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(WITH PLATE I AND TITLE-PAGE, TABLE OF CONTENTS,
AND INDEX TO VOL. XXII)

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CAMBRIDGE PHILOSOPHICAL SOCIETY
VOLUME XXII

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