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MANIFOLDS OF SMOOTH MAPS

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We show that the space of smooth maps from a compact smooth manifold into another smooth manifold can be endowed with the structure of a smooth manifold if we use the Γ-differentiation of Yamamuro. We then generalise the Smale Density Theorem to mappings between these manifolds.

The main purpose of this paper is to show that the space $C^\infty(X, Y)$ of smooth maps from a *compact* C^∞ manifold X into a finite-dimensional C^∞ manifold Y can be endowed with the structure of a smooth manifold if we use the Γ-differentiation of Yamamuro [*12*]. Here, for simplicity, we suppose that Y is finite-dimensional even though the result can be easily extended to an infinite-dimensional Y admitting a smooth spray [*1*]. Similar results have been obtained by Leslie [*8*] and Gutknecht [*4*] using the C_π^∞ differentiability of Keller and the C_Γ^∞ differentiability of Fischer respectively [*3*], [*7*]. The case of a non-compact X has also been investigated by Michor [*9*] using Keller's C_π^∞ differentiability.

The paper consists of three sections. In §1, we define general manifolds modelled on a Γ-family [*12*] of locally convex spaces, manifolds which we term as *Γ-manifolds*. Then corresponding to the $B\Gamma$-differentiability in [*12*] we have the subclass of *$B\Gamma$-manifolds*. Section 2 is for showing that $C^\infty(X, Y)$ is in general a Γ-manifold of class C^∞. We also give examples of $B\Gamma$-manifolds of class C^∞. The last section, §3, is devoted to a generalisation of the Smale Density

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Theorem [2] to the case of $B\Gamma$-maps between Γ-manifolds.

This paper depends heavily on [12] and some of its results have been announced in [10] and [11].

1. Γ- and $B\Gamma$-manifolds

Let F be a Γ-family of locally convex spaces [12] and X be a Hausdorff space. A Γ-*chart* on X is a triple (U, α, E) of an open set U of X, $E \in F$ and a homeomorphism α of U into E. Let $k \geq 1$ be an integer. Then, two Γ-charts (U, α, E) and (V, β, F) are said to be C^k_Γ-*compatible* if the transition map $\beta \circ \alpha^{-1} : \alpha(U \cap V) \to \beta(U \cap V)$ is a C^k_Γ-diffeomorphism [12].

A collection of Γ-charts $\{(U_\alpha, \alpha, E_\alpha)\}$ is called a Γ-*atlas of class* C^k if

(1) $\{U_\alpha\}$ is a covering of X,

(2) each member is C^k_Γ-compatible with every member of the collection.

It is said to be *maximal* if every Γ-chart that is C^k_Γ-compatible with all members of the atlas belongs to the atlas.

A Γ-*manifold of class* C^k modelled on the family F is a Hausdorff space X equipped with a maximal Γ-atlas of class C^k. Each Γ-chart in the maximal Γ-atlas will be called an *admissible chart*. A Γ-chart (U, α, E) is said to be *(centred) at* $x \in X$ if we have $x \in U$. If the model spaces coincide with a single $E \in F$ then the Γ-manifold is said to be *modelled on* E. When the transition maps are $C^k_{B\Gamma}$-maps, the Γ-manifold is called a $B\Gamma$-*manifold*.

Let X and Y be Γ-manifolds of class C^k modelled on the same family F. A map $f : X \to Y$ is said to be *of class* C^k_Γ or a C^k_Γ-*map* if, for each $x \in X$ and each admissible chart (V, β, F) on Y centred at

$f(x)$, there exists an admissible chart (U, α, E) on X centred at x , such that $f(U) \subseteq V$ and the *local representative* of f , $\beta \circ f \circ \alpha^{-1} : \alpha(U) \to \beta(V)$, is a C^k_Γ-map. In a similar manner, the $C^k_{B\Gamma}$-maps between $B\Gamma$-manifolds of class C^k modelled on the same Γ-family can be defined.

