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FEEDBACK PREDICTIVE CONTROL OF NONHOMOGENEOUS MARKOV JUMP SYSTEMS WITH NONSYMMETRIC CONSTRAINTS

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Abstract

We consider feedback predictive control of a discrete nonhomogeneous Markov jump system with nonsymmetric constraints. The probability transition of the Markov chain is modelled as a time-varying polytope. An ellipsoid set is utilized to construct an invariant set in the predictive controller design. However, when the constraints are nonsymmetric, this method leads to results which are over conserved due to the geometric characteristics of the ellipsoid set. Thus, a polyhedral invariant set is applied to enlarge the initial feasible area. The results obtained are for a more general class of dynamical systems, and the feasibility region is significantly enlarged. A numerical example is presented to illustrate the advantage of the proposed method.

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1. Introduction

In practice, especially for control engineering, system models are prone to abrupt changes. These are caused by environmental disturbances, temporary loss of communication or changes in interconnections of subsystems. Such situations are also found in economic systems, solar thermal central receivers and robotic manipulator systems. These systems belong to a class of systems, called Markov jump linear systems (MJSs), which are governed by a Markov chain [2] taking values in a finite set, where both time-evolving and event-driven mechanisms are involved. It has attracted a great deal of attention since the 1960s. The problems under investigation include Kalman filters [12], time delays [8, 13], gain scheduling [19], sliding mode control [14] and partly unknown transition probabilities [20, 21]. Typically, a MJS

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evolves according to a Markov stochastic process (or chain). The transition probability (TP) is a crucial factor in a Markov process (or chain), affecting the dynamical behaviour and performance of the MJS. Generally, TPs are assumed to be fully accessible or exactly known. However, TPs are often expensive to obtain and, hence, are not available for use in practice. Such situations can be found in communication networks [4], in which the packet dropouts are randomly distributed in different phases of the networks. In this case, the time-varying TP matrices become nonhomogeneous Markov chains. Another example is a helicopter system [10]; the airspeed variation in such system matrices can be modelled as a nonhomogeneous Markov process when the weather changes. A feasible approach to deal with such situations is to represent these time-varying uncertainties in terms of bounded sets. In this paper, a polytope set is applied to describe the characteristic of this time-varying uncertainty.

On the other hand, model predictive control (MPC) is well known for its capability to handle the constraints which are imposed on the control input and the state. The control algorithm is obtained by utilizing a prior-experimental process model to predict the future behaviour of the dynamics of the system. At each sampling instant, considering the current state of the system as the initial state, the MPC algorithm computes a sequence of control inputs, which are to be manipulated in the future by solving an optimal control problem. Then, only the first control input will be implemented to the process. At the next sequential sampling time, these actions are repeated. Many interesting results and their applications associated with MPC technology have been obtained in areas such as output feedback [18], system-on-chip implementation [15], dual decomposition [16] and actuator saturation [3]. Specifically, for the finite-horizon MPC, a standard MPC can be formulated as a compact quadratic program (QP) [9]. However, its closed-loop stability is hard to guarantee. On the other hand, for the case of infinite horizon, a worst-case performance cost algorithm was proposed [7] based on invariant sets (which restrains the states of the system in a certain range) as time evolves. In this way, the closed-loop stability is achieved. Bemporad et al. [1] explicitly characterized the solution of the constrained QP problem of MPC as a piecewise-linear state feedback law, based on the partition of the state space. The closed-loop control is realized through searching the corresponding control input on-line. Wan and Kothare [17] proposed an efficient off-line algorithm to obtain an on-line optimal control law by convex combination. However, in the existing literature, almost all the constraints considered are generally symmetric. In many engineering problems, these assumptions are often violated due to the fact that the machine is required to work at different operating points. Although ellipsoidal invariant methodology is widely applied to handle symmetric constraints, it is much too conservative for the estimation of the feasible area. The situation is much worse in the case of nonsymmetric constraints.

From the perspective of modelling and handling of different types of (symmetric or nonsymmetric) constraints to ensure meeting the performance requirements in the presence of abrupt changes in the system behaviour, one should consider MPC for nonhomogeneous Markov jump systems with nonsymmetric constraints.

