A conceptual three compartment hydraulic model, proposed to represent human bioenergetics by Margaria [2], was examined in detail (see Morton [3]) but found to be unsuitable. An empirical three component model was therefore developed. Biochemical (bioenergetic) power is supplied by oxygen uptake, by anaerobic glycolysis and by the depletion of a phosphagen based body energy

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store (alactic power). The model derivation for oxygen uptake is described by Morton [4] . The alactic power derivation is based on an assumption of Margaria [2] , that the supply rate is proportional to the amount of store remaining. Glycolytic power is obtained by difference.

The glycolytic process is of particular interest, since the resulting accumulation of lactic acid in the working muscle can induce fatigue. A two-compartment, working muscle and blood volume, physiological model developed by Zouloumian and Freund [6, 7] for post exercise conditions, is modified for use during exercise. Lactic acid produced during glycolysis is input to the muscle compartment and diffuses between compartments or is removed by biochemical breakdown in accordance with prevailing concentrations. The lactic acid concentrations are the modelled variables in this segment.

Finally a negative feed-back link in the whole model is provided by an empirically derived fatigue equation. The maximal muscular force exertable is constrained inversely by the lactic acid concentration increase in the working muscles. The runner can of course by choice operate at a force below the prevailing constraining level, in which case the feed-back link does not operate. Sooner or later however, except for the lower purely aerobic workloads, muscular lactic acid will build up to such a level as to invoke the feed-back.

Parameters of the whole model include initial body energy stores, maximal muscular strength, resistance to motion coefficient, diffusion constants for lactic acid circulation, bioenergetic parameters of the oxidative, glycolytic and alactic energy processes, biomechanical energy equivalents, body mass, fatigue coefficients, et cetera, twenty-four in number. These parameters bind the relationships between the nine variables of the whole model, expressed as nine simultaneous differential equations with respect to time. Parameter values were in some cases obtained by estimation from data on exercising subjects, specially collected for that purpose.

In other cases values determined experimentally by other researchers and published in the literature were utilised.

Computer simulation was performed using numerical integration methods provided by a routine from the NAG [5] library. Simulated results on all nine variables are realistic, conforming well to those observed in the laboratory on exercising subjects. A collection of simulations were performed to discover which parameters were of major importance in the determination of enhanced performance in a number of different running tasks. These results by and large confirm established experimental knowledge.

There remains scope however for future refinements, in the main improving the theoretical content of the whole model and in the process extending it to include the recovery period after exercise.

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