Let F be a Γ-family and X be a Γ-manifold of class C^k modelled on F . We shall always assume that $\mathbb{R} \in F$ and $\Gamma_{\mathbb{R}}$ consists of the absolute value norm. A *Γ-curve at* $x \in X$ is a C^1_Γ-map c from an open subset of $\mathbb{R}$ containing zero into X such that $c(0) = x$. By definition, if c is a Γ-curve at $x \in X$ and (U, α, E) is an admissible chart at x , $\alpha \circ c$ is a C^1_Γ-map into E . Since $L_\Gamma(\mathbb{R}, E) = E$, $(\alpha \circ c)'(0)$ is identified with an element of E .

We define the *Γ-tangent space* T_xX of X at x as the set of usual equivalent classes of Γ-curves at x [2]. Then, every admissible chart (U, α, E) at x defines a bijection of T_xX onto E and if (V, β, F) is another admissible chart at x , E and F are Γ-isomorphic; they are $B\Gamma$-isomorphic when X is a $B\Gamma$-manifold. By this bijection, we transplant the locally convex structure of E , including its calibration, onto T_xX . Therefore, the family $\{T_xX : x \in X\}$ becomes a Γ-family.

When X and Y are Γ-manifolds of class C^k $(k \geq 1)$ modelled on a family F and $f : X \to Y$ is a C^k_Γ-map, then we have the *Γ-tangent map* of f : $T_xf : T_xX \to T_{f(x)}Y$. Similarly the *$B\Gamma$-tangent map* can be defined for a $C^k_{B\Gamma}$-map between $B\Gamma$-manifolds of class C^k .

2. The Γ-manifold $C^\infty(X, Y)$

Let X, Y be finite-dimensional C^∞ manifolds without boundary, X being *compact*. In order to show that $C^\infty(X, Y)$ is a Γ-manifold of class C^∞ , we first prove the Γ-version of the Omega Lemma [1, Corollary 3.8].

Let E, F, G be Banach spaces, $X \subseteq E$ be *compact* and $Y \subseteq F$ be open. Then, for any integer $n \geq 0$ and $f \in C^{\infty}(X, F)$, define

(1) $$\|f\|_n = \sup\{\|f(x)\| + \|Df(x)\| + \ldots + \|D^n f(x)\| : x \in X\} < +\infty$$

and let $C^n(X, F)$ denote the Banach space of all C^n maps $X \to F$.

Consider $C^{\infty}(X, F)$ as the intersection of all $C^n(X, F)$, $n \geq 0$, and regard it as a locally convex space calibrated by the following sequence of increasing norms defined by (1):

(2) $$\{\|\cdot\|_n : n \geq 0\} .$$

Let $C^n(X, Y)$ (respectively $C^{\infty}(X, Y)$) be the subset of all $f \in C^n(X, F)$ (respectively $f \in C^{\infty}(X, F)$) such that $f(X) \subseteq Y$. Then it is clear that $C^n(X, Y)$ is open in $C^n(X, F)$ for each $n \geq 0$, and $C^{\infty}(X, Y)$ is open in $C^{\infty}(X, F)$ calibrated by (2).

Let $C^n(X, G)$ and $C^{\infty}(X, G)$ be similar spaces, where $C^{\infty}(X, G)$ is calibrated by a similar sequence of increasing norms

(3) $$\{\|\cdot\|_n : n \geq 0\} .$$

(2.1) (Γ-omega lemma). *Let E, F, G be Banach spaces, $X \subseteq E$ compact and $Y \subseteq F$ open. Then, for a fixed $g \in C^{\infty}(Y, G)$, the map*

$$g_* : C^{\infty}(X, Y) \subseteq C^{\infty}(X, F) \to C^{\infty}(X, G) : f \to g_*(f) = g \circ f$$

is C^{∞}_{Γ} with respect to the calibration

$$\Gamma = \{(\|\cdot\|_n, \|\cdot\|_n) : n \geq 0\}$$

for the pair $(C^{\infty}(X, F), C^{\infty}(X, G))$ where the first and second $\|\cdot\|_n$ are defined by (2) *and* (3) *respectively.*

Proof. This follows easily from [12, Remark, p. 26] and [6, Theorem 6, p. 117].