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The rest of the paper is organized as follows. In Section 2, the control problem involving MJSs with nonhomogeneous TPs is defined. In Section 3, an on-line optimal predictive controller design algorithm for MJSs with nonhomogeneous TPs is developed, and the initial feasible area is enlarged by using a polyhedral invariant set. In Section 4, a numerical example is provided to illustrate the validity of the results. A brief summary in Section 5 concludes the paper.

Throughout the paper, the notation R^n stands for an n-dimensional Euclidean space, the transpose of the matrix A is denoted by $A^\top$, $E\{\cdot\}$ denotes the mathematical statistical expectation of the stochastic process, a positive-definite matrix is denoted by $P > 0$, $(a|b)$ means a based on b, I is the unit matrix with appropriate dimension and $*$ means the symmetric term in a symmetric matrix. The expression $\mathcal{A}^\top\mathcal{B}(*)$ denotes $\mathcal{A}^\top\mathcal{B}\mathcal{A}$.

2. Problem statement and preliminaries

Let (M, F, P) be a probability space, where M, F and P represent the sample space, the σ-algebra of events and the probability measure defined on F, respectively. Consider the following discrete nonhomogeneous MJS with nonsymmetric constraints:

$$\begin{cases} x_{k+1} = A(r_k)x_k + B(r_k)u_k, \\ y_k = C(r_k)x_k, \end{cases} \tag{2.1}$$

where $x_k \in R^{n_x}$ is the state vector of the system, $u_k \in R^{n_u}$ is the input vector of the system and $y_k \in R^{n_y}$ is the controlled output vector of the system. The output parameter matrix is $C(r_k)$. A discrete-time Markov stochastic process is defined as $\{r_k, k \geq 0\}$, which takes values in a finite state set. The set $\Gamma = \{1, 2, 3, \ldots, \sigma\}$ contains σ modes of system (2.1), and r_0 represents the initial mode. The matrices of system parameters are denoted by $A(r_k)$ and $B(r_k)$. System inputs and outputs are subject to the following constraints:

$$-u_{\text{llim}} \leq u_k \leq u_{\text{ulim}}, \tag{2.2}$$

$$-y_{\text{llim}} \leq y_k \leq y_{\text{ulim}}, \tag{2.3}$$

where $u_{\text{ulim}}, -u_{\text{llim}}$ denote the upper limit and lower limit of u_k and $y_{\text{ulim}}, -y_{\text{llim}}$ denote the upper limit and lower limit of y_k.

Remark 2.1. In the literature, symmetry constraints are generally considered. Here, we consider nonsymmetric constraints, that is, $u_{\text{llim}} \neq u_{\text{ulim}}, y_{\text{llim}} \neq y_{\text{ulim}}$.

The transition probability matrix is defined as $\Pi(k) = \{\pi_{ij}(k)\}$, with $i,\ j \in \Gamma$, and $\pi_{ij}(k) = P(r_{k+1} = j|r_k = i)$ is the transition probability from mode i at time k to mode j at time $k + 1$, which satisfies $\pi_{ij}(k) \geq 0$ and $\sum_{j=1}^{\sigma} \pi_{ij}(k) = 1$. For given vertices $\Pi^s(k)$, $s = 1, \ldots, N$, the time-varying transition matrix $\Pi(k)$ of the nonhomogeneous Markov jump system is constructed as

$$\Pi(k) = \sum_{s=1}^{N} \alpha_s(k)\Pi^s(k), \tag{2.4}$$

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where $0 \le \alpha_s(k) \le 1$, $\sum_{s=1}^{N} \alpha_s(k) = 1$, that is, a polytope is applied to describe the time-varying transition probability matrix of system (2.1).

REMARK 2.2. It should be pointed out that if $\Pi(k)$ is a constant matrix, as considered by Iosifescu [5] and Kemeny and Snell [6], the system (2.1) becomes a homogeneous Markov jump system.

To proceed further, we need the following preliminary results.