Now let X be a compact C^{∞} manifold and $\pi : E \to X$, $\rho : F \to X$ be two C^{∞} (Banach) vector bundles over X. Denote by $S^{\infty}(\pi)$ and $S^{\infty}(\rho)$ the spaces of C^{∞}-sections of π and ρ respectively. Endow them with the following calibrations. Cover π and ρ by a finite number of

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pseudo-compact *VB*-charts $\left(U_i, \alpha_i^0, \alpha_i\right)$ and $\left(U_i, \alpha_i^0, \beta_i\right)$, $1 \le i \le n$, where $\left\{\left(U_i, \alpha_i^0\right) : 1 \le i \le n\right\}$ is an atlas of X [1, p. 15]. Then each $\gamma \in S^\infty(\pi)$ has the following principal part with respect to the *VB*-chart $\left(U_i, \alpha_i^0, \alpha_i\right)$:

(4) $\tilde{\gamma}_{\alpha_i} : \alpha_i^0\left(\overline{U}_i\right) \to E_{\alpha_i}$ $\left(1 \le i \le n\right.$, $\overline{U}_i$ being the closure of $\left.U_i\right)$

with $\tilde{\gamma}_{\alpha_i} \in C^\infty\left(\alpha_i^0\left(\overline{U}_i\right), E_{\alpha_i}\right)$ and $\alpha_i^0\left(\overline{U}_i\right)$ is compact.

For each $r \ge 0$, define

(5) $$\|\gamma_{\alpha_i}\|_r = \sup\left\{\|\tilde{\gamma}_{\alpha_i}(x)\| + \|D\tilde{\gamma}_{\alpha_i}(x)\| + \ldots + \left\|D^r\tilde{\gamma}_{\alpha_i}(x)\right\| : x \in \alpha_i^0\left(\overline{U}_i\right)\right\}$$

and, for each $\gamma \in S^\infty(\pi)$, define

(6) $$\|\gamma\|_r = \sum_{i=1}^{n} \|\tilde{\gamma}_{\alpha_i}\|_r .$$

Then endow $S^\infty(\pi)$ with the following calibration

(7) $$\Gamma_{S^\infty(\pi)} = \left\{\|\cdot\|_r : r \ge 0\right\} .$$

Similarly, $S^\infty(\rho)$ is endowed with the calibration

(8) $$\Gamma_{S^\infty(\rho)} = \left\{\|\cdot\|_r : r \ge 0\right\} .$$

Now, if $\Omega \subseteq E$ is an open set such that $\pi|\Omega : \Omega \to X$ is surjective, let $S^\infty(\Omega) \subseteq S^\infty(\pi)$ be the open set of sections with image contained in Ω.

If $f : \Omega \subseteq E \to F$ is a C^∞ fibre-preserving map, let

$$f_* : S^\infty(\Omega) \subseteq S^\infty(\pi) \to S^\infty(\rho)$$

be the induced map defined by $f_*(\gamma) = f \circ \gamma$ for all $\gamma \in S^\infty(\Omega)$. Then the local Γ-omega lemma in (2.1) can be globalised as follows.

(2.2) *Let* $\pi : E \to X$, $\rho : F \to X$, $f : \Omega \subseteq E \to F$ *be as above. Then* $f_* : S^\infty(\Omega) \subseteq S^\infty(\pi) \to S^\infty(\rho)$ *is* C^∞_Γ *with respect to the calibration* $\Gamma = \left\{\left(\|\cdot\|_r, \|\cdot\|_r\right) : r \ge 0\right\}$ *given by* (7) *and* (8) *for the pair*

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$(S^\infty(\pi), S^\infty(\rho))$ *, and, for any integer* $r \geq 0$ *, the* rth Γ*-derivative of* f_* *is given by*

(9) $$D^r f_* = (\partial^r f)_*$$

where ∂ *denotes the vertical derivative* [1].

Proof. Cover π and ρ by a finite number of pseudo-compact VB-charts as above. Apply (2.1) to the composite of the second projection and the local representative of f , and then use the definition of the vertical derivative.

Now, back to the space $C^\infty(X, Y)$. Let $s : TY \to T^2Y$ be a C^∞ spray on Y . Then there is an open neighbourhood $\mathcal{D}_s \subseteq TY$ of the zero-section and an open neighbourhood $F_s \subseteq Y \times Y$ of the diagonal such that $\mathrm{Exp}^s : \mathcal{D}_s \to F_s$ is a C^∞-diffeomorphism [1]. If $f \in C^\infty(X, Y)$ we have the diffeomorphism $s_f \equiv f^*\mathrm{Exp}^s : f^*\mathcal{D}_s \to \mathcal{D}_{f,s}$ where $\mathcal{D}_{f,s} \subseteq X \times Y$ is an open neighbourhood of the graph of f .

If $U_{f,s} \subseteq C^\infty(X, Y)$ consists of maps g such that $\mathrm{graph}(g) \subseteq \mathcal{D}_{f,s}$, then the map

(10) $$\phi_{f,s} : U_{f,s} \to C^\infty_f(X, TY) \equiv S^\infty(f^*TY)$$

defined by $\phi_{f,s}(g) = s_f^{-1} \circ \mathrm{graph}(g)$ is a homeomorphism of $U_{f,s}$ onto an open subset of $C^\infty_f(X, TY)$, the space of C^∞ vector-fields along f . We call the pair $(U_{f,s}, \phi_{f,s})$ a *natural* chart for $C^\infty(X, Y)$.

(2.3) *Let* X *be a compact* C^∞ *manifold and* Y *be a finite-dimensional* C^∞ *manifold. Then the family* $(U_{f,s}, \phi_{f,s})$ *of natural charts is a* Γ*-atlas of class* C^∞ *on* $C^\infty(X, Y)$ *if, for each* $f \in C^\infty(X, Y)$ *, we take as calibration for* $S^\infty(f^*TY)$ *the one defined by* (7). *Hence* $C^\infty(X, Y)$ *is a* Γ*-manifold of class* C^∞ *modelled on the* Γ*-family* $\{S^\infty(f^*TY) : f \in C^\infty(X, Y)\}$.

Proof. Let $(U_{f,s}, \phi_{f,s})$ and $(U_{f',s'}, \phi_{f',s'})$ be natural charts and suppose that $U_{f,s} \equiv U_{f',s'}$. It suffices to show that $\phi_{f',s'} \circ \phi_{f,s}^{-1}$

is a C^{∞}_{Γ}-diffeomorphism.

But it is clear that $\phi_{f',s'} \circ \phi^{-1}_{f,s} = F_*$ where

$$F = [f'^{*}\mathrm{Exp}^{s'}]^{-1} \circ [f^{*}\mathrm{Exp}^{s}] .$$

Since s, s' are C^{∞} sprays and f, f' are C^{∞}, it is clear that F is a C^{∞} fibre-preserving map. Thus, by (2.2), F_* is C^{∞}_{Γ}. Clearly, $F_*^{-1} = (F^{-1})_*$, so F_* is a C^{∞}_{Γ}-diffeomorphism.

As an immediate consequence, the space $\mathrm{Diff}^{\infty}(X)$ of all C^{∞}-diffeomorphisms of a compact C^{∞} manifold X is a Γ-manifold of class C^{∞}. Similarly, the space $\mathrm{Emb}^{\infty}(X, Y)$ of all C^{∞}-embeddings of X into a finite-dimensional C^{∞} manifold Y is also a Γ-manifold of class C^{∞}.