LEMMA 2.3. *Let $\mathcal{M}$ and $\mathcal{N}$ be positive-definite symmetric matrices. Then*

$$\mathcal{M} + \mathcal{M}^\top - \mathcal{N} \le \mathcal{M}\mathcal{N}^{-1}\mathcal{M}^\top.$$

PROOF. Since $\mathcal{N}$ is a positive-definite symmetric matrix, it follows that

$$(\mathcal{M} - \mathcal{N})\mathcal{M}^{-1}(\mathcal{M} - \mathcal{N})^\top \ge 0.$$

Consequently, the following inequality is derived:

$$\mathcal{M}\mathcal{N}^{-1}\mathcal{M}^\top - \mathcal{M} - \mathcal{M}^\top + \mathcal{N} \ge 0,$$

which completes the proof. □

DEFINITION 2.4. For a given initial state x_0 and an initial mode r_0, if

$$\lim_{T\to\infty} E\Big\{\sum_{k=0}^{T} x_k^\top x_k | x_0, r_0\Big\} < \infty,$$

then the discrete-time MJS (2.1) is said to be stochastically stable.

DEFINITION 2.5. Given a discrete MJS (2.1), a subset $\Theta = \{x \in R^{n_x} \mid x_k^\top P_k(r_k)x_k \le \gamma_k\}$ of the state space R^{n_x} is said to be an asymptotically stable mode-dependent invariant ellipsoid if the following property is satisfied:

$$\text{whenever } x_{k_0} \in \Theta, \quad x_k \in \Theta \quad \text{for all } k \ge k_0 \text{ and } x_k \to 0 \text{ as } k \to \infty.$$

THEOREM 2.6. *Consider system (2.1) (with $u_k = 0$). Suppose that there exists a set of symmetric positive-definite matrices $P_i^s > 0$ such that*

$$\mathcal{V}^+ = A_i^\top \mathcal{P}_{k+1} A_i - \mathcal{P}_k < 0, \tag{2.5}$$

where $\mathcal{P}_{k+1} = \sum_{j=1}^{\sigma}\sum_{s=1}^{N}\sum_{q=1}^{N} \alpha_s(k)\alpha_q(k+1)\pi_{ij}^s P_j^q$ and $\mathcal{P}_k = \sum_{s=1}^{N} \alpha_s(k) P_i^s$. Then this system with nonhomogeneous TP matrix (2.4) is stochastically stable.

PROOF. Define a Lyapunov function for system (2.1) (with $u_k = 0$):

$$V(x_k, r_k = i) = x_k^\top \mathcal{P}_k(r_k) x_k \quad \text{with } i \in \Gamma.$$

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Then

$$\begin{aligned}\Delta V(x_k, i) &= E\{V(x_{k+1}, r_{k+1})\} - V(x_k, r_k)\\ &= x_k^\top\Big(A_i^\top \sum_{j=1}^{\sigma}\sum_{s=1}^{N}\sum_{q=1}^{N} \alpha_s(k)\alpha_q(k+1)\pi_{ij}^s P_j^q A_i\Big)x_k - x_k^\top \sum_{s=1}^{N} \alpha_s(k) P_i^s x_k\\ &= x_k^\top \mathcal{V}^+ x_k.\end{aligned}$$

Clearly, condition (2.5) implies that $\Delta V(x_k, i) < 0$.

Denote $\delta = \min_k \lambda_{\min}(-\mathcal{V}^+)$ for all $i \in \Gamma$, where $\lambda_{\min}(-\mathcal{V}^+)$ is the minimal eigenvalue of $(-\mathcal{V}^+)$. Then $\Delta V(x_k, i) \le -\delta x_k^\top x_k$. Therefore, the following inequality holds:

$$E\Big\{\sum_{k=0}^{T} \Delta V(x_k, i)\Big\} = E\{V(x_{T+1}, T+1)\} - V(x_0, r_0) \le -\delta E\Big\{\sum_{k=0}^{T} x_k^\top x_k\Big\}$$

or, equivalently,

$$E\Big\{\sum_{k=0}^{T} x_k^\top x_k\Big\} \le \frac{1}{\delta}[V(x_0, r_0) - E\{V(x_{T+1}, T+1)\}] < \frac{1}{\delta} V(x_0, r_0),$$

which implies that $\lim_{T\to\infty} E\{\sum_{k=0}^{T} x_k^\top x_k\} \le (1/\delta)V(x_0, r_0) < \infty$.