We now give some simple examples of $B\Gamma$-manifolds of class C^{∞}. Let X be a compact C^{∞} manifold as always, and let Y be either the cylinder, or the cone, or the 1-sphere in $\mathbb{R}^3$ defined in [5, pp. 115-117]. Then we have explicit formulae for the corresponding exponential maps [5, pp. 116-118] and, using these formulae, it can be seen that $C^{\infty}(X, Y)$ is a $B\Gamma$-manifold of class C^{∞} [10].

More generally, let us denote by $\mathcal{C}$ the family of all Riemannian manifolds Y such that $C^{\infty}(X, Y)$ can be given a $B\Gamma$-manifold structure. Then it can be seen that

(i) every Euclidean space $\mathbb{R}^n$ belongs to $\mathcal{C}$,

(ii) if $Y \in \mathcal{C}$ and $Z \in \mathcal{C}$, then the product $Y \times Z \in \mathcal{C}$,

(iii) if $Y \in \mathcal{C}$ and Z is isometric to Y, then $Z \in \mathcal{C}$.

In particular, all flat manifolds belong to $\mathcal{C}$. The answer to the problem of whether $\mathcal{C}$ contains a non flat manifold is still not known.

3. The Smale Density Theorem

In this section, for the sake of generality and the possibility of application, we state and prove the Smale Density Theorem [2] in its general form which is due to Yamamuro (see [10]). The $B\Gamma$-version (5.1) in [11, p. 336] then follows immediately.

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Let $\mathcal{F}$ be a Γ-family, X, Y be Γ-manifolds of class C^r $(r \geq 1)$ modelled on $E, F \in \mathcal{F}$ respectively. Let $f : X \to Y$ be a mapping and $x \in X$ be a point. Then a pair of Γ-charts of class C^r, (U, α) and (V, β), of X and Y respectively is said to be a *pair of strong* Γ*-charts of class* C^r *for* f *at* x (or for short, a *pair of* SC^r_Γ*-charts for* f *at* x) if and only if $x \in U$, $f(x) \in V$, $f(U) \subseteq V$ and the local representative $f_{\alpha\beta} : \alpha(U) \to \beta(V)$ is a $C^r_{B\Gamma}$ map. We say that f is *strongly* C^r_Γ *at* x (for short *of class* SC^r_Γ *at* x) if and only if f is C^r_Γ at x, and if in addition there is a pair $\{(U, \alpha), (V, \beta)\}$ of SC^r_Γ-charts for f at x. f is a *strongly* C^r_Γ *map* (or *of class* SC^r_Γ) if and only if it is SC^r_Γ at every $x \in X$. Note that when X and Y are $B\Gamma$-manifolds of class C^r then SC^r_Γ maps $X \to Y$ coincide with $C^r_{B\Gamma}$ maps $X \to Y$.

Now consider a SC^r_Γ-map $f : X \to Y$ between Γ-manifolds of class C^r $(r \geq 1)$. We say that f *has the* $B\Gamma$*-Fredholm property at* $x \in X$ if, with respect to a pair of SC^r_Γ-charts $\{(U, \alpha), (V, \beta)\}$, the Γ-derivative $f'_{\alpha\beta}(\alpha(x)) : E \to F$ is a $B\Gamma$-Fredholm linear map [11], [12]. In this case, we define *the index of* f *at* x *with respect to the pair of* SC^r_Γ*-charts* $\{(U, \alpha), (V, \beta)\}$ by

$$\text{(11)} \qquad \operatorname{ind}\big(f; x, (U, \alpha), (V, \beta)\big) = \operatorname{ind} f'_{\alpha\beta}\big(\alpha(x)\big) .$$

We say that $f : X \to Y$ has the $B\Gamma$*-Fredholm property* if and only if it has the $B\Gamma$-Fredholm property at every $x \in X$.