From Definition 2.4, it now follows that system (2.1) (with $u_k = 0$) is stochastically stable. This completes the proof. □

3. MPC controller design for nonhomogeneous MJS with nonsymmetric constraints

3.1. MPC controller design The objective of this section is to derive an on-line optimal predictive control algorithm for the discrete nonhomogeneous MJS (2.1) with nonsymmetric constraints (2.2) and (2.3).

THEOREM 3.1. *Consider MJS (2.1) with nonhomogeneous TP (2.4) and nonsymmetric constraints (2.2) and (2.3). Suppose that there exists a set of matrices $F_k(r_k)$ such that the following optimization problem:*

$$\min_{u_k = F_k(r_k)x_k,\, r_k, r_{k+1} \in \Gamma} J_\infty(k),$$

subject to (2.2) and (2.3) and

$$E\{V(x_{k+1}, r_{k+1}|x_0, r_0)\} - E\{V(x_k, r_k|x_0, r_0)\} \le -E\{x_k^\top Q x_k + u_k^\top R u_k | x_0, r_0\}$$

has a solution. Then $J_\infty(k)$ has an upper bound, denoted as γ_k, at sampling time k, where $u_k = F_k(r_k)x_k$, $J_\infty(k) = E\{\sum_{k=0}^{\infty}(x_k^\top Q x_k + u_k^\top R u_k)|x_0, r_0\}$ and Q, R are weighting positive-definite matrices.

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Proof. Assume that at each sampling time k, a state feedback law $u(k+i|k) = F_k(r_k)x(k+i|k)$ is applied to minimize the value of $J_\infty(k)$. We shall derive an upper bound for $J_\infty(k)$. Define a quadratic Lyapunov function $V(x_k, r_k) = x_k^\top \mathcal{P}_k(r_k)x_k \le \gamma_k$, where $\mathcal{P}_k(r_k) > 0$. For MJS (2.1), suppose that $V(x_k, r_k)$ satisfies the following stability constraint:

$$E\{V(x_{k+1}, r_{k+1}|x_0, r_0)\} - E\{V(x_k, r_k|x_0, r_0)\} \le -E\{x_k^\top Q x_k + u_k^\top R u_k|x_0, r_0\}. \tag{3.1}$$

Summing the inequality (3.1) on both sides throughout for $k = 0$ to ∞ and noting that $\Delta V(x_k, r_k) = E\{V(x_{k+1}, r_{k+1}|x_0, r_0)\} - E\{V(x_k, r_k|x_0, r_0)\} < 0$, which implies the decrease of the Lyapunov function, leads to $\lim_{k\to\infty} x_k = 0$ or $\lim_{k\to\infty} V_k = 0$. Then, applying this result,

$$J_\infty(k) \le E\{V(x_k, r_k|x_0, r_0)\} = x_k^\top \mathcal{P}_k(r_k)x_k \le \gamma_k. \tag{3.2}$$

From inequalities (3.1) and (3.2), an upper bound on $J_\infty(k)$ is determined and this completes the proof. □

Remark 3.2. In Theorem 3.1, the problem of computing the minimum of $J_\infty(k)$ is transformed to that of obtaining the minimum of γ_k, which is a convex optimization problem and convenient to handle.

Theorem 3.3. *Consider MJS (2.1) with nonhomogeneous TP (2.4) and nonsymmetric constraints (2.2) and (2.3). Suppose that there exists a set of positive-definite matrices $g_k(r_k)$, Q_i^l and $Y_k(r_k)$ such that the following optimization problem has a solution:*

$$\min_{\gamma_k, g_k(r_k), Y_k(r_k)} \gamma_k, \tag{3.3a}$$

subject to

$$\begin{bmatrix} 1 & * \\ x_k & Q(r_k) \end{bmatrix} \ge 0 \quad \text{for all } r_k \in \Gamma, r_{k+1} \in \pi_{r_{k+1}}^{uk}, \tag{3.3b}$$

$$\begin{bmatrix} Z & Y_k(r_k) \\ * & g_k(r_k) \end{bmatrix} \ge 0, \quad Z_{tt} \le (u_{\lim}^t)^2, \tag{3.3c}$$

$$\begin{bmatrix} g_k(r_k) & * \\ C(r_k)\theta_k^\top(r_k) & M \end{bmatrix} \ge 0, \quad M_{hh} \le (y_{\lim}^h)^2, \tag{3.3d}$$

$$\begin{bmatrix} g_k(r_k)^\top + g_k(r_k) - H_k(r_k)^l & \theta_k^\top(r_k)\Xi(r_k)^l & g_k(r_k)^\top Q^{1/2} & Y_k(r_k)^\top R^{1/2} \\ * & \bar{Q}^m & 0 & 0 \\ * & * & \gamma_k I & 0 \\ * & * & * & \gamma_k I \end{bmatrix} > 0, \tag{3.3e}$$

where

$$\Xi(r_k)^l = \begin{bmatrix} \sqrt{\pi_{r_k 1}^l} I & \cdots & \sqrt{\pi_{r_k \sigma}^l} I \end{bmatrix}, \tag{3.3f}$$

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$$\bar{Q}^m = \begin{bmatrix} Q^m(1) & 0 & 0 & 0 \\ 0 & Q^m(2) & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & Q^m(\sigma) \end{bmatrix}. \tag{3.3g}$$

Here $\pi^{uk}_{r_{k+1}}$ *denotes the unknown transition probability at time* $k+1$, $P(r_k)^l = \gamma_k(H_k(r_k)^l)^{-1}$, $\theta_k(r_k) = A(r_k)g_k(r_k) + B(r_k)Y_k(r_k)$, Z_{tt} *denotes the tth diagonal elements of Z,* $u^t_{\lim}$ *denotes the tth element of the input constraints,* $t = 1, 2, \ldots, n_u$, M_{hh} *denotes the hth diagonal elements of M,* $h = 1, 2, \ldots, n_y$ *and* $y^h_{\lim}$ *denotes the hth element of the output constraints. Then, at current sampling time k,* $u(k+i|k) = F_k(r_{k+i})x_{k+i|k}$, $F_k(r_{k+i}) = Y_k(r_{k+i})g_k^{-1}(r_{k+i})$ *is the mode-dependent state feedback control law which minimizes the upper bound* γ_k *for the MPC objective* $J_\infty(k)$*, and stabilizes the closed-loop system within an invariant ellipsoid,* $\varepsilon = \{x_k^\top Q_k^{-1}(r_k)x_k \le 1\}$.

Proof. Denote $Q_k(r_k) = \gamma_k \mathcal{P}_k^{-1}(r_k)$. Then the condition $J_\infty(k) \le \gamma_k$ in (3.2) is implied by the following linear matrix inequalities (LMIs):

$$\begin{bmatrix} 1 & * \\ x_k & Q_k(r_k) \end{bmatrix} \ge 0 \quad \text{for all } r_k \in \Gamma,\ r_{k+1} \in \pi^{uk}_{r_{k+1}},$$

which will be used to construct an invariant ellipsoid. The input and output constraints are guaranteed by (3.3c) and (3.3d), while (3.3e) guarantees stochastic stability (for details, see the article by Wan and Kothare [17]).