(3.1) *Let* X, Y *be* Γ*-manifolds of class* C^r $(r \geq 1)$ *modelled on* $E, F \in \mathcal{F}$, E *being sequentially complete, and let* $f : X \to Y$ *be a* SC^r_Γ*-map having the* $B\Gamma$*-Fredholm property at a point* $x \in X$. *Then we can always find admissible* Γ*-charts of class* C^r, (U, α) *and* (V, β) *at* x

and $f(x)$ *respectively with the following properties:*

(i) $E = E_1 \oplus_{B\Gamma} E_2$ *where* E_1 *and* E_2 *are closed subspaces of* E, $\dim E_1 = n < +\infty$, $U \subseteq$ *domain of* f ; α *maps* U C^r_Γ*-diffeomorphically onto* $B_1 + B_2$ *with* B_i *closed, convex, circled neighbourhoods of* 0 *in* E_i $(i = 1, 2)$;

(ii) $F = F_1 \oplus_{B\Gamma} F_2$ *where* F_1 *and* F_2 *are closed subspaces of* F, $\dim F_1 = p < +\infty$, $f(U) \subseteq V$, β *maps* V C^r_Γ*-diffeomorphically onto an open subset of* $F_1 \oplus_{B\Gamma} F_2$;

(iii) *the local representative* $f_{\alpha\beta} : \alpha(U) \subseteq E \to \beta(V) \subseteq F$ *has the form* $f_{\alpha\beta} = \eta + \Phi \circ P_2$, *where* $\eta : \alpha(U) \subseteq E \to F_1$ *is* $C^r_{B\Gamma}$ *with* $\eta'(0) = 0$, Φ *is a* $B\Gamma$*-isomorphism of* E_2 *onto* F_2 *and* P_2 *is the second projection* $E = E_1 \oplus_{B\Gamma} E_2 \to E_2$.

Proof. Start with a pair of SC^r_Γ-charts and proceed as in the proof of [2, Theorem (1.7)] with the use of the Inverse Mapping Theorem [12, Theorem (5.2), p. 45].

(3.2) *Let* X, Y *be as in* (3.1) *and* $f : X \to Y$ *be a* SC^r_Γ*-map* $(r \geq 1)$ *having the* $B\Gamma$*-Fredholm property. Then* f *is locally closed.*

Proof. This follows quickly from (3.1).

Let $f : X \to Y$ be a C^r_Γ-map $(r \geq 1)$ between Γ-manifolds of class C^r. We say that $x \in X$ is a *regular point of* f if and only if the Γ-tangent map $T_x f : T_x X \to T_{f(x)} Y$ is surjective; x is a *critical point of* f if and only if it is not regular [2]. If C is the set of critical points of f, then $f(C) \subseteq Y$ is the set of *critical values* of f and $Y - f(C)$ is the set of *regular values* of f.

(3.3) *Let* X, Y *be as in* (3.1) *and* (3.2) *and let* $f : X \to Y$ *be a*

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SC^r_Γ-map having the $B\Gamma$-Fredholm property. Then the set of regular points of f is open in X, hence the set of critical points of f is closed in X.

Proof. This follows from the fact that the set $SL_{B\Gamma}(E, F)$ of $B\Gamma$-splitting surjections $E \to F$ is open in $L_{B\Gamma}(E, F)$ [12].

(3.4) (Γ-version of Smale Density Theorem). *Let F be a Γ-family, $E, F \in \mathrm{F}$ being sequentially complete. Let X, Y be Γ-manifolds of class C^r $(r \geq 1)$ modelled on E, F respectively with X Lindelöf. Let $f : X \to Y$ be a SC^r_Γ-map having the $B\Gamma$-Fredholm property and suppose that, for each $x \in X$, we can find a pair of SC^r_Γ-charts $\{(U, \alpha), (V, \beta)\}$ for f at x such that $r > \max\{0, \operatorname{ind}(f; x, (U, \alpha), (V, \beta))\}$. Then the set of regular values of f is residual in Y.*

Proof. Similar to the proof in [2, p. 43] with the use of (3.1), (3.2) and (3.3).

REMARK. The results in this section still hold if we define the critical points as in [12, p. 61].

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