From Lemma 2.3, $g_k(r_k)^\top (H_k(r_k)^l)^{-1} g_k(r_k) \ge g_k(r_k)^\top + g_k(r_k) - H_k(r_k)^l$. Then it follows from (3.3e) that

$$\begin{bmatrix} g_k(r_k)^\top (H_k(r_k)^l)^{-1} g_k(r_k) & \theta^\top(r_k)\Xi(r_k)^l & g_k(r_k)^\top Q^{1/2} & Y_k(r_k)^\top R^{1/2} \\ * & \bar{Q}^m & 0 & 0 \\ * & * & \gamma_k I & 0 \\ * & * & * & \gamma_k I \end{bmatrix} > 0$$

for all $l = 1, \ldots, N, m = 1, \ldots, N$ or, equivalently,

$$\begin{bmatrix} g_k(r_k)^\top & 0 & 0 & 0 \\ * & \bar{Q}^m & 0 & 0 \\ * & * & I & 0 \\ * & * & * & I \end{bmatrix} \begin{bmatrix} (H_k(r_k)^l)^{-1} & \theta^\top(r_k)\Xi(r_k)^l & g_k(r_k)^\top Q^{1/2} & Y_k(r_k)^\top R^{1/2} \\ * & (\bar{Q}^m)^{-1} & 0 & 0 \\ * & * & \gamma_k I & 0 \\ * & * & * & \gamma_k I \end{bmatrix} (*) > 0,$$

which is implied by

$$\begin{bmatrix} (H_k(r_k)^l)^{-1} & \theta^\top(r_k)\Xi(r_k)^l & g_k(r_k)^\top Q^{1/2} & Y_k(r_k)^\top R^{1/2} \\ * & (\bar{Q}^m)^{-1} & 0 & 0 \\ * & * & \gamma_k I & 0 \\ * & * & * & \gamma_k I \end{bmatrix} > 0.$$

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Note that $P(r_k)^l = \gamma(H_k(r_k)^l)^{-1}$ and $Y_k(r_k) = F(r_k)g_k(r_k)$. Let $\bar{P}^m = \gamma_k(\bar{Q}^m)^{-1}$. Then we get the inequality

$$(A(r_k) + B(r_k)F(r_k))^\top\Big(\sum_{r_{k+1}=1}^{\sigma} \pi^l_{r_k r_{k+1}} P^m_{r_{k+1}}\Big)(A(r_k) + B(r_k)F(r_k)) - P^l(r_k) \\ \leq -Q - F(r_k)^\top RF(r_k),$$

which, by multiplying appropriate coefficients, guarantees the stability of the system.

Next we shall show that the feedback law will stabilize the closed-loop system within the invariant ellipsoid $\varepsilon = \{x_k^\top Q_k^{-1}(r_k)x_k \leq 1\}$. Suppose that the optimal values at current sampling time, k, are

$$\mathcal{P}^*_k(r_k) = \gamma^*_k(Q^*_k(r_k))^{-1},$$
$$F^*_k(r_k) = Y^*_k(G^*_k(r_k))^{-1}.$$

From (3.3e), it follows that

$$E\{x_k^\top \mathcal{P}^*_k(r_k)x_k\} \geq E\{x_{k+1}^\top \mathcal{P}^*_k(r_{k+1})x_{k+1}\} + x_k^\top Qx_k + x_k^\top (F^*_k(r_k))^\top RF^*_k(r_k)x_k.$$

Since $\mathcal{P}^*_{k+1}(r_{k+1})$ is the optimal value at time $k+1$, $\mathcal{P}^*_k(r_{k+1})$ is a feasible value at time $k+1$. Thus,

$$x_{k+1}^\top \mathcal{P}^*_{k+1}(r_{k+1})x_{k+1} \leq x_{k+1}^\top \mathcal{P}^*_k(r_{k+1})x_{k+1}$$

and, hence,

$$E\{x_k^\top \mathcal{P}^*_k(r_k)x_k\} \geq E\{x_{k+1}^\top \mathcal{P}^*_{k+1}(r_{k+1})x_{k+1}\} + x_k^\top Qx_k + x_k^\top F_k^\top(r_k)RF_k(r_k)x_k. \tag{3.4}$$

Since the weighting matrices $Q > 0$ and $R > 0$ are positive definite, it follows from Theorem 2.6 that inequality (3.4) implies $\Delta V(x_k, r_k) < 0$. Thus, MJS (2.1) with nonhomogeneous TP (2.4) and nonsymmetric constraints (2.2) and (2.3) is stochastically stable. From (3.4), it follows that

$$E\{x_k^\top \mathcal{P}^*_k(r_k)x_k\} > E\{x_{k+1}^\top \mathcal{P}^*_{k+1}(r_{k+1})x_{k+1}\},$$

which means that $E\{x_k^\top \mathcal{P}^*_k(r_k)x_k\}$ is strictly decreasing. This indicates that the invariant ellipsoid is contracting and, hence, is an asymptotically stable mode-dependent invariant ellipsoid. This completes the proof. □

Remark 3.4. Here the problem of computing the minimum of $J_\infty(k)$ implies the infinite horizon of predictive control, which guarantees the stability of the system.

3.2. Enlargement of initial feasible area The ellipsoidal invariant set constructed in previous sections is applied to find an approximation for the polyhedral constraints. Since this set is symmetric and contains the origin, it tends to be rather conservative. The situation will be worse when the constraints are nonsymmetric. One effective methodology is to remove the redundant constraints leading to a less conservative polyhedral invariant set, which is illustrated in Figure 1. Pluymers et al. [11] have presented an efficient algorithm to remove redundant constraints. Here we extend this algorithm to nonhomogeneous MJSs to enlarge the initial feasible area.

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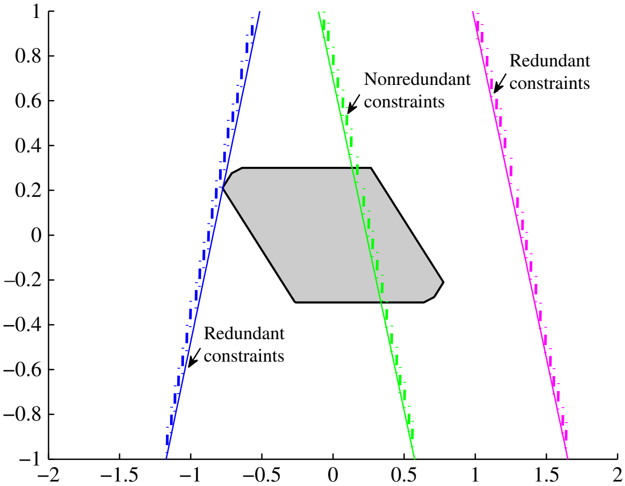

FIGURE 1. Redundant and nonredundant constraints.

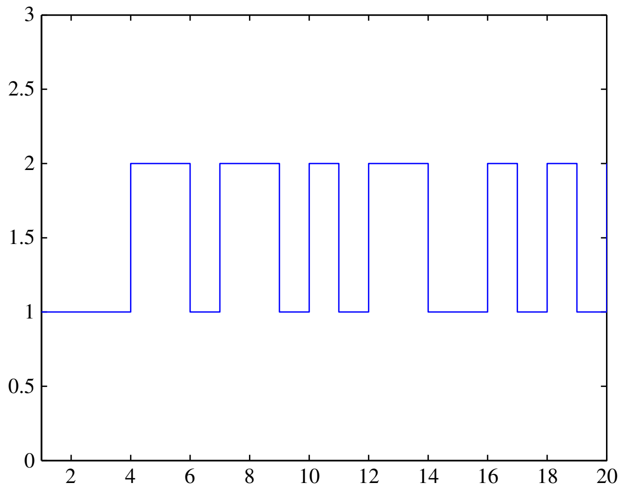

FIGURE 2. One sampled mode evolution.

4. An illustrative example

Consider the discrete-time MJS with two modes ($\sigma = 2$):

$$A_1 = \begin{bmatrix} 0.22 & -0.33 \\ 0.12 & 1.03 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.32 \\ 0.52 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.17 & -0.15 \\ 0.26 & 1.21 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.63 \\ 0.61 \end{bmatrix}.$$

The output parameter matrices $C(r_k)$ are $\left[\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right]$.

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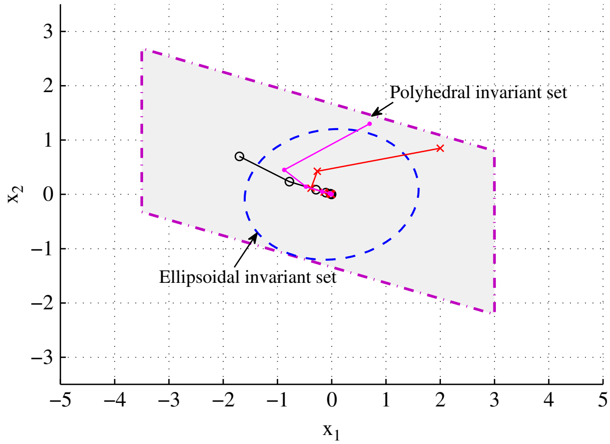

FIGURE 3. Trajectory of system state.

Algorithm 1 MPC using polyhedral invariant set.

Removal of redundant constraints

1. Compute the minimizer to obtain $\gamma(r_k)$, $Q(r_k)$, $X(r_k)$, $Y(r_k)$, $F(r_k)$ from Theorem 3.3. For each $F(r_k)$, the corresponding polyhedral invariant set is constructed by the following steps: let $S(r_k) = [C^\top(r_k), -C^\top(r_k), F^\top(r_k), -F^\top(r_k)]^\top$, $d(r_k) = [y^\top_{\text{ulim}}(r_k), y^\top_{\text{llim}}(r_k), u^\top_{\text{ulim}}(r_k), u^\top_{\text{llim}}(r_k)]^\top$.

2. Select row m from $(S(r_k), d(r_k))$ and check whether $S_m(r_k)(A(r_k) + B(r_k)F(r_k)) \leq d_m(r_k)$ is redundant by solving the following linear programming problem:

$$\begin{aligned} &\max \quad \rho_m \\ \text{such that} \quad &\rho_m = S_m(r_k)(A(r_k) + B(r_k)F(r_k))x - d_m(r_k) \\ \text{and} \quad &S(r_k)x \leq d(r_k). \end{aligned}$$

3. If $\rho_m > 0$, this means that the constraint $S_m(r_k)(A(r_k) + B(r_k)F(r_k)) \leq d_m(r_k)$ is nonredundant. Update the nonredundant constraints as $S(r_k) = [S^\top(r_k), (S_m(r_k)(A(r_k) + B(r_k)F(r_k)))^\top]^\top$, $d(r_k) = [d(r_k)^\top, d_m(r_k)^\top]^\top$.

The input and output constraints are $u_{\text{lmin}} = -2.2$, $u_{\text{umax}} = 1.6$, $y_{\text{lmin}} = -2.7$, $y_{\text{umax}} = 2.5$. The weighting matrices are $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $R = 0.00002$.

A sampled mode evolution is shown in Figure 2. The nonhomogeneous transition probability matrices are given as $\pi_1 = \begin{bmatrix} 0.25 & 0.75 \\ 0.36 & 0.64 \end{bmatrix}$, $\pi_2 = \begin{bmatrix} 0.16 & 0.84 \\ 0.45 & 0.55 \end{bmatrix}$, $\pi_3 = \begin{bmatrix} 0.3 & 0.7 \\ 0.4 & 0.6 \end{bmatrix}$, $\pi_4 = \begin{bmatrix} 0.2 & 0.8 \\ 0.3 & 0.7 \end{bmatrix}$. Three different initial states are $x_{01} = [-1.7\ 0.7]^\top$; $x_{02} = [-1.4\ 0.5]^\top$; $x_{03} = [2\ 0.85]^\top$. The corresponding three state trajectories are shown in Figure 3, where the polyhedral invariant set is much less conservative when compared with the ellipsoidal invariant set.

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5. Conclusions

We discussed the problem of feedback predictive control of nonhomogeneous MJSs with nonsymmetric constraints. The main advantage of this methodology is that it is applicable to a more general class of systems, while reducing the conservativeness of the method based on the ellipsoidal invariant set. From the numerical example, we observed the effectiveness of the method proposed. The developed results are expected to extend to issues such as output feedback and multi-objective predictive control of constrained nonhomogeneous MJSs in a future work.

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